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Demand and state estimation for perimeter control in large-scale urban networks

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1. Background and Aims

Cities are becoming increasingly congested, requiring the development of efficient control methods for management of large-scale traffic networks. Real-time control of traffic networks by perimeter flow control strategies are gaining popularity as they focus on developing network level (neighbourhood level) control measures. Partitioning a large scale heterogeneous network into several homogeneous regions (i.e. areas with compact shape and low variances in link densities) and optimal control of transfer flows between regions perimeters (boundaries) to minimize the congestion is the conceptual focus of perimeter flow control strategies as seen in (Kouvelas et al., 2017). Traffic signal times along the region perimeters are adjusted based on the perimeter control decisions such that region-level control decisions are transferred to intersection-level (or link level) control mechanisms (Kouvelas et al., 2017). Macroscopic fundamental diagram (MFD) (Geroliminis and Daganzo, 2008) which establish the relationship between network vehicle accumulations (network density) and network production (network flow) is the main modelling tool in perimeter control algorithms. MFD-based traffic models can track the non-linear behaviour in traffic state transitions from uncongested to congested conditions in urban networks (at the region-level) and derive transfer flows by adhering to the conservation of vehicle flows within the multi-region networks and boundary capacities (Mariotte et al., 2020). Feedback based control and rolling horizon approach taken in existing perimeter control algorithms allows to derive traffic responsive control measures by referring to current traffic states of the network and expected future demand inflows and vehicle outflows. Several solution algorithms are proposed in literature for perimeter control such as feed-back based controllers (Keyvan-Ekbatani et al., 2021) and, model predictive controllers (MPC) (Sirmatel and Geroliminis, 2018).

Many existing perimeter control methods assume full network observability and perfect demand information, which are rarely observed in real-world traffic networks. A sophisticated communication architecture that combines data form multiple sources like loop-detectors, probe vehicles, vision based identifications etc., is required to gather input variables for existing perimeter control methods such as observations on accumulation states in regions, existing route choices and current (or future) demand inflows. These are not necessarily available in practice due to measurement/estimation errors and difficulty in obtaining such detailed information in real-time. Sirmatel et al. (2021) suggest real-time estimation of traffic states to overcome the observability issue faced in feedback based perimeter control algorithms. Sirmatel and Geroliminis (2020) propose a nonlinear moving horizon estimator (MHE) to estimate accumulation states of the network prior to implementing an MPC algorithm. While these approaches are promising in estimating accumulation states, they assume adequate amount of probe vehicles are available in the network to understand the real-time route choice decisions and demand variations.

It is also essential to explore the variations in network-level demand flows when applying perimeter control algorithms, which is largely unexplored. Many studies in perimeter control either disregard demand flow variations or presume that they are distributed with white noise (with zero mean and finite variance) (Sirmatel and Geroliminis, 2020). Nevertheless, literature has revealed the importance of real-time (online) demand estimation for traffic control applications since the early works of Ashok (1993). Myriad of studies investigate many aspects of online demand estimation that are not limited to utilisation of various data sources (Carrese et al., 2017), observability (Yang et al., 2018), and effective solution algorithms (Bierlaire and Crittin, 2004) etc. Anyhow, most existing demand estimation methods face scalability issues when applied to large-scale networks and maintaining consistency of the estimation results in congested traffic conditions (Kumarage et al., 2021), which leads to sub-optimal results in control strategies. Considering these limitations in existing studies, we propose a novel framework for perimeter control with an MHE that can capture changes in route choice and demand flow variations in a multi-region network. In this study we propose; 1) a novel approach to define regional route choice in MFD based traffic models by incorporating historical information on route choice and 2) propose an economic MPC control strategy combined with a MHE that can capture both changes in route choice and regional demand variations. Preliminary results indicates the suitability of proposed control strategy being implemented with limited information in real-time.

2. Problem Definition and Formulation

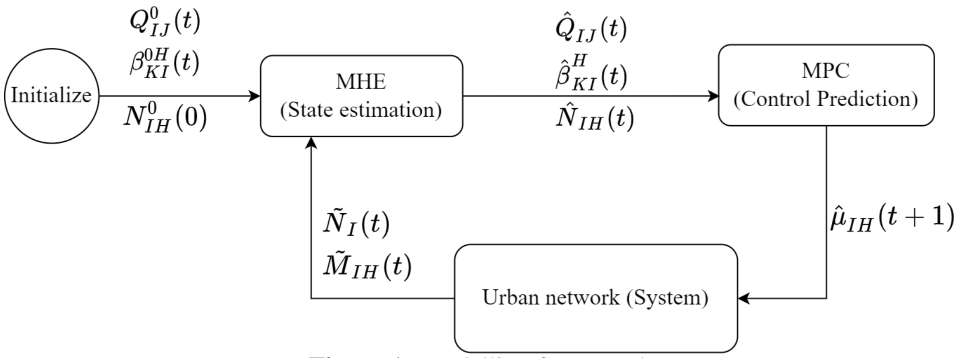

Figure 1: Modelling framework

We propose a feed-back based control method based on the state-space model of the urban network (system) in this study. Figure 1 presents the main components of this framework which assembles moving horizon estimator for state estimation and a model predictive controller to predict control decisions to be applied in the urban network. Contrasting from existing studies that require granular level real-time observations, our frameworks depend only on the region accumulations ($N_I(t)$) and boundary flows ($M_{IH}(t)$) observed in real-time. We incorporate historical route choice information and a *priori* demand flows (OD flows) as input variables together with observations to estimate the a *posteriori* OD flows, route choices and estimated accumulation states ($N_{IH}(t)$). Then the estimated states (outputs) are entered into the MPC to derive perimeter control actions for the next time step. The control decisions given by MPC are implemented in the urban network. This process is run iteratively where the feed-back observations will influence the control decisions in the next time step. the urban network is modelled using the multi-region trip based MFD model that account for boundary queues as introduced in Li et al. (2021). We utilize the accumulation based MFD model with improvements to definitions of route choice as explained in Section 2.1 to define traffic dynamics in MHE and MPC which are elaborated in Section 2.2 and Section 2.3 respectively.

2.1. Multi-region MFD dynamics

The urban network of the proposed framework is modelled using the multi-region trip based model that account for boundary flows similar to Li et al. (2021). The trip-based model captures the navigation of each vehicle in the network according to the region-level traffic conditions given by MFD. While

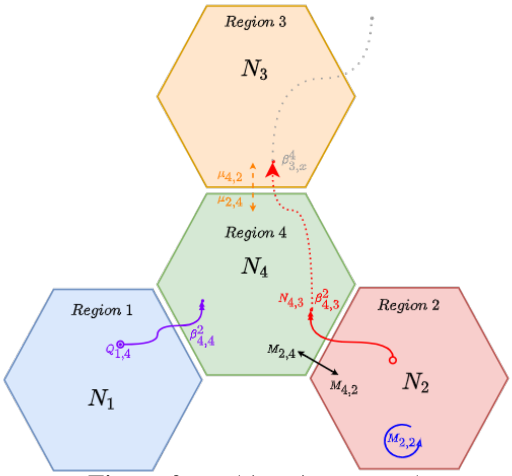

Figure 2: Multi-region network

trip-based MFD model is complex enough to capture the regional traffic behavior it is too detailed to be include in state estimation or control prediction. Hence, we use the multi-region accumulation based MFD model for control and estimation purposes. An urban network (R) could be partitioned into several regions ($R = \{1, 2, \cdots, I, H, J \cdots\}$) as shown in Figure 2. Let $N_I(t)$ be the total number of vehicles (accumulation) currently in region I, and $N_{I,H}(t)$ be accumulation currently in region I with next region H such that $N_I = \sum_{H \in G_I} N_{I,H}$, where G_I is the set of neighbouring regions to region I. Let $Q_{I,J}(t)$ be the demand inflow from region I to destination region J in current time step t. Then $Q_{I,H}(t)$ will be the demand inflow from region I to region H which will be given by $Q_{I,H} = \sum_{J \in R, J \neq I} \alpha_{J,H} \times Q_{I,J}$, where $\alpha_{J,H}$ is the ratio of trips with destination region J that with next region H (choice of next region H at the beginning of the trip). Let $M_{I,H}(t)$ be the transfer flow from region I to neighbouring region H and $M_{I,I}(t)$ be the exit flow in the region I. Then, the flow conservation equations for an R-region MFD network will be given by:

$$\dot{N}_{I,H}(t) = Q_{I,H}(t) + \sum_{K \in G_I} \beta^H_{K,I}(t) \times M_{K,I}(t) - \sum_{H \in G_I} M_{I,H}(t) \tag{1a}$$

$$\dot{N}_{I,I}(t) = Q_{I,I}(t) + \sum_{K \in G_I} \beta^I_{K,I}(t) \times M_{K,I}(t) - M_{I,I}(t) \tag{1b}$$

where, $\beta^H_{K,I}$ gives the ratio of trips that is headed for H (next region) among the trips that cross the boundary between K (previous region) and I (current region). This parameter allows us to capture regional route choice in modelling the state-space framework. Further details on this model with calculation of transfer flows using MFD will be elaborated in a full version of the paper.

2.2. Moving horizon estimator

The MHE used in state estimation could be formulated as follows;

$$\underset{Q,\beta}{\text{minimize}} \quad \sum_{t=t_k-n_e:t_k} \left\{ M^{\text{MFD}}(t) - M^{\text{OBS}}(t) \right\}^2 \tag{2a}$$

$$\text{subject to} \quad \text{for} \quad t = t_k - n_e \cdots \Delta_t \cdots t_k \quad ,$$

$$\left[N^{\text{MFD}}(t+1), M^{\text{MFD}}(t+1) \right] = f\left\{ N^{\text{MFD}}(t), Q, \beta, \mu(t) \right\}, \tag{2b}$$

$$N^0_I(t) \cdot \alpha_{lb} \leq N_I(t) \leq N^0_I(t) \cdot \alpha_{ub} \quad \forall I \in R, \tag{2c}$$

$$Q^0_{I,H} \cdot \lambda_{lb} \leq Q_{I,H} \leq Q^0_{I,H} \cdot \lambda_{ub} \quad \forall H \in G_I \quad \forall I \in R, \tag{2d}$$

$$\beta_{K,I}^{0,H} \cdot \gamma_{lb} \leq \beta_{K,I}^{H} \leq \beta_{K,I}^{0,H} \cdot \gamma_{ub} \quad \forall K,H \in G_I \quad \forall I \in R, \tag{2e}$$

$$\sum_{\forall H} \beta_{K,I}^{H} = 1 \quad \forall K,H \in G_I \quad \forall I \in R \tag{2f}$$

Eq 2a presents the objective function used in MHE which minimize the squared error between observed transfer flows ($M^{\text{OBS}}(t)$) and transfer flows estimated by the MFD model ($M^{\text{MFD}}(t)$) by taking demand inflow matrix (Q) and vector of route choice ratios (β) as decision variables. We look at observations from previous n_e time steps and optimize Q and β variables in a quasi-dynamic style where we assume the Q and β were constant throughout the estimation time span ($t_k - n_e : t_k$). However, the estimated parameters are valid for a single time-step as Q and β are estimated in the next time step again. Eq 2b defines the dynamics of the multi-region accumulation-based model, which gives the accumulation vector ($N^{\text{MFD}}(t+1)$) and transfer flow matrix ($M^{\text{MFD}}(t+1)$) for next time step ($t+1$) by function $f(\cdot)$ that consider the current accumulations ($N^{\text{MFD}}(t)$, Q, β and applied control variables ($\mu(t)$). Eq 2c defines the constraints for estimated region accumulation values to be within a range from observed region accumulations ($N_I^0(t)$) defined by α_{lb}, α_{ub}. Next, Eq 2d defines the boundary for decision variable Q such that all estimated demand flows ($Q_{I,H}$) to be closer to *a priori* demand flows ($Q_{I,H}^0$) defined by boundary $\lambda_{lb}, \lambda_{ub}$. Eq 2e set limits for the next decision variable β based on *a priori* route choice parameters. Note that route choice parameter is a ratio such that $\beta_{K,I}^{H} \in [0,1] \forall H,K,I$. The *a priori* route choice ratios could be obtained from historical sources including past GPS trajectories, Bluetooth detector data and vision based vehicle identification methods. Therefore, limits of γ_{lb} and γ_{ub} are dependent on the available data sources and sample size (penetration). Next, Eq 2f set the sum of route choice ratios to neighbouring regions to one. This constraint ensures all the vehicles that enter to region I from region K either exit in region I or transfer to next region H.

2.3. Model predictive controller

$$\underset{\mu}{\text{minimize}} \quad \sum_{t=t_k:n_p} \Delta_t \cdot \left\{ \sum_{I \in R} N_I^{\text{MFD}}(t) \right\} \tag{3a}$$

$$\text{subject to} \quad \text{for} \quad t = t_k \cdots \Delta t \cdots t_k + n_c \quad ,$$

$$N^{\text{MFD}}(t+1) = f\left\{ N^{\text{MFD}}(t), \hat{Q}, \hat{\beta}, \mu^0(t) \right\}, \tag{3b}$$

$$0 \leq \mu_{\min} \leq \mu_{I,H} \leq \mu_{\max} \leq 1 \quad \forall H \in G_I \quad \forall I \in R, \tag{3c}$$

$$0 \leq N_I(t) \leq N_I^{\text{crit}} \quad \forall I \in R \tag{3d}$$

Here, we present the MPC controller used to derive the control actions based on the estimated results of MHE. Eq 3a is the objective function which minimize the total time spent (TTS) in the the network. Here, μ is the decision variable which gives the vector of control variables ($\mu_{I,H}$) applicable to each region boundary. The MPC controller scans for n_c time steps forward (forward horizon) in deriving control actions, however assumes that the estimated outputs (Q and β) from MHE are kept constant for the forward horizon. This assumption related to the qasi-dynamic nature of traffic states highlighted in Cascetta et al. (2013) and helps to reduce the number of decision variables in the optimization. Although MHE and MPC assume quasi-dynamic behaviour, the control outputs and estimations are updated at each time step when the state space model is run. As a result, the quasi dynamic assumption contributes to the simplification of the optimisation problem without affecting the dynamic control architecture. Eq 3b defines the dynamics of the multi-region accumulation-based model used in MPC. Similar to MHE, the function $f(\cdot)$ gives the accumulation vector ($N^{\text{MFD}}(t+1)$) for next time step ($t+1$) by taking the current accumulations ($N^{\text{MFD}}(t)$, estimated demand ($\hat{Q}$), estimate route choice parameters $\hat{\beta}$ and *a priori* control variables ($\mu(t)$) as inputs. Eq 3c defines the boundaries for the decision variable based on the control requirements. μ. Eq 3d ensure region accumulations are non-negative and below the jam accumulation level.

3. Preliminary results

We test the proposed model with several numerical simulation scenarios. We set up a network of four regions as shown in Figure 2 as the test bed where each region is modelled with an MFD. The trip based MFD model that account for each vehicle movement and transfer at boundaries is used to model the test bed. The test bed is modelled with average trip length of 5040m, maximum boundary capacity of 2 veh/s, simulation time of 210 minutes and jam accumulation (N_I^{jam}) level of 10000veh. The real-time estimation and control strategy with MHE+MPC is implemented on the test bed. A 15% underestimated demand scenario is used in MHE+MPC where the *a priori* demand matrix is obtained by perturbing the ground-truth demand matrix. We assume that we can capture past route choice decisions of 15% of the total vehicles either by detectors or GPS trajectories such that *a priori* values for $\beta_{K,I}^{0,H}(t)$ could be obtained. We have set parameters $[\mu_{lb}, \mu_{ub}] = [0.95, 1.05], [\lambda_{lb}, \lambda_{ub}] = [0.80, 1.20]$ and $[\gamma_{lb}, \gamma_{ub}] = [0.80, 1.20]$ for the MHE in the scenario. The boundary of control parameter is set as $\mu_{\min}, \mu_{\max}] = [0.3, 1]$ for the MPC.

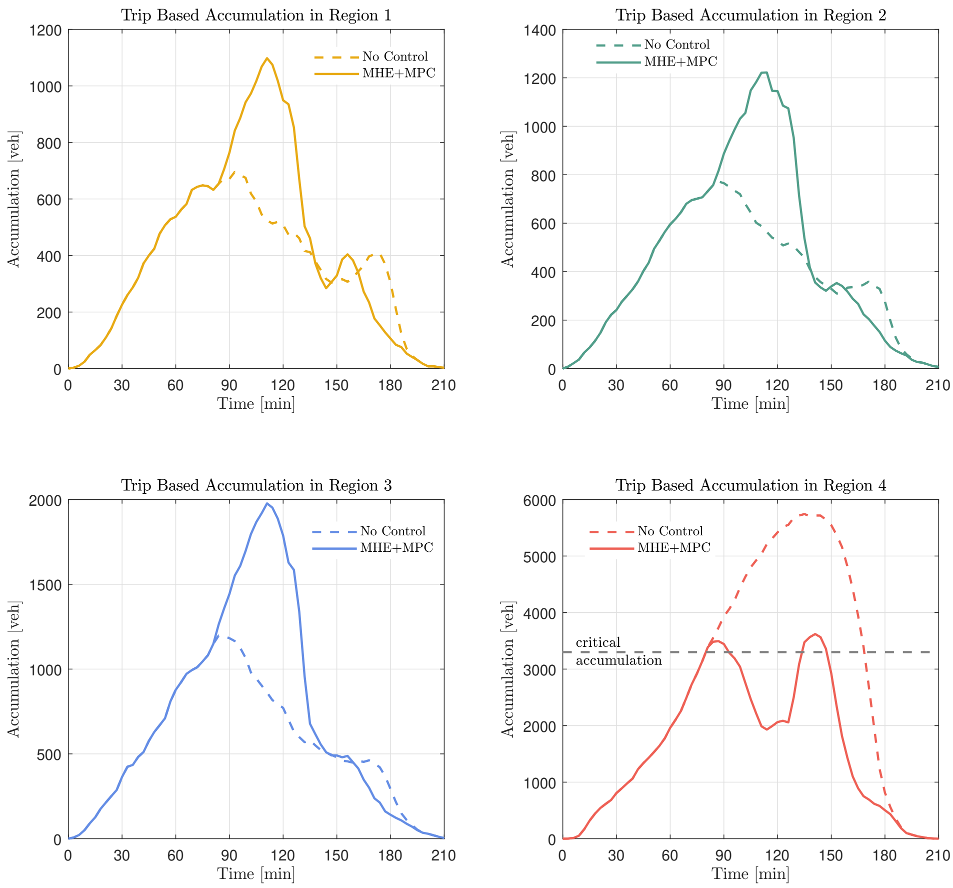

Figure 3: Preliminary results

Figure 3 presents preliminary results for the real-time estimation and control strategy presented in this study. The accumulation profiles of each region are presented in the four graphs respectively. The observed accumulation profile in the no control scenario is shown in dashed lines. It indicates that the accumulation region-4 (the center region) goes beyond the critical accumulation levels in no-control conditions creating congested traffic conditions. However, the other three regions (surrounding regions) demonstrate lower accumulations levels indicating less congested traffic conditions. The implementation of MHE+MPC framework has resulted in favourable traffic conditions in all regions as shown by the accumulation profiles given by solid lines. The MHE+MPC controller has reduced the accumulation in region-4 below the critical accumulation level by controlling the transfer flows from the boundary. It should be emphasised that the perimeter control strategies optimize the transfer flows to minimize the total time spent in the network. Hence, vehicle accumulations in surrounding regions are increased to protect the critical region. As a result, the accumulation levels in boundary regions (region 1, 2 and 3) have risen than the levels observed in the no control scenario but they have not reached critical accumulation levels. The implementation of the MHE+MPC has resulted 20% improvement in the total time spent in the network creating desirable traffic conditions in the whole network. The MHE was able to adjust the underestimated demand matrices and capture route choice changes efficiently. We will further conduct experiments on several scenarios involving demand variations, comparison of control methods, and report the results in a future publication with further elaborations on theory.

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