Australasian Transport Research Forum 2023 Proceedings
29 November – 1 December, Perth, Australia
Publication website: http://www.atrf.info

Vulnerability of the Sydney train network using access, graph centrality, and spectral-graph centrality measures

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Abstract

Operational incidents are a significant cause of unreliability on rail transit n etworks. These incidents cause major delays in services, impact passenger travel time, and have knock-on effects that interrupt other public transport services. Consequently, the vulnerability of the rail transit network is a crucial concern for managers and operators. This paper employs network vulnerability analysis to characterise individual critical railway stations in the transit network. The concepts of classic graph theory, spectral graph theory, and person-weighted access are implemented to identify the critical nodes in the Sydney train and metro network, and the results are compared. In the first method, weighted and unweighted centrality measures are computed to find the most critical s tation. In the second approach, critical nodes are identified by scoring all nodes in the network using eigenvectors and their associated eigenvalues. In the last approach, stations are ranked by the reduction of access before and after an incident. Finding of this study may have implications not only for the train operators and managers but also for the transit network planners to enhance the resilience of the public transport network.

1. Introduction

Operational incidents are one of the significant threats to the smooth running of railway systems. Derailments, vehicle faults, power breakdowns, signal and shunting failures, intentional shutdowns, and emergencies disrupt the operation of the network temporarily or permanently. These incidents delay services both on and off-the affected route and consequently delay many passengers. These direct and side-effects are more consequential when the system is operating at its capacity (during peak hours) and vulnerable to disruption due to operational loads. Hence, the vulnerability of the rail transit network is a concern of managers and operators, and identifying critical stations, links, or regions under major incidents is crucial.

There is extensive research assessing network vulnerability and resilience using graph theory concepts, which abstract the physical structure of the network into links and nodes. Many of them study the vulnerability and resilience of road networks (Bell, Kanturska, et al., 2008; Berdica, 2002; Jenelius, Petersen, and Mattsson, 2006; Luping and Dalin, 2012; El-Rashidy and Grant-Muller, 2014; Scott et al., 2006) and some assess the reduction of network access before and after a system failure (Cui and D. Levinson, 2018; Chen et al., 2007; Michael AP Taylor, Sekhar, and D'Este, 2006; Michael AP Taylor and D'Este, 2007). Some research investigates the vulnerability

of public transport networks (Nassir et al., 2016; Cats and Jenelius, 2012; Cats and Jenelius, 2014; Jiang, Lu, and Peng, 2018; Rodriguez-Nunez and Garcia-Palomares, 2014) and measures the lost access by transit.

In terms of transport networks, critical components include bottlenecks and restrictions which are at risk of failure or degradation (Michael A.P. Taylor and Susilawati, 2012) and whose closure produces cascading failures. Different approaches exist to identify critical components in a transit network. One evident approach is to address critical nodes (or links) with or without the network's demand and supply dimension (Bell, Kurauchi, et al., 2017). To identify critical nodes, topological measures have been introduced such as degree, betweenness, and closeness centrality, and node clustering by converting the system into a weighted graph (where nodes are stations and links are the in-between segments with travel cost weights) (Ferber et al., 2007). Other measures such as hub centrality are developed to identify the traffic hubs more comprehensively (Shi et al., 2019).

An other approach is to measure the loss of access loss when certain nodes or links fail (Cui and D. Levinson, 2018) and rank them to find the crucial network elements. Hypothetical network disruption can be simulated by removing stations one at a time and, at each removal, measure the cumulative opportunity access before-and-after the removal. This approach considers not only the operational services but also the demographic layer of land use covered by the transit network. This implies that purely topological indices are not always a sufficient means to analyse the vulnerability of a network, and that other physical characteristics of the system have a crucial role to play too.

Classic graph theory is one of the main methods to assess network vulnerability and resilience. For example, Cats and Jenelius (2012) considers the stochasticity of public transport network vulnerability and extended the betweenness centrality to measure the importance of nodes and links from the perspectives of both operators and passengers. The developed measures are applied in a case study for the Metro network of Stockholm, Sweden. The result indicates that betweenness centrality may not be a good indicator of link importance and real-time information may have both positive and negative influence on disruption impacts.

On the other hand, some studies applied spectral graph theory methods, employing the information contained in the eigenvalues and eigenvectors of a network matrix, in the vulnerability assessment of the transport network. Gutiérrez-Pérez et al. (2013) used spectral graph theory to investigate the relative importance of regions in water supply networks. Two famous ranking algorithms, PageRank, and HITS were used to rank the nodes in a Laplacian graph matrix. Results indicate that clustering is an efficient way to identify critical points in the water supply network. Bell, Kurauchi, et al. (2017) study the vulnerability of the road network by capacity-weighted spectral analysis. They employed the Laplacian matrix of a network in order to find the best (critical) cut to bi-partition the network into sub-networks using the second smallest eigenvalue and corresponding eigenvector to identify potential capacity bottlenecks. Their method considers only the capacity, not the origin-destination demands.

To date, spectral graph theory based methods have not been compared against other network index-based methods, or access-based methods. However, there are overlaps and inherent similarities in the aims and objectives of applying these methods. Specifically, spectral graph theory methods provide 'global' information on the network structure, taking into consideration the entire network structure while ranking a single node. On the other hand, many other network indices, such as centrality measures, provide 'local' information on the graph structure, taking into consideration a single node and its neighbours, for example. Access-based methods take into consideration a whole host of information on dynamic as well as static network structure, still in a 'local' way, but taking

into consideration not only network structure, but travel times, origin-destination properties, and other physical factors. Specifically, to date, no comparison has been made between the reduction of access, when a station collapses, and the corresponding change in station rank in spectral graph theory and access analyses. To bridge this gap, this paper first defines disruption in the network and secondly, compares these methods. This paper also considers both the network structure and operational services to identify the critical links and nodes using spectral graph theory and compares the importance of critical nodes with their system-wide population-weighted access loss when an incident occurs.

2. Methods

The concepts of classic graph theory, spectral graph theory, and person-weighted access are implemented to identify the critical nodes in the Sydney train and metro network. In the first method, weighted centrality measures are computed to find the most critical station. In the second approach, critical nodes are identified by scoring all nodes in the network using eigenvectors and their associated eigenvalues. In the last approach, stations are ranked by the reduction of access before and after an incident. The following subsections define the failure in train network and explain the proposed metrics.

2.1. Defining service-based network, and network failure

The railway network is a complex structure with different components including tracks, crossovers, and signals, and running services adds more complexity to finding the critical locations in the system. For example, regular and express services using the same set of tracks makes some of the stations more important than just taking the physical structure into account. To overcome that, this study uses service-based networks to define an abstracted graph (nodes and links). A service-based network has information about both the physical connections between stations and the scheduled services. Figure 1 illustrates the difference between structure-based networks and service-based networks.

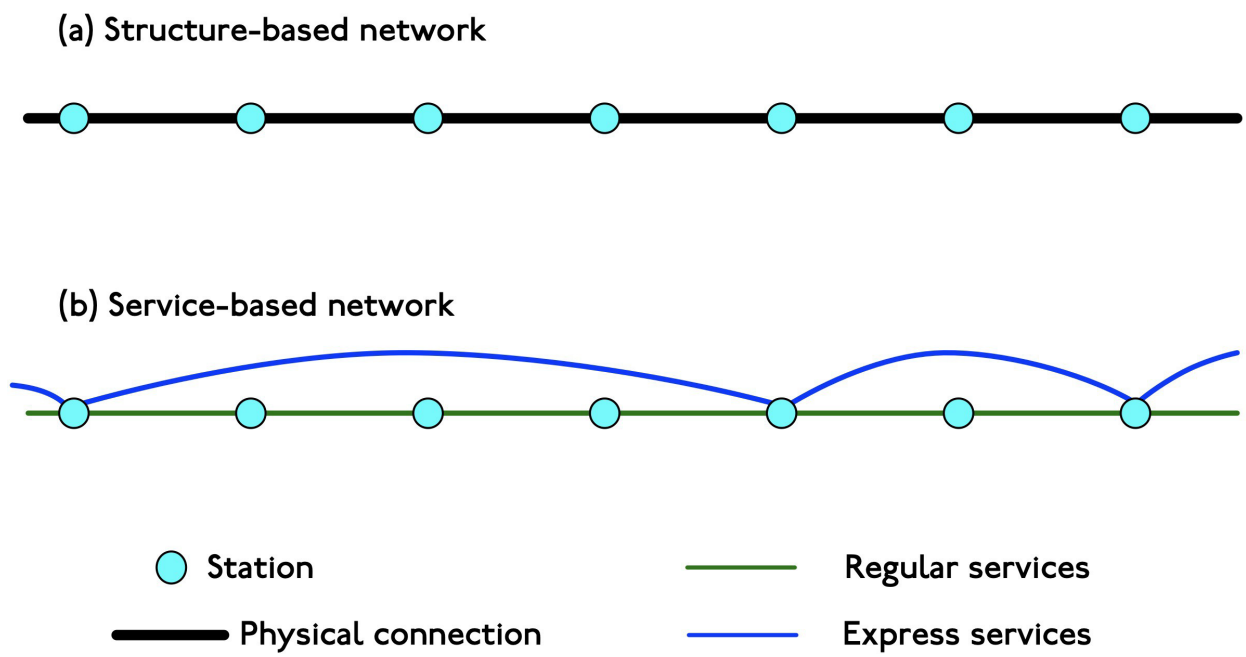
In a railway network, in general, failure can happen in stations (and platforms) and along the connected tracks. To generalize the potential failures, we assume that a station failure means a discontinuity in the services (i.e., no through services) and that no trains can load/unload passengers at that station. In the access analysis, this has a significant impact on travel time between stations and thus between origins and destinations. However, the rest of the network can still operate and affected services can adjust to the new circumstances. This strategy can reduce the effect of cascading failures and mitigate access loss. Figure 2 displays an example of normal operation and adjusted operation before and after a failure in the network.

In the adjusted operation, it is assumed that there are crossovers preceding and following the failed stations. Consequently, vehicles from upstream and downstream can return to their original terminal without completing their scheduled trips. In this study, a 3-minute delay is considered for shunting the vehicle into the opposite track route and initiating service in the opposite direction (1 minute for going forward, 1 minute for switching, and 1 minute for vehicle return). This strategy will be implemented to establish new services for measuring travel time in the access calculation.

2.2. Notation

Variables, parameters, and coefficients that will appear throughout this article are notated in Table 1.

Figure 1: The difference between structure-based and service-based rail network. The service-based network can be used in both classic and spectral graph theory.



2.3. Trip ratio index

The ratio of arrivals to the number of platforms at a station is one of the most straightforward ways to identify the most vulnerable stations in a network. The ratio illustrates the average load on each platform (i.e., how busy a station's platforms are) and how a disruption in one track of rails (each track serves a platform) could affect station arrivals. The greater the ratio, the greater the station's susceptibility to network failure and diverging trips. Equation 1 formulates the trip ratio index.

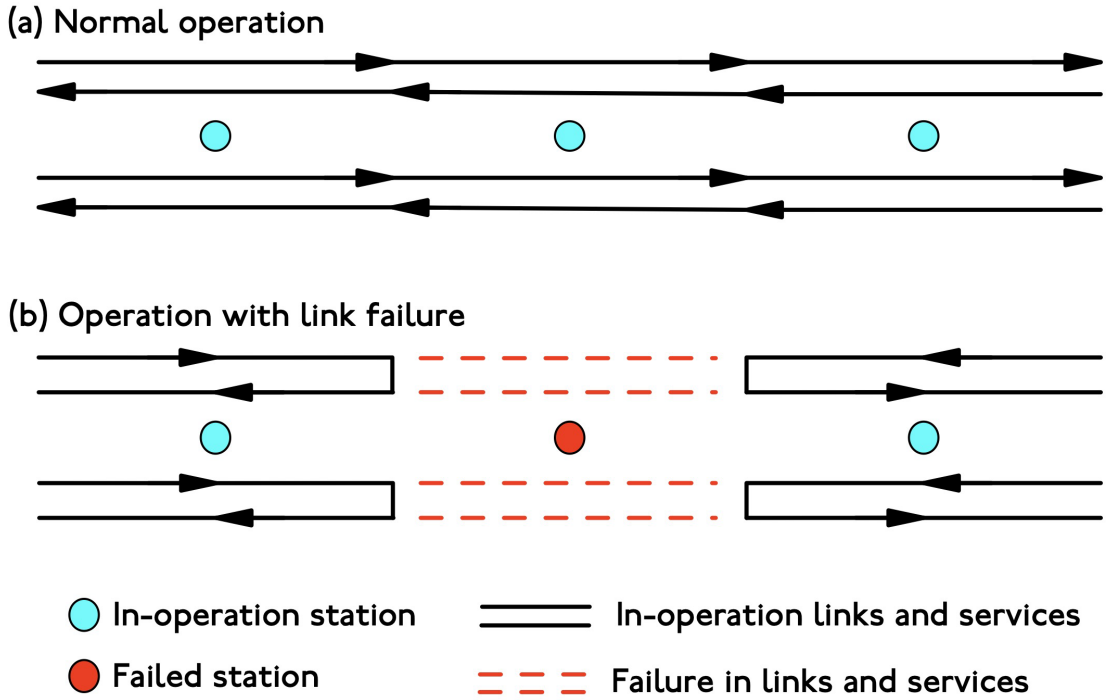
$$a_i = \frac{A_{r,i}}{P_{l,i}} \tag{1}$$

where a_i is the trip ratio of station i; $A_{r,i}$ is the number of daily arrivals into station i; and $P_{l,i}$ is the number of operational platforms in station i.

2.4. Classic graph theory

The service-based network is converted to an undirected graph representation $G(V, E, W)$. The undirected graph demonstrates a bi-directional connection between each pair of nodes which compromises both single track with bi-directional services and double track with one-direction services. In the following, three graph-theoretic centrality measures used for evaluating the network vulnerability analysis are defined. It is important to note that the following measurements pertain to planar graphs (a planar graph contains edges that intersect only at their endpoints).

Figure 2: Comparison between normal and adjusted operation. In the access analysis, the person-weighted access for the adjusted operation will be compared against the normal operation.



2.4.1. Degree Centrality

The degree centrality of a station represents the number of connections with other stations, and thus illustrates the connectivity and the importance of a station in the network. The higher the degree, the more central the station is (Golbeck, 2013; Scheurer and Porta, 2006). Degree centrality (normalized by dividing by the maximum possible degree) can be formally defined as Equation 2.

$$d_i = \frac{\mathrm{Deg}(v_i)}{|V| - 1} \quad \forall i \in V \tag{2}$$

2.4.2. Betweenness Centrality

Betweenness centrality measures how important a station is to the shortest paths through the rail network. It is the fraction of length (or number of links) of those shortest paths that include station i and all the other paths (Cats, 2017). The higher the betweenness, the more important a station is in travelling on the network. The betweenness centrality is written as Equation 3.

$$b_i = \sum_{j \neq k \in V} \frac{n_{j,k}(i)}{n_{i,j}} \quad \forall i \in V \tag{3}$$

Table 1: Notation

Symbol	Description	Unit/Type
a_i	trip ratio index of station i	–
φ	eigenvector	–
λ	eigen value	–
b_i	betweenness centrality of station i	–
c_i	closeness centrality of station i	–
d_i	degree centrality of station i	–
$n_{i,j}$	length of shortest path between node i and node j	links or km
x_i	spectral centrality score of station i	–
A	locational access	ppl
A_r	number of train arrivals	#
A'	adjacency matrix	–
C	generalized travel cost	minutes
D	diagonal matrix of node degrees	–
E	edges (links)	#
G	a graph including links and nodes	–
L	Laplacian matrix	–
P_l	number of platforms	#
P	population	ppl
T	travel time threshold	minutes
V	vertices (nodes)	#
W	graph weights	# trips

2.4.3. Closeness Centrality

Closeness centrality of a station is the average length of the shortest path between the station and all other stations in the graph. Thus the more central a station is, the closer it is to all other nodes (stations). The shortest path length can be measured in number of links or unit of distance. Equation 4 shows the formal definition of closeness centrality.

$$c_i = \sum_{/i=j \in V} \frac{1}{n_{i,j}} \quad \forall i \in V \tag{4}$$

2.5. Spectral graph theory

A transit network with the geographic structure and operational services can be demonstrated as a graph with weight attributes (there may be some levels of abstraction). Thus, the system is a

weighted network $G = (V, E, W)$ where V is set of nodes (transit stations), E is set of links, and W the associated weight values such as distance or the number of arrivals.
The adjacency matrix for network G can be described as $A'_G = (a_{uv})_{N \times N}$ where $N = |V|$. The elements of the adjacency matrix (A'_G) are:

$$a_{uv} = \begin{cases} w_e & \text{if } e = (u, v) \in E \\ 0 & \text{otherwise} \end{cases}$$

where $w_e \in W$, nodes $u, v \in V$ and link $e = (u, v) \in E$. The Laplacian graph is the difference between the diagonal and the adjacency matrix:

$$L = D - A'_G$$

where D is the diagonal matrix of node degrees. In the Laplacian matrix, the off-diagonal elements are the negative of the corresponding off-diagonal elements of the adjacency matrix.
The eigenvalues and the associated eigenvectors (spectra) of the Laplacian matrix can be formulated as Equation 5.

$$L\varphi = \lambda\varphi \tag{5}$$

where, φ is the eigenvector and λ is their associated values. Eigenvectors are mutually orthogonal and unit vectors.

2.5.1. Eigenvector centrality

Eigenvector centrality indicates that a station's (node's) importance depends on both the degree and importance of its neighbouring stations. PageRank and HITS are eigenvector centrality measure derivatives. The relative centrality score of node i can be defined as Equation 6.

$$x_i = \frac{1}{\lambda'} \sum_{j=1}^{N} a_{ij} x_j \tag{6}$$

Where λ' is a constant. The computation of eigenvector centrality is an iterative procedure until a stable value is reached.

2.6. Person-weighted access

Calculating the access is a way to measure the number of opportunities reachable in a specific time threshold. A person-weighted access measure is the number of opportunities (i.e. population in this study) at destinations reachable to the population at each origin. This index allows comparing the system-wide access by transit (D. M. Levinson, Giacomin, and Badsey-Ellis, 2016).
The cumulative opportunities of block i is represented in Equation 7.

$$A_{i,T} = \sum_{j=1}^{J} P_j f(C_{ij}) \tag{7}$$

where P_j is the population of region j, C_{ij} is the generalised travel cost from region i to region j, and $f(C_{ij})$ is the impedance function which:

$$f(C_{ij}) = \begin{cases} 1 & \text{if } C_{ij} \leq T \\ 0 & \text{otherwise} \end{cases}$$

Person-weighted access could be formulated as the Equation 8.

$$A_{pw,T} = \sum_{i=1}^{I} A_{i,T} P_i \tag{8}$$

where A_i is the cumulative opportunities of block i to every other blocks reachable in time T, and P_i is the population within region i.
The advantage of using person-weighted access is considering not only the physical structure of the train network at a large scale but also the covered land use layer.

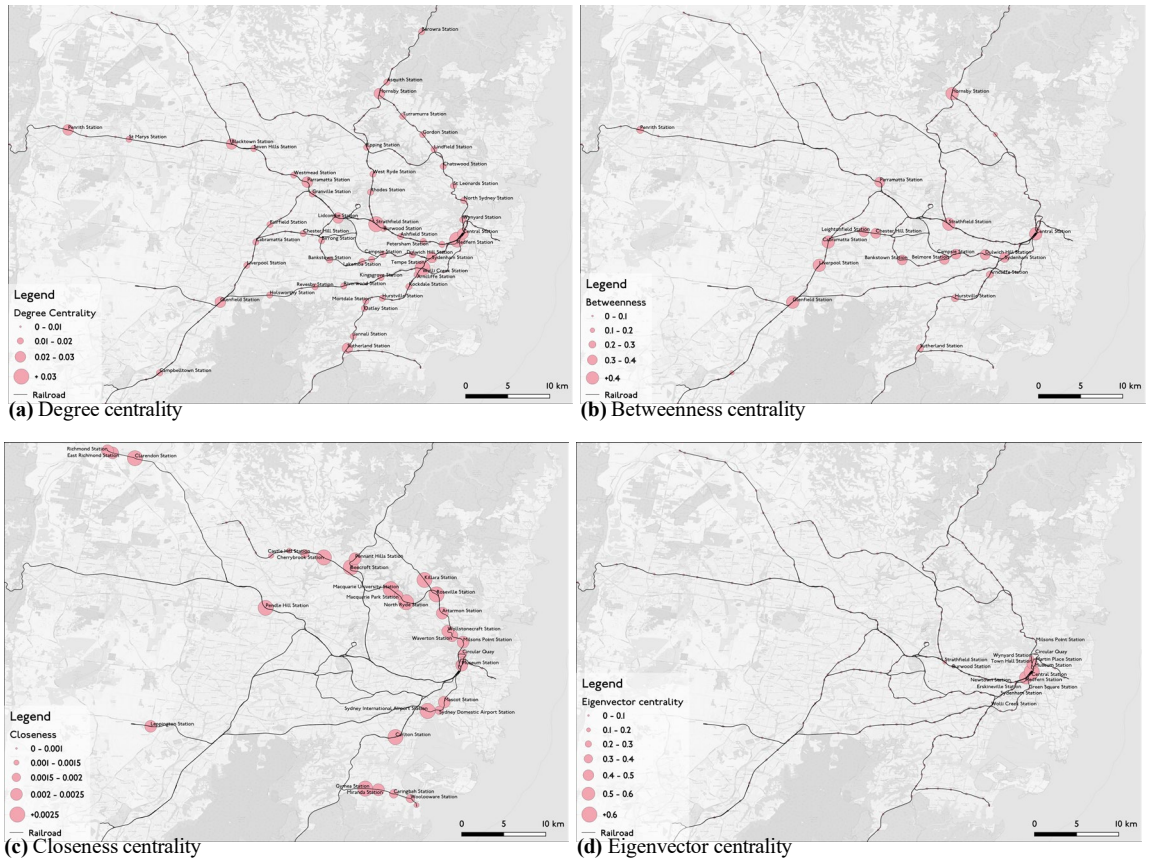
2.7. Summary of measures

This study uses four methods to evaluate and compare the critical nodes in a transit network: (I) service characteristics: the ratio of arrivals per number of platforms; (II) classic graph theory: traditional centrality measures of a planar graph to identify critical nodes; (III) spectral graph theory: eigenvectors and associated eigenvalues of a network's Laplacian matrix to identify critical nodes using spectral graph theory. (IV) access: the person-weighted access measure to rank the stations based on their access loss during a disruption. These measures are calculated for the Sydney train and metro network provided in the standard GTFS format.

3. Results

For calculating the graph centrality measures, the Sydney train has been transformed into a weighted graph. The weights represents the number of trips between each node pair. Then, the importance (rank) and centrality measures are calculated for each station on the network. Figure 3 demonstrates the four centrality indexes of each station, including degree, betweenness, closeness, and eigenvector centrality. The relative measures are illuminated with different shades of red. The analysis shows the stations in the city regions and inner west have high degrees of centrality, while stations with less centrality are farther out. The closeness centrality is higher for stations between transfers, and on the other hand, transfer stations have a higher betweenness than stations serving single lines.
The access analysis considers the normal operation and failure of each node (before and after an incident). The assumption is that the network will adjust the trips in the upstream and downstream of the failure location as outlined in Figure 2. Tevaluates the system failure of a multimodal transit network by comparing 30- and 45-minute transit access times. i.e other modes of transport continue to operate, and passengers can transfer between modes (where applicable) if their desired trips are disrupted. The stations are ranked by the person-weighted access loss (both 30- and 45-minute), and results indicate that Town Hall, Wynyard, Museum, Circular Quay, St James, and Strathfield Stations are the most vulnerable stations. Figure 4 depicts the transit access for two time thresholds during normal operation (before the incident) and failure (after the incident) at Town Hall station. Table 2 ranks Sydney train stations based on seven proposed measures, namely trip ratio, degree centrality, betweenness centrality, closeness centrality, eigenvector centrality, 30-min person weighted access loss, and 45-min person weighted access loss. The ranking in the last column is

Figure 3: Centrality measures of Sydney train network. The network is weighted by the number of arrivals (trips) at each station.



(a) Degree centrality **(b)** Betweenness centrality

(c) Closeness centrality **(d)** Eigenvector centrality

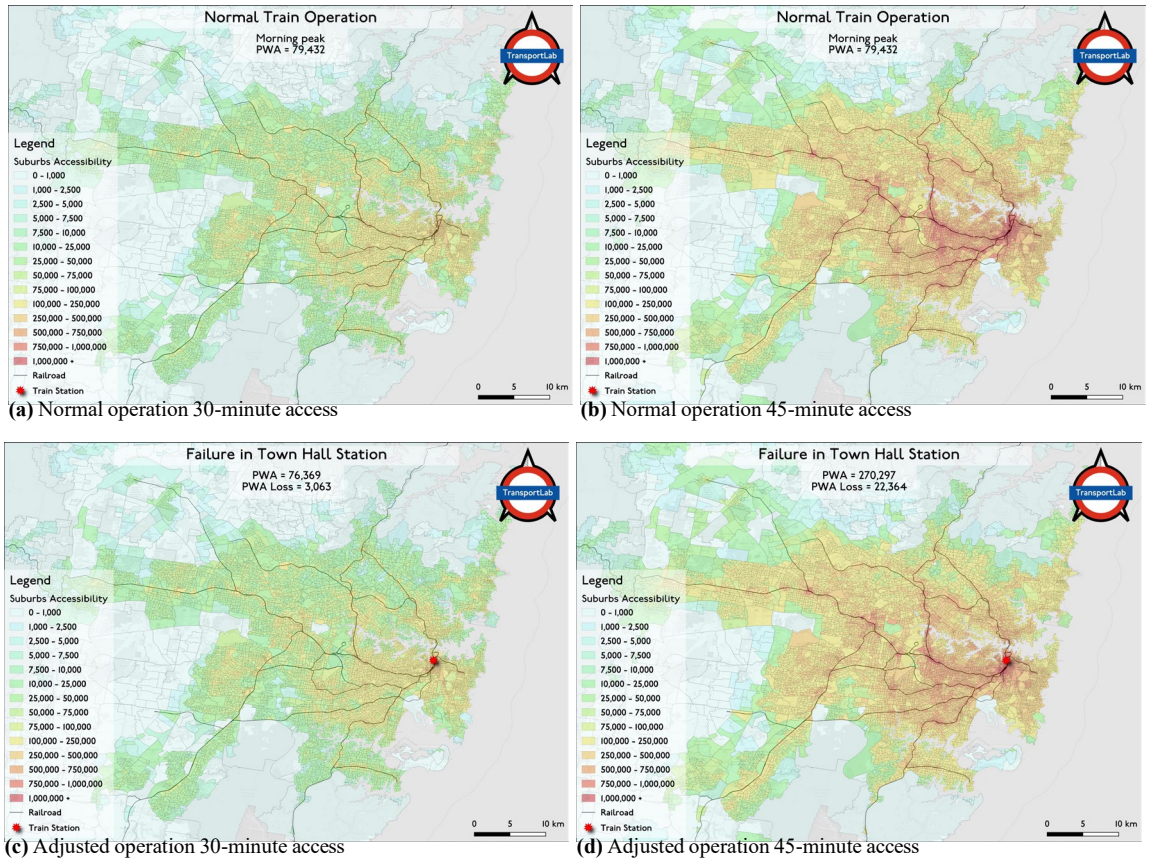
based on the average score of all methods. The rankings vary significantly based on the employed metric. For instance, St James ranks first in terms of trip ratio, whereas Circular Quay ranks first in terms of average score. Central, Redfern, and Strathfield are, based on the average score, the top three stations. These stations have high rankings across most measures, indicating their importance in the transport network.

Table 2: Station ranks with different criteria. Number in the parentheses are the measures value.

Rank	Trip ratio	Degree centrality	Betweenness centrality	Closeness centrality	Eigenvector centrality	30-min PWA loss	45-min PWA loss	Average score
1	St James(264.5)	Wolli Creek(0.035)	Central(0.627)	Central(0.655)	Central(6.39E-3)	Town Hall(3063)	Town Hall(22364)	Strathfield
2	Museum(263.5)	Redfern(0.032)	Glenfield(0.559)	Redfern(0.515)	Glenfield(6.383E-3)	Wynyard(2657)	Wynyard(18245)	Redfern
3	Circular Quay(262)	Strathfield(0.032)	Strathfield(0.495)	Town Hall(0.428)	Liverpool(6.332E-3)	Museum(2441)	Museum(15960)	Central
4	Wynyard(252.3)	Sydenham(0.029)	Hornsby(0.445)	Wynyard(0.179)	Campbelltown(6.32E-3)	Circular Quay(2384)	Circular Quay(15710)	Hornsby
5	Milsons Point(241)	Hornsby(0.027)	Cabramatta(0.409)	Museum(0.17)	Parramatta(6.316E-3)	St James(2383)	St James(15691)	Parramatta
6	Town Hall(238.7)	Parramatta(0.024)	Liverpool(0.408)	Strathfield(0.14)	Penrith(6.289E-3)	Strathfield(1331)	Strathfield(13472)	Wolli Creek
7	St Leonards(232.5)	Central(0.024)	Chester Hill(0.392)	Sydenham(0.091)	Cabramatta(6.281E-3)	Burwood(911)	Redfern(10411)	Town Hall
8	Wollstonecraft(212)	Sutherland(0.024)	Bankstown(0.378)	Green Square(0.087)	Chester Hill(6.263E-3)	Redfern(849)	Wolli Creek(9317)	Wynyard
9	Waverton(212)	Lidcombe(0.021)	Leightonfield(0.375)	Martin Place(0.076)	Fairfield(6.242E-3)	Auburn(810)	Lidcombe(7709)	Museum
10	Artarmon(211.5)	Lidcombe(0.021)	Campsie(0.363)	Burwood(0.073)	Carramar(6.224E-3)	Lidcombe(808)	Burwood(6635)	Sydenham

Degree centrality and betweenness centrality measures provide similar rankings, with Central and Strathfield consistently ranking high. In contrast, closeness centrality provides a different ranking, with stations such as Town Hall and Wynyard ranking high. Eigenvector centrality measures the importance of a station based on its connections to other important stations, and its rankings are

Figure 4: 30- and 45-minute access by transit; normal operation and failure in Town Hall Station. The person-weighted access represents the multi-modal performance.



(a) Normal operation 30-minute access **(b)** Normal operation 45-minute access

(c) Adjusted operation 30-minute access **(d)** Adjusted operation 45-minute access

similar to closeness centrality. The 30-min PWA loss and 45-min PWA loss measures provide information on the impact of a station's closure on the train network. These measures do not correspond to the rankings provided by the other measures and provide unique insights into the transport network's resilience.

4. Conclusion

Operational incidents are one of the significant causes of unreliability on rail transit networks. These incidents cause major delays in services, impact passenger travel time, and have knock-on effects that interrupt other public transport services. Consequently, the vulnerability of the rail transit network is a crucial concern for managers and operators. This paper employs network vulnerability analysis to characterise individual critical railway stations in the transit network. The concepts of classic graph theory, spectral graph theory, and person-weighted access are implemented to identify the critical nodes in the Sydney train and metro network, and the results are compared.

Results from the evaluations highlight the importance of considering multiple measures when analysing and ranking transport networks. Different measures provide different insights into the network's structure and resilience, and a comprehensive analysis requires considering multiple measures simultaneously. It also highlights the importance of stations such as Central, Redfern, and Strathfield, which consistently rank high across most measures, indicating their crucial role in

the Sydney transport network.

The findings of this study may have implications not only for train operators and managers but also for transit network planners seeking to enhance the resilience of public transport networks. Additionally, the study's results could be useful for authorities who design land-use development schemes around rail stations. Moreover, the vulnerability of other rail networks can be assessed, and necessary actions can be taken to minimise potential consequences.

There are several research suggestions that need to be discussed. First, in this study, the graph is weighted by the number of trips on each link. Other weights, such as those based on distance and inverse distance, may offer additional insights into the network's topology and lead to different rankings. Second, this study only considers a daily service-based network. Defining peak and off-peak service-based networks may provide temporal insights into the rankings. Moreover, measuring the Cheeger constant would serve as a valuable reference point for future investigations, particularly when undertaking comparative analyses of analogous studies with transit networks (Bell, Kurauchi, et al., 2017). Additionally, the access analysis was conducted for only one departure time. Conducting the analysis for an average access time of 15 minutes could increase the accuracy of the rankings for stations. Finally, in this study, the PWA ranks were developed based on access to the population. However, measuring access to employment opportunities may enhance the indicator's performance in identifying critical stations.

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