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Two phase Monte Carlo simulation approach for multimodal network routing problem

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Abstract

Multimodal transportation is being increasingly adopted by a large number of freight companies. Multimodal transportation refers to using combinations of various transportation modes to move commodities from origin to destination. In this paper, a two phase Monte Carlo simulation approach is applied to find the best route(s) in a multimodal network for given origin and destination. The Monte Carlo model developed in this paper integrates total costs, including transfer costs from one mode of transportation to another, duration of routes and the availability of each mode for each leg to generate the most preferred routes. The results of the Monte Carlo simulations are then analysed to extract the pareto optimal front solutions to offer various routes having respective advantages in terms of duration and/or costs. The proposed approach is then applied to a simple situation to demonstrate its simplicity, versatility and practicality.

1 Introduction

Freight companies, both global and local, have started employing multimodal transportation to optimise the movements of their goods from an origin to a destination. Multimodal transportation is defined as the combined and collective use of different modes of transportation (rail, road, water and air) to move commodities to a destination. Compared to the ordinary single mode transportation, multimodal transportation potentially could improve efficiency, costs, safety and flexibility for the transportation industry.

One of the operational problems that needs to be solved in the multimodal setting is the freight routing problem. That is, to find and/or select the best route(s) while using the best available mode to move commodities from their origins to destinations through the transportation network.

This paper tackles the multimodal freight routing problem using a two phase Monte Carlo simulation (MCS) approach. The Monte Carlo model developed in this paper integrates total costs, which incorporates transfer costs from one mode of transportation to another, duration of trip and the availability of each mode for each leg to generate the most preferred routes from origin to destination. Subsequent to getting the MCS results, these are analysed to extract the pareto optimal front solutions to offer various routes having respective advantages in terms of duration and/or costs. In the presence of several pareto optimal front solutions, a known approach is tested to extract the best solution out of the ones provided.

The results of the Monte Carlo simulations can be used by freight companies to optimise their costs and/or efficiency, and also to present alternatives to clients so that they can make informed decisions on which option best suits their needs.

The simplicity, versatility and practicality of the Monte Carlo simulation approach is illustrated by applying to a simple situation.

2 Literature review

Determining the optimal means of transporting goods from shippers to receivers is a common problem for logistics service providers. Whilst trucks are commonly used, other transport modes such as rail, air and sea can have advantages with respect to financial costs, safety, fuel consumption and emissions, but may have longer travel times and lower levels of reliability.

Multimodal transport involves at least two modes being used to transport goods (Steadieseifi *et al.*, 2014). This creates the need to develop routes that require consideration of a range of possible transport and terminal options. This can be challenging due to the number of options available.

Typically, only one objective such as minimising financial cost is considered when only one mode, such as truck transportation, is available. However, other objectives such as minimising travel time are often considered when there are other modes available. Different transport modes will generally have different financial costs as well as different travel times between terminals (Sun and Lang, 2015).

Multi-objective optimisation methods utilising mathematical programming and network analysis can be used to determine feasible and optimal solutions for multi-modal transport problems. Feasible solutions consist of routes comprising a path of transport modes that can be used to carry goods between terminals linking the shipper and the receiver.

Multi-modal transport problems can be formulated and solved by combining the objectives within a single objective function, wherein each objective is weighted according to preferences. It is often difficult to determine the relative weighting of each objective and only a single solution is provided, where often a set of solutions is desirable in practice.

Numerous procedures based on genetic algorithms have been developed for identifying pareto optimal solutions, including the Vector Evaluated Genetic Algorithm (Schaffer, 1985), the Non-dominated Sorting Genetic Algorithm (Srinivas and Deb, 1994), the NSGA-II (Deb *et al.*, 2002), the Normalized Normal Constraint Method (Messac, Ismail-Yahaya and Mattson, 2003) and the bi-level multi-objective Taguchi genetic algorithm (Xiong and Wang, 2014).

This paper introduces an alternative approach that used Monte-Carlo simulations to identify pareto optimal solutions for multi-modal transport networks. To the best of authors' knowledge, there is no previously published work applying the proposed approach for multimodal network routing problem utilizing the extracted Pareto optimal solutions from the initial results of MCS and into the next set of iterations.

3 Pareto optimality

The origin of the term Pareto optimality goes back to the year 1906 applied in the area of economics and later on found its application in the field of Mathematics especially multi-objective optimisation (Arora, 2017). Multi-objective optimization problems (MOPs) are a branch of mathematical optimisation which involves having to optimize more than one objective function simultaneously. Usually MOPs don't have a single optimal solution that optimises each objective function in which case Pareto optimal solutions may be used to represent the solution set. Pareto optimal solutions are solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives. The set of Pareto optimal outcomes is often called the Pareto front or Pareto boundary. Solutions which do not lay on the Pareto front are called Pareto dominated solutions. See Fig. 1.

Let $X \subset \mathbb{R}^n$ be a non-empty set of feasible solutions and $F = [F_1\ F_2, \dots, F_p]: \mathbb{R}^n \dashrightarrow \mathbb{R}^p$ be a of objective functions. Feasible solution, $\hat{x} \in X$ is called a Pareto optimal solution of the

MOP: $\min_{x \in X} F(x) = [F_1(x), F_2(x), \dots, F_p(x)]$,

if and only if there does not exist any $x \in X$ such that $F(x) \leq F(\hat{x})$.

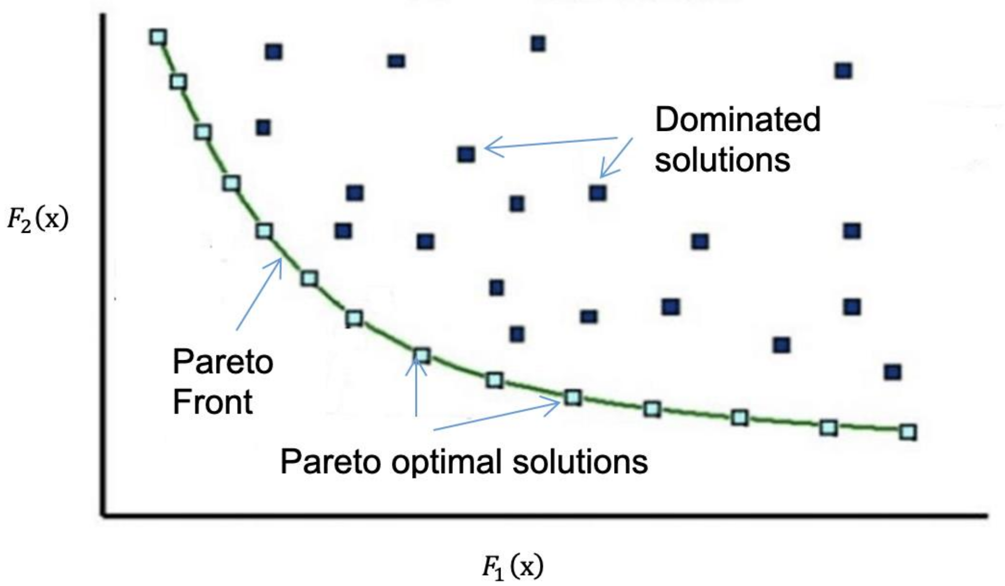

Fig 1: Pareto Front

4 Monte Carlo simulation

Monte Carlo simulation is a computerized mathematical technique that approximates solutions to quantitative problems through statistical sampling. This technique is used by professionals in fields of finance, project management, energy, manufacturing, engineering, research and development, insurance, oil & gas, transportation, and the environment, to approximate solutions in sectors including project cost estimation, project schedule estimations, risk assessments, benefit cost analysis and selecting risk response strategies, see for example (Prakash and Jokhan, 2017; Prakash and Jokhan, 2016; Prakash, 2018; Prakash and Mitchell, 2015) to name a few.

This method is useful for obtaining numerical solutions to problems which are too complicated to solve analytically. Monte Carlo simulation furnishes the decision-maker with a range of possible outcomes and the probabilities of the possible outcomes. Also the reason for its wide usage is its applicability and also for the simplicity in which one can construct models as compared to certain optimisation models, which would require expert knowledge.

The technique was first used by scientists working on the atom bomb (Kochanski, 2005).

Monte Carlo simulation involves building models of possible results by substituting all of the input values having inherent uncertainties, with probability distributions. It then calculates results repeatedly, each time using a different set of random values from the probability distributions.

The results of Monte Carlo simulation are not single values but distributions of possible outcome values (Vose, 2008).

Generally, the following steps are involved in performing a Monte Carlo simulation:

- Step 1: Create one (or more) parametric Model(s), $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots, F_{\boldsymbol{p}}(\mathbf{x})]$**,** where p is the number of objective functions and, input $\mathbf{x} = (x_1 x_2, \ldots, x_m)$, where m is the number of possible inputs.
- Step 2: Represent the inputs $(x_1 x_2, \ldots, x_m)$ using probability distributions.
- Step 3: Generate a set of random inputs$(x_{k1}, x_{k2}, \ldots, x_{km})$ from the distributions for each iteration k, $k = 1\ to\ t$, where t is the total number of iterations.
- Step 4: Evaluate the model using the random inputs, $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots, F_{\boldsymbol{p}}(\mathbf{x})]$ for each iteration, k.
- Step 5: Analyse the results of $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots, F_{\boldsymbol{p}}(\mathbf{x})]$, obtained for all the iterations, $k = 1\ to\ t$.

5 Model formulation

The multimodal freight route selection problem involves choosing a combination of various routes from a selected origin to a particular destination, taking into consideration the effects of implementing these combinations such as, at the very least, travel costs, time taken, availability of routes and available modal options for each available route.

Let $G = (V, E, M)$ denote the multimodal transportation network with the set of V vertices, a set of E edges and a set of M transportation modes. Let each edge, $e \in E$, connecting two vertices u and v, be denoted by (u, v) and the associated weight of the edge be denoted by $\delta(u, v)$.

Let $\emptyset(u^i, v^j)$be the transfer cost at vertex v from mode i to j, $i, j \in \{1{:}\,road, 2{:}\,rail, 3{:}\,water\}$, where i is the mode arriving at u and j is the mode arriving at v and hence the mode of transportation of (u, v). Note if $i = j$, then $\emptyset(u^i, v^j) = 0$, i.e. there is no cost if no mode transfer occurred. Additionally, $\emptyset(u^i, v^j) = 0$ if u is the starting vertex.

<u>Definitions:</u>

- u is adjacent to (or is a neighbour of) v, if $(u, v) \in E$.
- The set of all neighbours of u is the neighbourhood of u and is denoted $N(u)$.
- A path is defined as an ordered set of vertices $(v_1, \ldots \ldots, v_t), t > 1$, such that $(v_h, v_{h+1}) \in E$ for $h = 1, \ldots, t - 1$, and the vertices are not repeated.

To formulate the Monte Carlo simulation model, given the origin, A_O and the destination, A_D towns, the task is to generate possible intermediate towns, from origin, until the destination is obtained using the available modal options between each town. That is:

<u>Phase1 of MCS</u>

Step 1: Construct parametric model: $\mathbf{F}(\mathbf{x}) = \left[F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots, F_{\boldsymbol{p}}(\mathbf{x})\right]$, where:

- $\left[F_{\mathbf{1}}\ F_{\mathbf{2}}, \ldots, F_{\boldsymbol{p}}\right]$ are various applicable objective functions such as criteria, payoff functions, cost functions, time and value functions;

- $\mathbf{x} = (A_O, A_2{}^m, A_3{}^m, \ldots\ldots, A_D{}^m)$ are the towns to be traversed to reach destination, A_D from the origin, A_O ; and $m \in \{1\text{:}road, 2\text{:}rail, 3\text{:}water\}$ denoting the mode of transportation used to arrive at the vertex.

Step 2: For each iteration ($k = 1\ to\ t$), starting with the town of origin, A_O, randomly select the next town to be traversed from range of neighboring towns ($A_e, \ldots, A_f$) represented using probability distribution. These input distributions can be derived from the adjacency matrix for the network. After the next town is selected, also randomly select a mode from the list of available modes of transportation to this town from the preceding town. If this selected town is the desired destination town, A_D then stop, otherwise select the next town visited from respective range of neighboring towns represented by appropriate probability distribution until the desired destination town, A_D is reached. If the next town does not exist, that is, the current town has no neighbor, then stop and restart next iteration. The result of this step would generate, $\boldsymbol{x^k} = (A_O, A_2{}^m, \ldots\ldots, A_D{}^m)^{\boldsymbol{k}}$, for each iteration k.

Step 3: Evaluate the model, $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots.., F_{\boldsymbol{p}}(\mathbf{x})]$, using the random inputs of $\mathbf{x}$**,** generated in step 2, for each iteration, k.

Step 4: Analyse the results of $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x}), \ldots.., F_{\boldsymbol{p}}(\mathbf{x})]$, obtained for all the iterations, $k = 1\ to\ t$. We will analyze the results using the Pareto optimal front approach.

Phase 2 of MCS

Step 5: Extract the Pareto optimal front solutions of traversed towns denoted by $\mathbf{OF}(\dot{\boldsymbol{x}}) = [\dot{\boldsymbol{x}}_{\mathbf{1}}, \dot{\boldsymbol{x}}_{\mathbf{2}}, \ldots.., \dot{\boldsymbol{x}}_{\boldsymbol{q}}]$, for $g = 1$ to q unique solutions and $\dot{\boldsymbol{x}}_{\boldsymbol{g}} = (A_O, \ldots\ldots, A_D)^g$.

Step 6: For each unique extracted solution of $\mathbf{OF}(\dot{\boldsymbol{x}})$, regenerate random modes, $m \in \{1\text{:}road, 2\text{:}rail, 3\text{:}water\}$ of traversal to obtain $\ddot{\mathbf{x}}^{\boldsymbol{l}} = (A_O, A_2{}^m, \ldots\ldots, A_D)^{\boldsymbol{l}}$ for each iteration $l = 1\ to\ r$.

Step 7: Analyse the refined results of $\mathbf{F}(\ddot{\boldsymbol{x}}) = [F_{\mathbf{1}}(\ddot{\boldsymbol{x}}), F_{\mathbf{2}}(\ddot{\boldsymbol{x}}), \ldots.., F_{\boldsymbol{p}}(\ddot{\boldsymbol{x}})]$, obtained for all the iterations, $l = 1\ to\ r$, using the Pareto optimal front approach.

The pseudocode for the two phase MCS simulation is provided below:

```
Phase 1 MCS
import network G
construct adjacency matrix, distance matrix and mode matrix
origin town_O
destination town_D
total_iterations_phase1
total_iterations_phase2
src = town_O
path = src
mode = []
MCS_phase1_cost = []; MCS_phase1_time = []; MCS_phase2_cost = []; MCS_phase2_time = []
for (counter =1 to total_iterations_phase1)
        while (src is not town_D)
                next_town  = randomly choose a neighbour of src using adjacency matrix
                while loop created
                        next_town = randomly choose another neighbour of src
                        if next_town = null
                                break;
                next_mode = randomly choose a mode from SRC to next_town using mode matrix
                path = array ( path + next_town)
                mode = array( mode + next_mode)
                src = next_town
        MCS_phase1_cost = array (MCS_phase1_cost + F1(path, mode))
        MCS_phase1_time = array (MCS_phase1_time + F2 (path, mode))
Pareto_optimal_front1 = extract Pareto optimal front solutions ( MCS_phase1_cost, MCS_phase1_time)
Phase 2 MCS
for each solution PO1 in Pareto_optimal_front1
        for (counter =1 to total_iterations_phase2)
                mode = randomly select a mode for each connecting towns in PO1
        MCS_phase2_cost = array (MCS_phase2_cost + F1(PO1, mode))
        MCS_phase2_time = array (MCS_phase2_time + F2 (PO1, mode))
Pareto_optimal_front2 = extract Pareto optimal front solutions (MCS_phase2_cost, MCS_phase2_time)
Display Pareto_optimal_front2 solutions
Finish
```

6 Applied model

In this section, an example adapted from (Xiong and Wang, 2014) is presented to demonstrate the use of the two phase Monte Carlo simulation approach for best freight route selection in a multimodal network.

In this example there are 35 vertices and 136 edges as shown in Fig. 1. There are at most 3 possible modes of transportation which corresponds to A (road), B (rail) and C (water) between any vertices. Table 1 provides the edge weights which are transportation distances between vertices for various modes if available. A dash (-) denotes that there is no connection between the associated vertices for the respective mode. Table 2 provides the speed and the costs per unit load for each transportation mode while Tables 3 and 4 give the transfer costs per unit load and time per unit load for each change of mode.

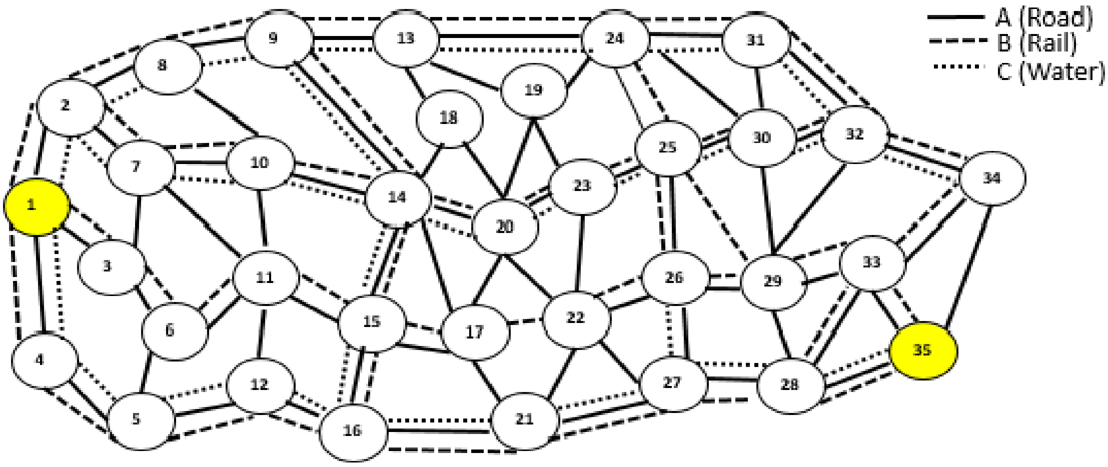

Fig.2 Example multimodal transport network (adapted from (Xiong and Wang, 2014))

Table 1: Vertex distances with respect to available modes

	Modes				Modes		
Edge	**A**	**B**	**C**	**Edge**	**A**	**B**	**C**
(1,2)	2	3	8	(18,20)	1	–	–
(1,3)	10	16	–	(19,20)	4	–	–
(1,4)	3	5	11	(19,23)	3	–	–
(2,7)	3	5	9	(19,24)	9	–	–
(2,8)	9	15	29	(20,22)	3	–	–
(3,6)	7	12	–	(20,23)	8	13	24
(3,7)	1	–	–	(21,22)	10	–	–
(4,5)	2	3	8	(21,27)	10	16	32
(5,6)	9	–	–	(22,23)	8	–	–
(5,12)	2	4	7	(22,26)	4	6	–
(6,11)	3	5	–	(22,27)	5	–	–

(7,10)	7	11	21	(23,25)	2	5	8
(7,11)	3	–	–	(24,25)	6	11	–
(8,9)	5	9	16	(24,30)	3	–	–
(8,10)	1	–	–	(24,31)	10	15	31
(9,13)	10	16	31	(25,26)	7	12	–
(9,14)	6	10	19	(25,29)	–	6	–
(10,11)	4	–	–	(25,30)	7	12	22
(10,14)	5	9	16	(26,27)	3	–	9
(11,12)	3	–	–	(26,29)	4	7	–
(11,15)	4	8	–	(27,28)	9	15	27
(12,16)	2	4	8	(28,29)	7	–	–
(13,18)	6	–	–	(28,33)	2	3	–
(13,19)	8	–	–	(28,35)	8	14	26
(13,24)	5	8	16	(29,30)	6	–	–
(14,15)	6	11	19	(29,32)	1	–	–
(14,17)	2	–	–	(29,33)	2	4	–
(14,18)	4	–	–	(30,31)	6	–	–
(14,20)	8	12	25	(30,32)	6	10	–
(15,16)	7	11	22	(31,32)	4	6	13
(15,17)	5	8	–	(32,34)	6	9	19
(16,21)	6	10	20	(33,34)	5	8	–
(17,20)	8	–	–	(33,35)	8	13	–
(17,21)	1	–	–	(34,35)	6	–	–
(17,22)	–	9	–				

Table 2: Speed (distance per unit time) and Costs (cost per unit load) with respect to transportation modes

	Road	Rail	Water
Speed	4.5	3	1
Costs	6	3	1

Table 3: Transfer cost rates (per unit load) when changing transportation modes

	Road	Rail	Water
Road	0	4	7
Rail	4	0	7
Water	7	7	0

Table 4: Transfer time rate (per unit load) when changing transportation modes

	Road	Rail	Water
Road	0	0.0067	0.0113
Rail	0.0067	0	0.0113
Water	0.0113	0.0113	0

The task at hand is to find the best route(s) from town 1 to town 35 carrying $\mu = 20$ units, utilising available modes such that the objective functions $\mathbf{F}(\mathbf{x}) = [F_{\mathbf{1}}(\mathbf{x}), F_{\mathbf{2}}(\mathbf{x})]$, are minimized:

Cost function: $F_{\mathbf{1}}(\mathbf{x}) = \mu * \sum_{h=1}^{t-1}[\delta\left({A_h}^i, {A_{h+1}}^j\right) * \alpha^j + \emptyset({A_h}^i, {A_{h+1}}^j)]$;

and

Time function: $F_{\mathbf{2}}(\mathbf{x}) = \sum_{h=1}^{t-1}[\delta\left({A_h}^i, {A_{h+1}}^j\right) / \vartheta^j + \mu * \theta({A_h}^i, {A_{h+1}}^j)]$;

where:

- $\mathbf{x} = (A_O, {A_2}^m, \ldots\ldots, {A_D}^m)$ are the towns to be traversed to reach destination, $A_D = 35$ from the origin, $A_O = 1$; and $m \in \{1{:}road, 2{:}rail, 3{:}water\}$

- t >0 is number of traversed town and $A_1 = 1$ and $A_t = 35$;

- $i, j = \{1{:}road, 2{:}rail, 3{:}water\}$ are the modes of transportation used arriving at respective vertices;

- $\delta\left(u^i, v^j\right)$ is the distance of two towns u and v traversed using mode j;

- $$\emptyset\left(u^i, v^j\right) = \begin{cases} 0; & i = j \\ 4; & i = 1\ and\ j = 2\ or\ i = 2\ and\ j = 1 \\ 7; & i = 1\ and\ j = 3\ or\ i = 2\ and\ j = 3 \end{cases}$$ (Transfer rate of cost as provided in Table 3)

- $$\theta\left(u^i, v^j\right) = \begin{cases} 0; & i = j \\ 0.0067; & i = 1\ and\ j = 2\ or\ i = 2\ and\ j = 1; \\ 0.0113; & i = 1\ and\ j = 3\ or\ i = 2\ and\ j = 3 \end{cases}$$ (Transfer rate of time as provided in Table 4)

- $$\vartheta^j = \begin{cases} 4.5; & j = 1 \\ 3; & j = 2 \\ 1; & j = 3 \end{cases}$$ (as provided in Table 2).

- $$\alpha^j = \begin{cases} 6; & j = 1 \\ 3; & j = 2 \\ 1; & j = 3 \end{cases}$$ (as provided in Table 2).

7 Results and Discussion

The Pareto optimal front solution routes obtained after phase 1 of MCS, as described in section 4, were:

- 1-4-5-12-16-21-27-28-35
- 1-4-5-12-16-21-17-22-26-29-33-35
- 1-2-7-11-15-17-22-26-29-33-35
- 1-4-5-12-16-21-22-27-28-35

These routes were then used to run phase 2 of MCS, as described in section 4, where now only the modes are randomly chosen for these fixed vertices.

The Pareto optimal front solution of phase 2 MCS is provided in Table 5 and shown in Fig 3.

Table 5: Pareto optimal front solution after using the obtained results of phase 1 MCS with phase 2 MCS

#	Route	Modes	Cost	Time
1	1-4-5-12-16-21-27-28-35	C-C-C-C-C-C-C-C	2780	139.00
2		B-C-C-C-C-C-C-C	3000	129.89
3		B-B-C-C-C-C-C-C	3020	122.89
4		B-B-B-C-C-C-C-C	3120	117.23
5		B-B-B-B-C-C-C-C	3200	110.56
6		B-B-A-A-C-C-C-C	3280	108.92
7		A-A-A-A-C-C-C-C	3320	107.23
8		B-B-B-B-B-C-C-C	3400	93.89
9		A-B-B-B-B-C-C-C	3540	93.03
10		B-B-A-A-B-C-C-C	3560	92.38
11		B-B-A-A-A-C-C-C	3600	90.25
12		A-A-A-A-A-C-C-C	3640	88.56
13		B-B-B-B-B-B-C-C	3720	67.23
14		A-B-B-B-B-B-C-C	3860	66.36
15		B-B-A-A-B-B-C-C	3880	65.72
16		A-A-A-A-B-B-C-C	3920	64.03
17		B-B-B-B-B-B-B-C	4080	45.23
18		A-A-A-A-A-A-A-A	5040	9.33
19	1-4-5-12-16-21-17-22-26-29-33-35	B-C-C-C-C-A-B-B-B-B-B	3980	58.47
20		B-B-C-C-C-A-B-B-B-B-B	4000	51.47
21		C-C-C-C-B-A-B-B-B-B-B	4040	51.05
22		C-C-C-B-B-A-B-B-B-B-B	4120	44.38
23		C-C-C-A-A-A-B-B-B-B-B	4160	41.36
24		B-B-B-B-B-A-B-B-B-B-B	4180	22.16
25		B-B-A-A-A-A-B-B-B-B-B	4300	18.38
26		A-A-A-A-A-A-B-B-B-B-B	4340	16.69
27		A-A-A-A-A-A-B-B-B-A-B	4500	16.07
28		A-A-A-A-A-A-B-B-A-A-B	4560	14.62
29		A-A-A-A-A-A-B-B-B-A-A	4600	13.38
30		A-A-A-A-A-A-B-B-A-A-A	4660	11.93
31		A-A-A-A-A-A-B-A-A-A-A	4780	10.82

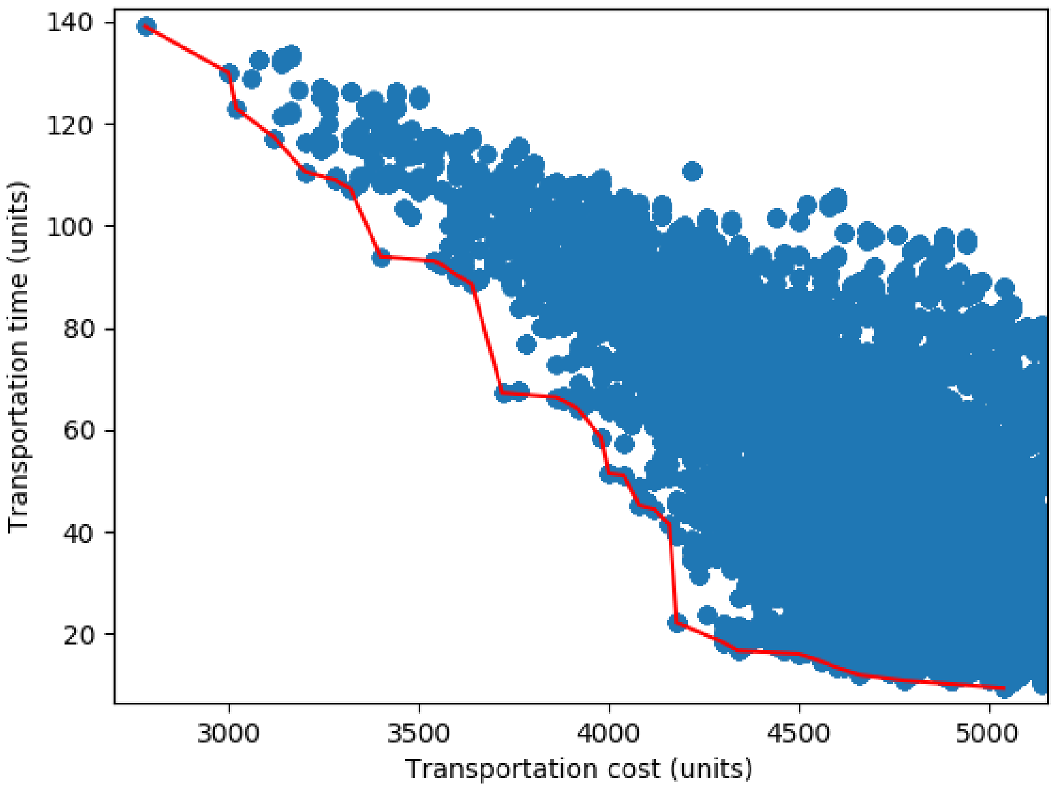

Fig.3 MCS phase 2 results showing Pareto optimal front.

Note that out of the 4 prominent routes obtained after phase 1 MCS, only two routes as shown in Table 5 form solutions on the Pareto optimal front solution set of phase 2 MCS. These solutions can be presented to the decision makers including the freight company or the client to choose which suits them the best.

If, in some circumstances, it is difficult to give a dollar value to time and hence decide on the best combination of cost and time, the approach below can be used to choose one route and mode combination from the Pareto optimal front solution set. The approach described below is called normalization and according to (Marler and Arora, 2004), this is the most robust approach to transforming to non-dimensional objective functions. The transformation is obtained by:

$$F_i(\boldsymbol{x})^{trans} = \frac{F_i(x) - F_i^{min}}{F_i^{max} - F_i^{min}}$$

where $F_i^{min} = \min_x\{F_i(x)|x \in X\}$ and $F_i^{max} = \max_x\{F_i(x)|x \in X\}$.

For the purposes of this paper, the Pareto optimal front solution set of phase 2 MCS (Table 5) were normalized using the approach above and the route and mode which gave a minimum of the sum of normalized Cost function was selected: $F_1(\mathbf{x})$ and Time function: $F_2(\mathbf{x})$. The solution thus obtained was solution # 24, which is route:
1-4-5-12-16-21-17-22-26-29-33-35 using modes B-B-B-B-B-A-B-B-B-B-B. This combination yielded a cost of 4180 units and time of 22.16 units as shown below.

#	Route	Mode	Cost	Time
24	1-4-5-12-16-21-17-22-26-29-33-35	B-B-B-B-B-A-B-B-B-B-B	4180	22.16

8 Conclusion

In this paper, a two phase Monte Carlo method was developed to effectively extract the best routes of travel from an origin to a destination in a multimodal transportation network. To generate preferred routes, this method integrates total costs (including costs associated with transferring between modes of transportation), duration of routes and the availability of respective modes in respective legs of a route. The two phase Monte Carlo method was demonstrated in an example situation, which showed its efficient, practical and versatile nature. The results of this were then analysed to present Pareto optimal solutions from which a choice can be made.

References

Arora, J. S. (2017) 'Chapter 18 - Multi-objective Optimum Design Concepts and Methods', in Arora, J. S. (ed.) *Introduction to Optimum Design (Fourth Edition)*. Fourth Edition. Boston: Academic Press, pp. 771–794. doi: https://doi.org/10.1016/B978-0-12-800806-5.00018-4.

Deb, K. *et al.* (2002) 'A fast and elitist multiobjective genetic algorithm: NSGA-II', *IEEE Transactions on Evolutionary Computation*. doi: 10.1109/4235.996017.

Kochanski, G. (2005) *Monte Carlo Simulation*. Available at: http://kochanski.org/gpk/teaching/0401Oxford/MonteCarlo.pdf (Accessed: 30 May 2019).

Marler, R. T. and Arora, J. S. (2004) 'Survey of multi-objective optimization methods for engineering', *Structural and Multidisciplinary Optimization*. doi: 10.1007/s00158-003-0368-6.

Messac, A., Ismail-Yahaya, A. and Mattson, C. A. (2003) 'The normalized normal constraint method for generating the Pareto frontier', *Structural and Multidisciplinary Optimization*. doi: 10.1007/s00158-002-0276-1.

Prakash, S. (2018) 'Alternative approach to estimating crash costs for cost-benefit analysis using Monte Carlo simulation', in *Australasian Transport Research Forum 2018 Proceedings*, pp. 1–16.

Prakash, S. and Jokhan, A. (2016) 'An Optimal Cane Delivery Scheduling using the Monte Carlo Method', in *Australasian Transport Research Forum 2016 Proceedings*, pp. 1–10.

Prakash, S. and Jokhan, A. (2017) 'Monte Carlo for selecting risk response strategies', in *Australasian Transport Research Forum 2017 Proceedings*.

Prakash, S. and Mitchell, D. (2015) 'Probabilistic Benefit Cost Ratio – A Case Study 2 . Probabilistic Benefit Cost Ratio (BCR)', in *Australasian Transport Research Forum 2015 Proceedings*, pp. 1–14.

Schaffer, J. D. (1985) 'Multiple objective optimization with vector evaluated genetic algorithms', *The 1st international Conference on Genetic Algorithms*.

Srinivas, N. and Deb, K. (1994) 'Muiltiobjective Optimization Using Nondominated Sorting in Genetic Algorithms', *Evolutionary Computation*. doi: 10.1162/evco.1994.2.3.221.

Steadieseifi, M. *et al.* (2014) 'Multimodal freight transportation planning: A literature review', *European Journal of Operational Research*. doi: 10.1016/j.ejor.2013.06.055.

Sun, Y. and Lang, M. (2015) 'Bi-objective optimization for multi-modal transportation routing planning problem based on pareto optimality', *Journal of Industrial Engineering and Management*. doi: 10.3926/jiem.1562.

Vose, D. (2008) *Risk Analysis: A Quantitative Guide*. Wiley (Canada, Origins and Options Series). Available at: http://www.amazon.fr/Risk-Analysis-A-Quantitative-Guide/dp/0470512849.

Xiong, G. and Wang, Y. (2014) 'Best routes selection in multimodal networks using multi-objective genetic algorithm', in *Journal of Combinatorial Optimization*. doi: 10.1007/s10878-012-9574-8.