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Lessons for Adopting Microeconomic Land Use Models at the City Scale: Perth Case Study

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Abstract

Major transport infrastructure has the potential to significantly influence land use development, with increased development likely along its corridors. The land use (population and employment) inputs into the Perth Strategic Transport Models are currently produced at the small area level using a rule-based approach, Metropolitan Land Use Forecasting System (MLUFS). This approach does not explicitly take accessibility into account and limits the analysis of transport infrastructure programs and their impact on land-use intensification. An improved behaviour-based strategic land-use model to predict the potential impacts of major transport infrastructure on land-use development patterns was identified as a modelling imperative for Western Australia's planning and transport agencies (Taplin *et al*., 2014). CUBE Land was selected because it is behaviourally sound and specifically addresses the land-use and transport interaction. This paper presents an active research case study, undertaken by researchers at PATREC, on the implementation of an equilibrium land use model for Perth. In addition the paper reports a short survey of the use and experience of Cubeland completed by other jurisdictions, worldwide.

1. Introduction

It is crucial that the land-use model can estimate behavioural responses through the incorporation of location choices made by households and businesses. Land-use models with a strong grounding in real estate market economic theory thus currently dominate international best practice. CUBE Land is based on Martinez (1992, 1996) combining bid-rent and discrete choice theory within an applied land-use model Estimation models of household and business demand for properties are iterated with developers' profit maximisation objective to arrive at a land-use forecast, the equations are set out in section 2.2 below. To integrate the land-use component with a transport modelling stage, these forecasts underpin a trip (or tour) based transport models, particularly at the generation (household location choices) and distribution (employment) stages (Martinez, 1996). A feedback loop passes accessibility measures back to the land-use models, providing adjusted land rents.

Whilst a comprehensive land use and transport integration (LUTI) model is promising in theory, practical implementation is challenged by the very nature of planning in that land releases and zoning are often uncoordinated or driven by decisions other than integrated land-use and transport development (e.g., political decisions). Whilst regulators may consider market forces, ultimately land-use planning is under the direction of state and local government authorities. These regulations “impose discontinuities in the city space that are difficult to handle with models” (Martinez, 2018: 9). Adding the history of planning decisions that will have shaped the city to date, the top-down approach to land-use means the system dynamics of urban growth is highly constrained and is at odds the market driven equilibrium assumption of models like CUBE Land.

The aim of this paper is to present the lessons learned when during an active research case study, undertaken by researchers at the Planning and Transport Research Centre (PATREC), on the development and implantation of a large scale land use model for Perth and Peel Regions in Western Australia. The paper opens with an overview of microeconomic equilibrium land use models (Section 2) and the presents the Perth and Peel case study (Section 4) in which researchers at PATREC worked with staff at the Department of Planning, Heritage and Land (DPLH) WA. As part of the research an international survey of CUBE Land practitioners was undertaken and this is presented in section 3. Summaries and Conclusions are given in Section 5.

2. Land use and transport models

The interaction between transport and land-use systems has been recognised as a critical issue (e.g., Litman & Steele, 2014; Soares Lopes et al., 2019; Wegener, 2020). Whilst a consensus on both methodologies and operational applications has not been met, Miller (2018b: 387) suggests that a small number of commercial software package “UrbanSim, PECAS, Transun, DELTA and CUBE Land” represent some convergence in methods. Real estate market-based equilibrium models, such as CUBE Land, are able to model activity demands and spatial distribution patterns, while considering transport as exogenous (Soares Lopes et al., 2019). This is considered essential, as it reflects the behaviour of the agents.

Furthermore, Miller (2018) notes that LUTI models in general have failed to achieve widespread adoption, which he puts down to previous failures, onerous resource and data requirements and the tendency for agencies to silo land-use planning away from transport modelling. Although not raised as a potential reason for low levels of adoption, it is worth noting that multi-year timeframes present an obstacle to comprehensive and frequent model validation. In particular Miller (2018: 393) suggests that models may calibrate well against base conditions, but they may not perform well in forecast applications.

LUTI models aim to provide a decision support platform that explains, simulates and forecasts land-use responses to economic and population growth, planning policies, land release or infrastructure programs – including transport provision. Micro-economic equilibrium models (e.g., CUBE Land) forecast future land-use based on micro-economic representations of household and business location choices subject to: current land-use supply and predetermined provision of reserves (green space); regulation and zoning decisions; and physical (land form) constraints. The purported advantage of the package is that the aggregate land-use is inferred from representations of micro-economic agents (households and firms). The model is based on a **market-clearing equilibrium** approach for which an analogue could be drawn to the four-step transport models based on disaggregated choice data. In essence it sits somewhere between non-representative **rule-based planning support systems** that incorporate a set of heuristics to explain aggregate market response, and **micro-simulation** land-use models that adopt a dynamic disequilibrium paradigm for which synthetic (i.e., behaviourally coded) agents trade real estate under different planning scenarios.

Microeconomic equilibrium models are designed to incorporate transport planning activities through: Supplying transport models with household and business location forecasts (i.e., land-use responses to transport or planning policies); and accounting for transport accessibility (costs) outputs from the transport model in the land-use module (Vorraa, 2004). The combination of the land-use modelling and transport integration aims to provide a useful and trusted strategic decision support system used to analyse and evaluate:

- Infrastructure programs (large developments, road and public transport provision) with long planning horizons;
- Metropolitan or local planning policies (i.e., strategic corridors, re-zoning);
- Localised (small) provision of infrastructure.

2.1. Microeconomic equilibrium models

Equilibrium models assume that all consumers in the real estate market, either households or commercial activities, are located on land for which they were the 'highest bidders'. Real estate is assigned as if an auction had taken place and that the bidding process determines the price of the real estate. The demand component of the model introduces an estimable bid function – defined as the inverse of the utility function for rents – to represent household preferences, subject to budget restrictions. The supply component is a representation of profit maximising developers who look to provide housing options that will yield the highest rent. However, in a real-world application the market clearing activity is heavily reliant on the existing built environment and on planning regulations. Consequently, for each forecast year, the model provides the distribution of real estate property by zone and type of dwelling/activity, to simulate changes in policy regulations (banning types of activities or the concentration of certain types of activities). Land-use policies may also use subsidies and taxes to influence demand, including location choice. These policy tools may be used to influence both the physical supply of land and its location.

The demand and supply components are integrated by way of a sequence of three estimating equations. Following Martínez (1992, 1996) the demand side – **the bid function** – estimates a willingness-to-pay for location attributes and housing quality, and **the rent function** – in essence – is an estimate of the monetary value of the winning bid. On the supply side, the (implicit) **cost function** is a measurement of the developers' most profitable mix of properties for each location. It is worth noting that the input components of regional economic and regional demographic models are non-trivial in a real application and require allocation of resources and potential coordination across modelling branches within the agency.

2.2.1. Demand side equations

The bid-rent model starts with the demand for housing and economic activity. This means that the observed dwelling characteristics (with the attributes of the families living in them) and the businesses (with the profile of their activity and employees) are the main input, using the current rents for the dwellings (by segments) or businesses (by their typology). Citilabs (2018: 5) enumerates the demand model of 'auction probability' as follows:

$$H_{hvi} = S_{vi} P_{h/vi} \quad [1]$$

$$P_{h/vi} = \frac{H_h \phi_{hvi} \, exp(\mu_m B_{hvi}(P_{h/vi}))}{\sum_{g\varepsilon H(m)} H_g \phi_{gvi} \, exp(\mu_m B_{gvi}(P_{h/vi}))} \quad [2]$$

where S_{vi} is the real estate supply for zone i and real estate typology v. Within this auction probability demand model, S_{vi} is fixed, but determined by a separate model from the supply side equations (equation [4] below). H_{hvi} represents the number of agents allocated in category *h* to zone *i* for real estate typology *v*. Whereas, $P_{h/vi}$ is the probability that agents of category *h* being the highest bidder in zone *i* of real estate typology *v*. H_h represents the number of agents in category H, whereas $H^{(m)}$ is the total number of agents competing in the auction (for zone *i* and real estate typology *v*).

ϕ_{gvi} is a "cutoff function" used to consider constraints on the agents, such as budget, whereas μ_m is a scale factor, which is assumed to be calibrated in the bid function. The bid function, B_{hvi}, represents the bid of the consumer of category *h* for the real estate typology *v* and in zone *i*. and is estimated using a multinomial logit model using maximum likelihood (ML). The logsum from the bid function feeds into the rent function.

$$r_{vi} = \frac{LS_{vi}}{\mu_m} + \frac{\gamma}{\mu_m} + G_{vi}(X_{vi}, X_i, Z_i) = \frac{1}{\mu_m}(LS_{vi} + \gamma) + \sum_k \alpha_{km} x_{kvi}^{\theta km} y_{kvi}^{\eta km} \quad [3]$$

r_{vi} represents the rent, γ is the location parameter for an extreme value type I distribution[1] an, μ_m represents the scale factor[2], G_{vi} is the rent function accounting for features and amenity x_{kvi} and y_{kvi}, LS is the logsum, θ and η are parameters for the attributes of real estate type *v* and zone *i*. This function can be either estimated as an OLS regression model or jointly in a combined likelihood function.

Two important comments need to be made in relation to the various markets: a) the location data is generally more reliable than rents and thus the sequential estimation is preferred; b) the analyst may require other proxy measures for rental values, land or equivalent of property price.

2.2.2. Supply side equation

The supply model calculates the *'Cost function'*, which is actually presented as land developer profit, representing the difference between the received rent and the 'production' cost for a specific type of property and location, through the proportion of real estate units of a certain type built and located in each zone. This calculation may require using other proxy measures for rental values: land or equivalent of property price.

Citilabs (2018: 9) describes a real estate supply model as follows to calculate supply (S) of typology *v* in zone *i*:

$$S_{vi} = H^{(m)} P_{vi} = H^{(m)} \frac{\rho_{vi}\, exp\lambda(r_{vi}(S_{.i}) - C_{vi}(S_{vi}))}{\sum_{v'i'} \rho_{v'i'}\, exp\lambda(r_{v'i'}(S_{.i}) - C_{v'i'}(S_{v'i'}))} \quad [4]$$

where the fraction corresponds to the probability of constructing properties of typology *v* in zone *i*. λ is a scale factor, common to all markets, that should be estimated as part of the MNL process, whereas ρ is a 'cutoff function' for typology *v* representing things such as zoning restrictions for the typology.

From this, the profit (π) for a specified typology *v* in zone *i* can be calculated and extracted as equivalent to:

$$\pi_{vi} = r_{vi} - C_{vi} \quad [5]$$

where r_{vi} is the rent received and C_{vi} is the production cost.

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[1] Extreme Value Distribution Type I (EVI) is found by the difference between two logistic distributions. Theoretically it holds some importance but can be ignored for the purpose of this report.

[2] The scale factor is unknown when estimating equation [2]. The analyst will set this to one. The implication of doing so is that the log-sum is in some unknown utility scale rather than in dollar values (i.e. reflecting the market rents). It is possible to estimate equations [2] and [3] simultaneously, but typically the estimation is undertaken sequentially with $\frac{1}{\mu_m}$ being estimated in equation [3].

The estimation is either using ML or the 'Berkson and Theil' approach, such that a reference group is chosen and the supply model [4] may be expressed as a linear function of the ratio between all other groups and the reference group.

2.2.3 Equilibrium

The bid (location) and rent models are estimated sequentially in the bid (choice model). These estimated parameters are used in CUBE Land to be solved as equation structures, balancing the demand and supply. An important feature is treating location as a different good/bundle, with unique properties, which makes comparability between spaces in spatially extensive systems very distinct from other economic markets. The land-use market is a system with multiple and heterogeneous agent interactions in processes that occur over space, with the traded objects being the real estate properties.

CUBE Land applies several adjustments, corrections, to replicate the market situation. There are three correction terms: $BAdj_{hvi}$, $RAdj_{vi}$, $CAdj_{vi}$, for bids, for rents, and for costs. The experience of DPLH in working with CUBE Land is that the cost adjustments are the largest, which poses a challenge for model application (potentially being unresponsive to forecasted changes or implementation of various measures). The CUBE Land user manual recommends that the equilibrium/convergence process will be repeated with potentially superior functions for bid-rent and cost to capture the location processes, in such a way that the differences between model results and observed input are minimised. This suggests that while the tenets of the model are theoretically sound, data availability and quality may hinder CUBE Land calibration, which is a resource-intensive process.

2.2.4 Constraints

CUBE Land uses constraints for demand (Location Model) and supply (Cost/Supply Model) mostly endogenously. Soft constraints are implemented via 'cut-off' functions (binomial probability distributions) and additional exogenous hard constraints are currently under development. Demand restrictions require a minimum level of housing, whereas supply restrictions are given by land availability. The functional design of CUBE Land enables gradual adjustments of the land-use allocations when constraints are not met. However, without intervention, the forecasts may exceed the constraints, which is highly undesirable from planning policy perspectives. In the post-processing enhancement of the Cube Land II model[3], zones where the number of types of properties are exceeded are made less attractive by increasing the cost (positive adjustment). Conversely, zones where there is reserve of capacity could become more attractive if the cost decreases.

3. An International survey of CUBE Land practitioners

As part of the collaborative efforts PATREC undertook an international survey of current CUBE Land users. Following Avin and Cambridge Systematics (2016: 4-1) , the survey aimed to review CUBE Land software applications using three major tool attributes: *conceptual* ('what kind of tool'), *functional* ('how the model does it work'), and *implementation* ('what does it take it to apply it'). *Conceptual* attributes cover the theory, assumptions, and the limitations of the model; the *functional* aspects refer to data requirements, the modular structure

[3] CUBE Land II is a new release that has incorporated the learning for the case study reported in this paper.

and the ability for planning teams to design and evaluate scenarios; and *implementation* primarily regards access to and distribution of the models functionality and results.

3.1 Survey Results

Thirteen responses were recorded, 10 from private consultancies and researchers and the remainder from government. Four respondents were analysts, three were domain experts (in planning or related domain) and three senior managers. Seven respondents claimed to be proficient. An accurate response rate cannot be provided, as the ‘population’ of CUBE Land users is unknown and for privacy reasons we cannot divulge our recruitment frame.

In line with Miller (2018) observation that LUTI models have failed to achieve widespread adoption, only one respondent claimed to have a fully developed model in use, three respondents reported partially developed/implemented models and three indicated that use of CUBE Land has previously been investigated, but did not progress to implementation. Additionally, one respondent clarified that while the model has been applied in smaller regional areas, the intention is to use the accumulated experience to develop models for larger areas or major cities.

When asked about what they appreciate the most in a modelling platform such as CUBE Land, the respondents indicated that rigour, accuracy, and validation with real data are their top requirements. This was followed by producing meaningful outputs that can be communicated to wide audiences, and by easy and transparent assumptions, possibility to apply the models to large areas, as well as to assess local policy and regulations influence. This demonstrates the value of the *conceptual* and *functional* features of a software modelling platform, over the *implementation* aspects, which are not considered substantial barriers in adoption and application

3.2 Visualising the relative importance of the model’s aspects

A multi-dimensional scaling (MDS) analysis was applied to the best-worst responses, to visualise the features sought/preferred in a modelling platform such as CUBE Land. The MDS technique assists analysts to reveal key dimensions underlying respondents’ evaluations of various entities/objects (products/services, events, phenomena, and here software features etc.) and to understand the clustering of entities in the perceptual map. The axes of the map represent the dimensions that the analyst infers. MDS is based on distance measures, which means that entities/objects/respondents far from each other in this multidimensional space are more dissimilar than others.

The results of our aggregate analysis indicate adequate representation of the preferred features (s-stress = 0.149 and RSQ=0.888, akin to R^2 in regression analysis) and the two-dimensional map suggests less emphasis on the licensing, distribution and administration, strong focus on accuracy and transparent assumptions, and relevance for policy and practice (as shown along the Dimension 1, which could be labelled as ‘key requirements’).

Additionally, we notice that relying on the minimum data that is required to produce meaningful and accurate results is substantially more important than other data and assumptions, with the lowest score along Dimension 2 (reflecting 'data and integration with other platforms') being recorded for links to other platforms.

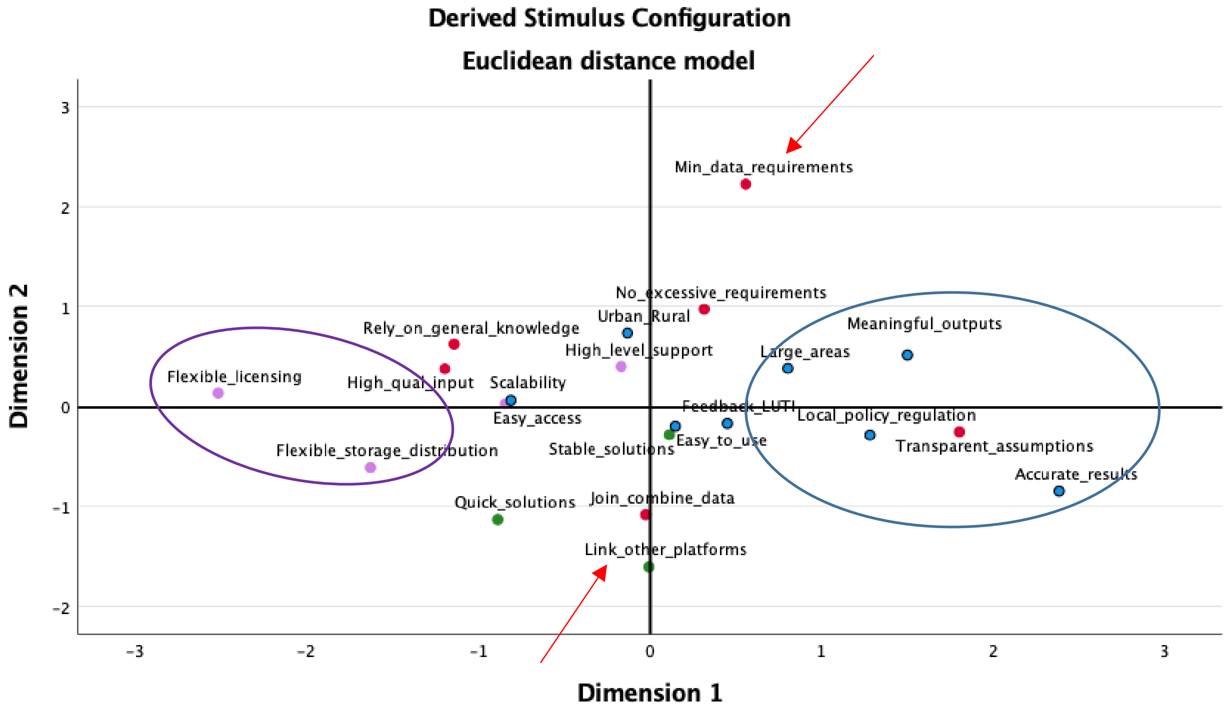

Figure 1: Perceptual map of desired software features

4. Case study: implementing a microeconomic land use model for Perth

This section presents results from the joint PATREC-DPLH investigation of the CUBE Land features and limitations and sets out to explain the challenges in applying the model and practical solutions tested and actioned. The purpose is to highlight real-world learning that may be of value to practitioners and their academic partners.

4.1. Aligning the tool with organisational strategy

DPLH purchased CUBE Land with the expectation that the software would provide a robust method to provide routinely updated forecasts and to test growth scenarios. The acquisition of this software (or software that serves the same purpose) aligned with the department's strategic direction to provide evidence-based decision support by leveraging its existing data resources. The immediate benefits identified by DPLH was that CUBE Land would operate in a GIS framework (compatible with ArcGIS) and the software would integrate with CUBE and CUBE Voyager (the software currently being used for Perth's Strategic Transport Model, STEM). A specific value of the software was that it could provide a behaviourally-driven support tool to assist long term land-use planning. This would enhance DPLH's position of being the principal source for evidential support on population and employment growth for all of government's needs.

The deliverables and metrics for CUBE Land implementation were given as (Nayton and Zheng 2017):

Deliverables

- Forecast residential and commercial land development by type and zone;
- Forecast household and employment location choices by type and zone;
- Estimate equilibrium real estate prices.

Success Measures

- Client acceptance of forecasts;
- Transparency and justification for forecasts;
- Timeliness of delivery.

Outlining the modelling procedure Figure 1, acts a useful base on which to discuss the model implementation status at DPLH. The modelling stages can be broadly classified into MODEL SET-UP (Steps 1-3), MODEL SPECIFICATION (Steps 4-6) and MODEL APPLICATION (Steps 7-8).

Figure 1: Steps towards application of an equilibrium land use model

Despite its sequential presentation, the implementation process is not 'linear' and involves several feedback loops from Steps 6 and 7 to Steps 3, 4, and 5. This iterative process requires numerous trial-and-error steps, which inevitably lengthens the duration of the implementation until convergence is reached.

4.1. Model implementation

4.2.1 Application area (Step 1) and zonal system (Step 2)

Given the aim of implementing CUBE Land to be Perth and Peel's strategic land-use model, the model study area and zoning systems were identified as being the Perth and Peel region (i.e., greater metropolitan Perth) with zonal partitions meeting those for Perth's strategic transport models with 1200 TAZ. However, because the Census data is a principal resource the zoning system currently implemented at the 254 Statistical Areas level 2 (SA2). To correct for spatial variations in zoning, a spatial analysis based on land capacity is used to allocate households to traffic zones and similarly to a weighted average of accessibility measures from the transport modes to CUBE Land.

4.2.2 Data requirements (Step 3)

Data requirements are notably a challenge for land-use modelling and long-term forecasting. However, to simply identify the sources of data undersells the considerable resources required to source, clean and validate data. Miller (2018) raises high resource needs and the considerable risk (of partial implementation or project abandonment) as being notable barriers to large scale integrated models.

> *A rough approximation is that 75% of the effort and an even higher percentage of the time involved in developing model applications is due to the difficulty of developing the data for the model system. (*Waddell, 2011)

DPLH's implementation to date has managed to incorporate multiple sources of data for the residential location choice component of the model. Initial efforts to estimate choice models based on aggregate shares (i.e. proportion of household categories) failed to yield a good model. The significant advancement was to match records from the land-use data from the Valuer Generals Office (VGO) with unit household records from the Australian Bureau of Statistics (ABS). Enriching the data meant that the inputs into the model had a higher level of precision on both the supply side (i.e., household records) and the supply side (i.e., land use data). Given that these data are highly sensitive, access to the unit records from ABS meant that DPLH and PATREC personnel underwent data security training and accreditation. The process from raising a request to data readiness took a little over 12 months.

4.2.3 Model estimation (Step 4) and model calibration (Step 5)

The theoretical bid-rent demand and cost supply model was conceived to be internally consistent. Such an assumption is ideal for an academic exercise but presents challenges to practical implementation on a large scale. The principle challenge for real-world application is calibration against observed market rents and existing stocks of residential buildings. At each stage market observations were included in the estimating equations to either replace the internally estimated value or to provide a calibration adjustment factor. To summarise, estimating equations [1-5] provide a system of behaviourally consistent models, but for applications the closer the model reflects this internally consistent market behaviour the more difficult it is for the results to calibrate baseline conditions or to meet imposed constraints.

4.2.4 Model validation (Step 6) and sensitivity analysis (Step 7)

Validation of CUBE Land may be considered a type of 'docking[4]' (Olaru *et al.*, 2014), achieving alignment between models estimated outside CUBE Land (e.g., logistic models and regressions estimated in R) and the CUBE Land allocation and 'recovery' of the spatial patterns

[4] Docking, also known as alignment or replication is a cross validation approach for complex dynamic models in which the results from two or more model drawn from different theoretical underpinnings are compared

of the real estate market. As per the literature (Axtell *et al.*, 1996, cited in Olaru *et al.*, 2014), three performance criteria may be used:

- Numerical identity: Results are numerically the same for the two models;
- Distributional equivalence: Distributions of results are statistically indistinguishable, and;
- Relational alignment: The same patterns of interactions exist in the models.

Although many docking (and replication) experiences demonstrate either relational or distributional equivalence, very few reach numerical identity and successful validation requires considerable time and effort to undertake, sufficient descriptions of the models, and similar interpretations of the models' theoretical underpinnings. In the case of CUBE Land at DPLH, it is too early to ascertain whether distributional equivalence and numerical identity were achieved using the 2016 Census data, given the on-going process for checking the model parameters and correction terms. In the intermediate term cross-validation is to be performed against results from MLUFS for household locations and alongside the department's, internally developed, commercial location projection model.

Sensitivity analysis can be considered as part of the validation process. 'What-if' scenarios ascertained the effect of inputs on the model's output. Up-to-date, changes in accessibility as a result of the METRONET planned improvements and major land developments such as the Airport-Forrestfield were tested, with results which are defensible in qualitative terms.

4.2.5 Model application (Step 8)

To date CUBE Land has been tested alongside existing rule based planning models and is yet to be fully adopted as the standard modelling asset. The view of the department is that residential location choice model outputs are defendable and current efforts to integrate with Perth's strategic transport models are underway. Additionally, ongoing work in modelling employment densities through business location choice models will need to advance for the model to become the primary land use forecasting tool.

5. Discussion, Conclusion and Recommendations

Behavioural models, explaining choices made by households and businesses, are undoubtedly superior to sketch models relying heavily on simplifying assumptions. Nevertheless, when applied microeconomic equilibrium models are heavily constrained by the existing built form and by planning regulations which in turn means that calibration and validation is a critical part for implementation. An outcome from our participation in the implementation phase is that these models should be continually validated through a number of processes: sensitivity analysis and scenario development, testing various models with distinct inputs, and cross-validation with outputs from existing land use projection models.

The case study at DPLH Western Australia and the limited survey information from other practitioners adds to the literature (Miller 2018; Wegener, 2020) that adoption of a large scale behavioural LUTI model is complex process and requires ongoing support throughout the development stage that may span years rather than months.

In conclusion, the case highlights the need for the modelling team to **stress the importance of quality data inputs.** However, it is also incumbent on the modelling team to **report meaningful and defendable outputs** that have been examined by way of **sensitivity analysis** and that ongoing **validation exercises** are performed. This will increase the confidence in the model capabilities and adjust expectations of the users 'downstream' of planning. Unsurprisingly, this was echoed by the international user survey results. The survey confirmed the high value of the *conceptual* and *functional* features of a software-modelling platform, over the *implementation* aspects, which are not seen as hurdles in adoption and application.

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