

# Applying the radiation model to long-distance trip patterns in Western Australia

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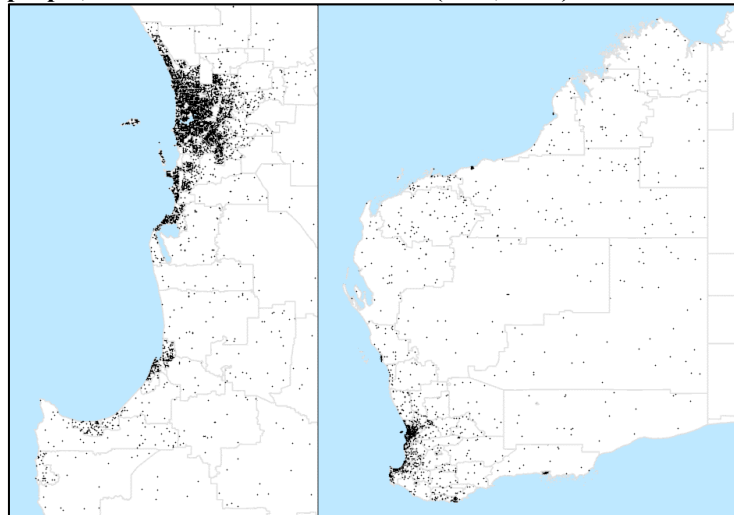
## 1. Introduction

Trends towards urbanisation mean that transport infrastructure projects are mostly located in urban environments. This trend results in transport modelling assumptions biasing urban environments. With less rigour placed on understanding how the models work outside of urban contexts. This paper challenges the assumptions of the gravity model, in particular its relationship to variations in land-use density.

The state of Western Australia provides an extreme case-study due to the large differences in population density (Figure 1). The state contains both the dense urban environment of Perth (approx. 3000 ppl/km<sup>2</sup>) and the sparsely populated outback regions between the Kimberley and Goldfields (>1 ppl/km<sup>2</sup>).

The state road agency (Main Roads WA) maintains a strategic transport model of the entire state, known as the Western Australian State-wide Transport Model (WASTM). This model employs a traditional ‘four-step’ methodology. This includes a trip distribution step which utilises the gravity model. This paper reviews the gravity model, including a critique of its theoretical basis. It then introduces an alternative distribution method, known as the radiation model. It goes on to apply the radiation model to WASTM and report on its findings. In doing so, this paper aims to provide a case-study for the application of the radiation model to strategic transport modelling and highlight the short-comings of the gravity model in non-urban environments.

**Figure 1: Population density of Western Australia and (inset) Perth, Peel and South-West region, each dot represents 250 people, as counted in the 2021 census (ABS, 2021)**



## 2. Literature review

Within the field of transport modelling, the most common technique for distributing trips between origin and destination pairs is the gravity model (Ortúzar & Willumsen, 2011). The gravity model determines the number of trips between two points as a function of the demand located at the origin and destination zones, as well as the proximity of the two locations. In this way locations with larger total demands can be assigned trips that travel greater distances than those zones with smaller total demands.

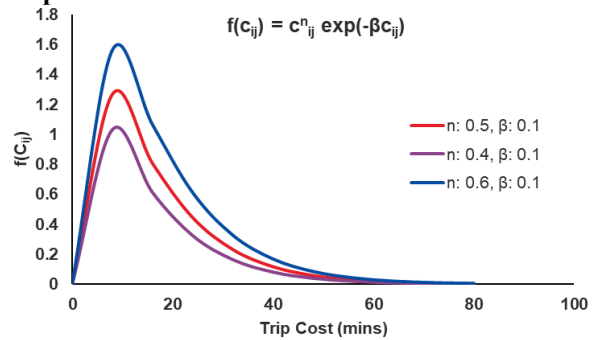
The development of the gravity model has occurred over many decades. Its initial form proposed a simplistic relationship between origin-destination trip distance and the demand volumes. Subsequent development of the gravity model sought to implement a more complex cost function. This adjusted function seeks to weight different travel distances to match variations in observed trip lengths. This form of the gravity model is provided in Equation 1. Wherein  $T_{ij}$  represents the number of trips between an origin ( $i$ ) and destination zone ( $j$ ),  $O_i$  and  $D_j$  represent the total origin and destination trip demands respectively,  $f(C_{ij})$  is a cost function based on the movement between the zone pair, and  $\alpha$  is a scaling factor.

$$T_{ij} = \alpha O_i D_j f(C_{ij}), \quad [1]$$

The cost function is adjusted so that the output trip matrix matches available data of trip length distribution. An example of the cost function is shown within Figure 2. In this example parameters  $n$  and  $\beta$  are used to vary the cost function magnitude to favour trips of approximately 10mins.

When defining the problem of trip distribution within an isolated system, it should be noted that the number trips originating from all zones needs to equal the number of trips that terminate across all zones. To satisfy this requirement, the gravity model requires a secondary processing step in which the matrix is submitted to the ‘furness’ process (or Iterative Proportional Fitting). This adjusts values in the trip matrix until both origin and destination trip totals are equal. In practice multiple iterations are required to adjust the matrix values until an equilibrium is reached between origin and destination totals.

**Figure 2: Examples of variation in cost function response based on changes to the calibration parameters. These parameters determine the strength of the gravity model response at different trip costs**



In critiquing the gravity model, Simini (2012) proposes six analytic inconsistencies, these are:

1. It is not possible to determine the gravity model from a purely theoretical standpoint. Specifically, whilst it is possible to use entropy maximisation theory to generate a simplified gravity model equation, there is no way to determine parameter weights without empirical data.
2. The deterrence function is replaced with a range of context specific functions, suggesting a lack of universality.
3. Empirical evidence requirements for the cost function mean that calibration is a necessity. This requires comprehensive datasets.
4. Once constructed the gravity model has ingrained discrepancies that cannot allow for variations in land-use density across the model area.
5. In its initial form it can increase trips based solely on increases at the origin or destination, leading to imbalances in the matrix.
6. It is a deterministic model, which cannot include variance.

Ortúzar & Willumsen (2011) also note that the requirement to calibrate against observed trip length distributions carries forward bias into forecast scenarios. This is despite a lack of evidence that the observed trip length distributions will be maintained by future populations. Both Ortúzar & Willumsen (2011) and Simini (2012) are observing that trip length

distributions are not a fixed parameter, but a response to the available travel choices which are themselves linked to underlying land-uses.

An alternative is proposed by Simini (2012) which resolves their proposed analytic inconsistencies of the gravity model. Known as the radiation model, it apportions trips from an origin zone  $T_i$  to a given origin-destination pair based on; the ratio between the origin population  $m_i$ , destination population  $n_j$ , and the sum of population within a radius about the origin. This radius is equal to the distance between  $i$  and  $j$ . This population sum is noted as  $s_{ij}$ . The relationship between these parameters is shown in Equation 2. The model is based on the radiation and absorption rates of wavelengths that are randomly discharged from a given source. Within the transport context the magnitude of the radiation and absorption rates are set by the magnitude of the population in each location.

$$\langle T_{ij} \rangle = T_i \frac{m_i n_j}{(m_i + s_{ij})(n_j + s_{ij})}, \quad [2]$$

The radiation model provides a variance-based output centred on the average number of trips between a given origin and destination pair.

The theoretical basis for the radiation model is similar to the intervening opportunities model in that they both link movement to land-use density. This contrasts with the gravity model which is concerned only with the cost of moving between two points. The radiation model and intervening opportunities model consider the alternative destinations between the two locations. However, the radiation model implementation moves away from a deterministic framework which requires calibration. Instead, its implementation is based on the spatial and demographic information only.

Recent application of the radiation model by McCulloch et al (2021) found it can capture a wide range of trip patterns when compared against other distribution models. They also found it tended to over-estimate long-distance trip patterns. When implemented within an urban context Piovani et al (2018) found that the radiation model needed a significant amount of amendment, with authors introducing a calibration factor to match observed trip length distributions.

Whilst the gravity model maintains significant incumbent advantage over other methods of trip distribution it's longevity should not preclude it from critique. The WASTM model presents a unique opportunity to challenge the gravity model as many of the assumptions which underpin it do not apply. Specifically, the extreme variance in land-use density, and the resulting variance in trip length distribution, mean that a gravity model would be difficult to implement. However, the literature suggests that the radiation model not be applicable to transport modelling applications without modification to allow for some level of calibration.

This study implements the radiation model within WASTM to understand the extent to which the radiation model can resolve trip distribution issues for models with large variances in land-use density. It presents a case-study that explores the impact on the wider model methodology and the model outputs.

### 3. Methodology

The key input into the radiation model is population and employment land-use data. This information is provided by transport model zone within the existing WASTM input data. This model methodology utilises a simplified set of trip purposes to generate the total demand. These are: Commuting trips (as per the original formulation of the radiation model) and

business trips. To determine commuting trip population zone totals were used for the origin demand, and employment totals for the destination. Business trips were assigned employment zone totals for both origin and destination demand. The radiation model requires a trip generation factor to convert the distribution values to trips volumes. These trip generation rates are shown in **Table 1**. The rate for business trips has been determined as an average of the business trip rates used in the original WASTM model. The commuting rate is determined from the ABS census Journey to Work data.

**Table 1: Trip rates used within the radiation model application.**

| Trip type | Trip rate                    |
|-----------|------------------------------|
| Commuting | 0.32 (trips/population)      |
| Business  | 0.88 (trip/total employment) |

The original WASTM methodology utilised two parallel gravity models calibrated for long-distance and short-distance trips. For this case study, the radiation model has been applied in place of both the long and short trip distribution models. This has been done as the structure of the radiation model requires intervening destinations to determine the distribution.

The WASTM structure has been maintained for the mode choice and highway assignment component of the model. It should be noted that the model determines mode choice through fixed proportions as opposed to a logit choice model. For the highway assignment module additional vehicle trips are maintained as per the original WASTM framework for the external zones, airport trips and heavy good vehicles. Only car trips previously determined by the gravity model have been replaced with those generated by the radiation model.

Evaluation of the model outputs utilises existing validation datasets and reporting tools included within WASTM. Additional data has been extracted from open datasets on traffic volumes maintained by Main Roads WA's TrafficMap data-portal.

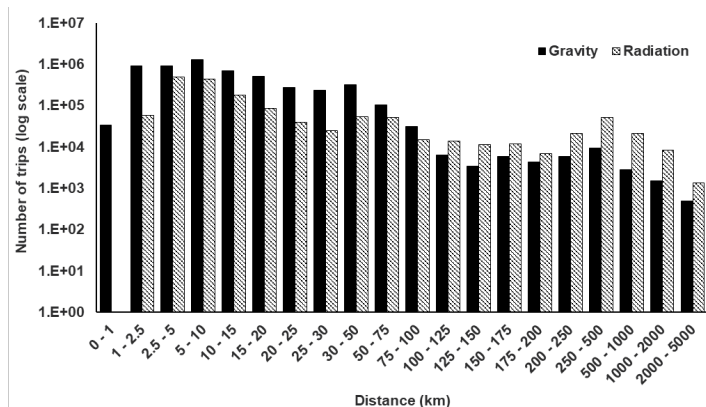
## 4. Results

**Table 2: Matrix totals for car trips produced by each distribution method.**

|                         | Gravity model    | Radiation model  |
|-------------------------|------------------|------------------|
| Total Trips             | 5,358,248        | 1,592,440        |
| Intrazonal Trips        | 1,862,417        | -                |
| <b>Interzonal Trips</b> | <b>3,495,831</b> | <b>1,592,440</b> |

Initially, it seemed reasonable to maintain the total number of car trips produced by the gravity and radiation models respectively. However, large differences in the distribution pattern became apparent, and this criterion was dropped. Table 2 highlights the total number of car trips used within each method. It can be observed that the radiation model is based on significantly less trips than the gravity model.

**Figure 3: The trip length distribution of car trips produced by the gravity and radiation models.**



When considering the impact on the highway network it is important to note that intrazonal trips (where trips remain inside a single zone) do not enter the model network. These trips have been removed from the totals to provide the interzonal trip for comparison. The production of intrazonal trips is a major differentiator between the two distribution methods as the radiation model does not calculate intrazonal trips.

The total number of trips does not provide a comprehensive picture of vehicle movement. This is because of the variation in distance between different zone pairs. Looking at the trip length distribution produced by the two models, Figure 4, it can be seen that the radiation model favours long distance trips. This finding matches observation made by Simini and McCulloch.

**Finding 1, As observed by others the Radiation model is biased towards long distance trips.**

The need to undertake this adjustment of the input trips highlights the interrelated nature of each of the steps within the four-step model, in particular trip generation and trip distribution. The radiation model by its simplicity highlights this direct relationship as well as more subtle relationships caused by demand classification processes related to zone structure, trip purpose segmentation and the apportionment of trip rates.

**Finding 2, The distribution of trips and resulting demands is highly dependent on the application of demand classification related to the zone system, trip purposes and trip rates.**

To understand the impact on the highway demand we must observe the total vehicle distance travelled (VDT), this is the distance travelled between each zone pair multiplied by the number of vehicles undertaking the trip. This result is shown in Table 3. The lack of vehicle trips within the radiation model matrix is compensated by its bias towards long distance trips to produce a total VDT difference of approximately 2%.

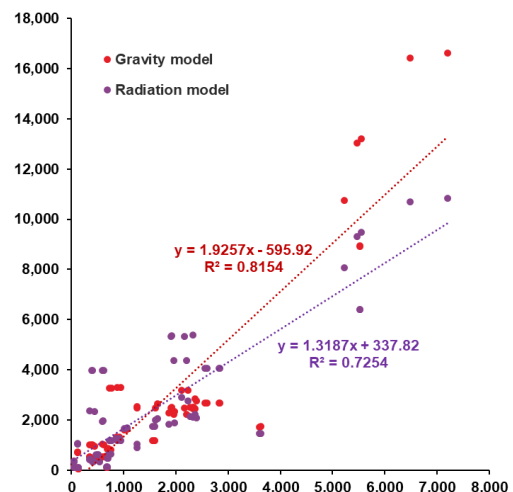
**Table 3: Difference in Vehicle Distance Travelled (VDT) in km for car trips between the gravity and radiation models**

|                 | Gravity model     | Radiation model   |
|-----------------|-------------------|-------------------|
| <b>VDT (km)</b> | <b>77,805,413</b> | <b>75,904,432</b> |

When comparing the assigned trip volumes to counts (Figure 5) it can be seen that the radiation model produces a relationship with gradient 1.31, as opposed to 1.93 produced by the gravity model. This indicates that the order of magnitude of the response is closer to observed. However, there is greater variation in the radiation model response, with a  $R^2$  value of 0.72 compared to 0.82 for the gravity model.

The final demand comparison looks at the differences between the outputs of the two distribution methods through a plot of the differences in car vehicle volumes has been produced, Figure 6. From this plot it is clear that the radiation model is producing more

**Figure 4: Correlation of highway count volumes for cars vs the radiation model and gravity model assigned volumes.**



long-distance trips than the gravity method and less short distance trips within the urban setting of Perth.

A final observation on the radiation model implementation is the impact on model run-times. The gravity model and required matrix furnishing step can be a time-consuming process. The need to iterate the matrix to balance the trip end totals is dependent on convergence criteria and therefore difficult to predict processing time. The fixed nature of the radiation model results in quicker and less variable run times. The radiation model processing time was found to be approximately 1 min. This compares to approximately 5 mins for the gravity model implementation.

**Finding 3, the radiation model has faster, and more consistent compute times compared to the gravity model.**

## 5. Discussion

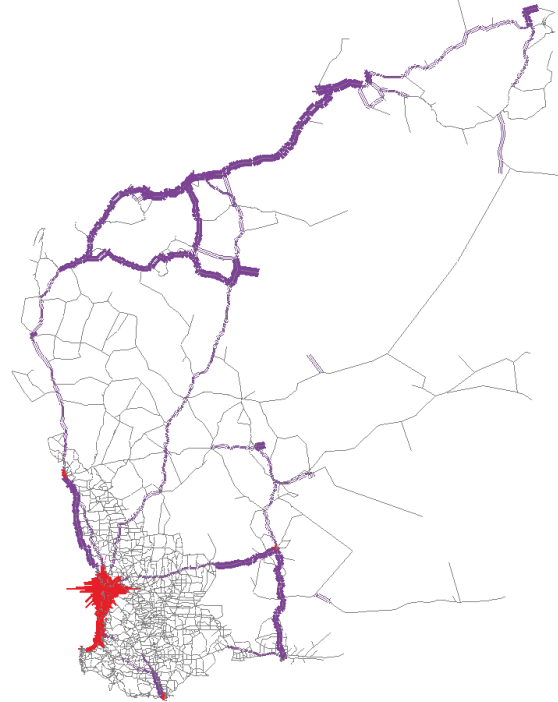
The process of implementing the radiation model into an existing modelling framework has highlighted the limitations of the four-step methodology, specifically in relation to classification. This classification problem, whereby initial steps create classifications of demand through the zone system, trip purpose and in the case of WASTM trip length, impacts the structure of subsequent model processes. Each subsequent step must make adjustments through calibration parameters to account for changes in the data structure. However, this can hide inconsistencies (or bias) in the underlying structure of the model.

The radiation model challenges this approach by providing a framework based on the underlying land-use information only. The modeller is then forced to consider the composition of demand classification when constructing the model. For example, how does the zone sizing impact the generation and distribution of trips, specifically in relation to intra-zonal versus interzonal trips.

The final observation is that implementing the radiation model can have a dramatic improvement on model build and run times. The fast run times and straight forward linkage between the underlying demographic information mean that debugging and revision is significantly less complex than gravity model applications. Saving model development time, model run times and model debugging time.

In conclusion, this research has shown that the radiation model can be retrospectively applied to an existing four-step model. Its implementation has highlighted its strength at capturing long-distance trips. However, further work is recommended to investigate the radiation model response to changes in zone topographies and alternative demographic inputs. This would help determine calibration strategies for the radiation model, including the scaling of different classification parameters. Observing the impact of zone structure and demand inputs should be explored further to understand the impact on its trip distribution pattern.

**Figure 5: A comparison of highway volumes produced by gravity model (red) and the radiation model (purple) distributions.**



## References

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