

Efficient OD matrix estimation based on metamodel for nonlinear assignment function

David Masip¹, Tamara Djukic¹, Martijn Breen¹, Jordi Casas¹

¹Aimsun SL, Ronda Universitat 22B, Barcelona 08007, Spain
E-mail for correspondence: tamara.djukic@aimsun.com

Abstract

In this work, the problem of dynamic origin-destination (OD) matrix estimation, using traffic observations and its assimilation into traffic assignment models, is addressed. In the past decades, a rich body of literature, has been devoted to development of the heuristic methods to reduce high computational effort of traffic simulation without significant trade-off of result's accuracy. In this paper we propose a metamodel as a function that maps the nonlinear relationship between traffic observations and OD flows. Derived metamodel framework is consistent with all the estimated demands through iteration steps, which results in non-constant, demand dependent, assignment matrix. Further, metamodel requires the same amount of traffic simulations as the classic approach, to maintain a low computational costs. In addition to proposed metamodel, a stochastic gradient method, that is more efficient to escape from local minima, is applied to solve the formulated dynamic OD matrix estimation problem. The performance of the proposed metamodel and solution approach has been evaluated for four OD estimation methods formulation. Least square OD matrix estimation model solved with gradient descent algorithm has been selected as a complementary method, widely used in practice, whose limitations and assumptions we aimed to solve. Numerical experiments are performed on real network, (Vitoria, Basque Country, Spain) with real data and two historical OD patterns to evaluate the performance of the proposed solution approach. Experimental results show that the proposed stochastic gradient metamodel method demonstrates stable decrease of the cost function with the lowest value through iteration steps. The experiments indicate that the proposed method based on metamodel yields to robust performance for both quality of the initial OD matrices, compared to benchmark method.

1 Introduction

Estimating the dynamics of traffic demand over space and time is essential for many applications over the entire transport modeling industry, from operations, control and management; to planning and policy assessment. Since we do not directly observe origins and destinations of the trips, for each traveler, we have to estimate origin-destination (OD) flows from partially direct and/or indirect data available. The OD matrix estimation is (still) one of the toughest problems to address in transportation to date, whose solution would improve the realism of traffic models significantly.

The inability of providing high quality OD matrix estimates makes prediction with advanced simulation models simply impossible, regardless of how well these models have been calibrated. In this respect estimation of OD matrix with sufficient data of sufficient granularity are critical in establishing the credibility of simulation tools for traffic planning and management purposes. As simulations are used to make decisions in transport planning and transport management, this is an important problem that needs to be solved.

In this paper, the problem of dynamic origin-destination (OD) matrix estimation, using traffic observations and its assimilation into traffic assignment models, is addressed. This is the common meaning of the term OD matrix estimation, sometimes referred to as adjustment or calibration problem, and there is a long record of contributions in this area as well (e.g. Van Zuylen & Willumsen (1980), Cascetta (1984), Cremer & Keller (1987), Bell (1991), Yang et al. (1992), Ashok & Ben-Akiva (2000), Zhou & Mahmassani (2007), Cascetta et al. (2013), Antoniou et al. (2006) to name a few). There are many challenges in solving the dynamic OD matrix estimation problem: it is *high-dimensional* problem (number of non-zero trips in OD matrix is often in order of thousands), *under-determined* problem (small set of traffic observations in the network), *unreliable prior OD matrix* information (usually available from census or previous studies), *non-convex* problem (it contains many local minima), and *traffic simulation dependent* (lack of tractable analytical models or metamodels). For example, high resolution traffic simulation software are computationally very expensive. Hence, function evaluations for each given point through iteration steps is very computationally expensive. As a result, derivative-free solutions that do not require estimation of derivatives of the simulation based OD cost function, are commonly adopted in practice. This derivative-free solution is commonly reflected in linear approximation of the assignment matrix (mapping function between OD demand and traffic counts). However, the linear mapping between counts and OD flows is valid only, and only if, route choice is fixed and known, and the network is uncongested, which means that under most conditions more complex simulation models are required. For example, recent studies by Toledo & Kolechkina (2013), Frederix et al. (2013), Shafiei et al. (2017), Djukic et al. (2018), rely on linear approximation of the assignment matrix with non-separable response in every iteration, which relaxes the assumption of constant link-flow proportions and explicitly accounts the congestion effects. This definition requires the computation of the marginal effects of demand flow change on the link-flow proportions at the current solution of each iteration. Thus, it requires, in every iteration of the gradient solution, to perturb each element in the OD demand vector, one at a time, leading to 2RDT runs, where D is the number of OD pairs in the network, T is the number of time intervals for the simulation period and R is the number of simulation runs. To overcome computational overhead, authors proposed heuristic-based approaches. However, all these approaches rely on strong heuristic assumptions such as ignoring the effect of OD demand changes outside of congested area or have been tested on relatively small and medium sized networks. Another assumption commonly adopted in practice to reduce computational costs of traffic simulation is to keep assignment matrix constant (fixed) while computing gradient steps. The main criticism of this approach lies in fact that it only uses information from traffic assignment given by the last simulated demand. It is obvious that such solution approach will result in bias approximation of mapping function, \mathcal{A} , and consequently of expected traffic counts, that has to be overcome. Thus, there are no results in the bibliography showing robust and efficient algorithms, that perform well in every situation and require few function evaluations (Osorio (2017)).

Hence, in this paper we propose a metamodel as a function that maps the nonlinear relationship between traffic observations and OD flows. Derived metamodel framework is consistent with all the estimated demands through iteration steps, which results in non-constant, demand dependent, assignment matrix. Further, metamodel requires the same amount of traffic simulations as the classic approach, to maintain a low computational costs. In addition, this metamodel is non-linear, in the sense that the derivative of the traffic counts with respect to the demand is not constant. In addition to proposed metamodel, a stochastic gradient method, that is more efficient to escape from local minima, is applied to solve the formulated dynamic OD matrix estimation problem. This solution algorithm is independent of the metamodel. A thorough analysis of the proposed methods and their computational efficiency using real data from a real network is provided. We demonstrate the reliability and accuracy improvement of OD matrix estimates based on metamodel with stochastic gradient solution approach as opposed to the traditional, gradient approach widely used in practice.

Paper is organized as follows. Section 2 describes the overall OD matrix estimation problem; the consistent metamodel formulation in Section 2.2; the framework to obtain metamodels (Section 2.3); and stochastic gradient algorithm to solve the quadratic OD optimization problem. In Section 3, we outline how we assess the performance of the proposed metamodel and its solution. We do this on real-size network with two demand patterns that reflect regular daily and highly congested traffic fluctuations, in order to demonstrate method's feasibility. These results are presented in Section 4. We offer conclusions and a critical discussion on further research avenues in Section 5.

2 Methodology

2.1 Problem formulation

In general, dynamic OD matrix estimation problem can be formulated as an optimization problem to find an estimate of OD demand matrix by effectively utilizing traffic and demand observations. In this work, we focus on formulation that considers the most widely available (often only available) type of traffic data: link traffic counts. We introduce the following notation, to formulate the OD matrix estimation problem:

- m is the number of detectors.
- n is the number of origins and destinations.
- T is the number of time intervals.
- d is the matrix representation of the demand.
- y^{rds} is the representation of the real data set in a vector form.
- f is the function that maps demand to a set of simulated counts.
- δ is a parameter that controls the importance given to the historical demand.
- d^{hist} is the matrix representation of the historic demand.

The dynamic OD matrix estimation problem is formulated by a cost function that aims to minimize the square distance between estimated and observed traffic counts, and estimated and historical OD demand. The cost function is defined as:

$$\text{cost}(d) = \frac{1}{m} \sum_{t=1}^T \sum_{i=1}^m \left(f(d)_{i,t} - y_{i,t}^{rds} \right)^2 + \delta \frac{1}{n^2} \sum_{t=1}^T \sum_{i=1}^n \sum_{j=1}^n \left(d_{i,j,t} - d_{i,j,t}^{hist} \right)^2 \quad (1)$$

$$d \geq 0$$

By flattening d into a vector, Equation 1 can be rewritten as:

$$\text{cost}(d) = \frac{1}{m} \left\| f(d) - y^{rds} \right\|_2^2 + \frac{\delta}{n^2} \left\| d - d^{hist} \right\|_2^2 \quad (2)$$

When solving the optimization function in Equation (1), the function $f(d)$ between unobserved OD demand, d , and observable traffic counts has to be defined, implicitly or explicitly. The function needs to map decision demand vector, d , in combination with the exogenous network parameters u_n (e.g., network graph, prevailing traffic control and management strategies, time of day or weather conditions) and endogenous simulation parameters u_s (e.g., route-choice and driving behavior of travelers) to observable traffic counts y^{rds} (and other traffic supply variables, e.g., speeds, travel times, etc). The most adopted way to model this complex function from OD demand to supply is to use a traffic simulation model of the form:

$$\mathbf{y}^{rds} = \mathcal{A}(\mathbf{d}, \mathbf{u}_n, \mathbf{u}_s) \quad (3)$$

in which \mathcal{A} may represent any micro-, meso-, or macroscopic- traffic simulation model, and \mathbf{u}_s represent the route choice (often in equilibrium conditions) and driving behavior models. Clearly, mapping function \mathcal{A} to estimate traffic counts for a given demand, is a nonlinear function that lacks sound mathematical properties, such as convexity, and has no closed-form expression available. A standard approach is to linearly approximate mapping function \mathcal{A} in the form of assignment matrix, A , by running multiple dynamic traffic assignment (DTA) simulation runs, each of which is computationally costly to evaluate. Hence, the common assumption is $f(d) = Ad$ to optimize the cost function. This method is based on defining a *pseudo* cost function that is equal to the cost function in the last simulated point. Suppose that by simulating a point d_0 an assignment matrix A can be obtained. Then, this method defines the pseudo cost function:

$$\text{cost}_A(d) = \frac{1}{m} \left\| Ad - y^{rds} \right\|_2^2 + \frac{\delta}{n^2} \left\| d - d^{hist} \right\|_2^2 \quad (4)$$

Straightforward methods can be used to minimize the pseudo cost function, as it is a quadratic function. Since cost function is quadratic (a paraboloid), gradient descent algorithm will converge to the unique and global minimum, and its gradient is given by:

$$\nabla \text{cost}_A(d) = \frac{2}{m} A^T \left(Ad - y^{rds} \right) + \frac{2\delta}{n^2} \left(d - d^{hist} \right) \quad (5)$$

The pseudo cost function is equal to the cost function when evaluated at the point d_0 . This method expects $f(d)$ to be similar to Ad in a neighborhood of d_0 .

The algorithmic implementation of the gradient based method to solve optimization problem defined in Equation (4) is given as follows:

Algorithm 1 Gradient-based method

- 1: set $d = d^{hist}$
 - 2: **while** stopping criteria are not met **do**:
 - 3: simulate the network with d as input, set to A the simulated assignment matrix.
 - 4: **for** i from 1 to $internal_steps$ **do**:
 - 5: $d \leftarrow d - stepSize \cdot \nabla cost_A(d)$
-

In general terms, in every external iteration of the algorithm, a traffic simulation is carried to obtain an assignment matrix. Then, many gradient steps are executed with that assignment matrix, minimizing the pseudo cost function. After new demand is estimated in the inner loop, next traffic simulation is performed to obtain a new assignment matrix. This process continues iteratively until some convergence criteria is met.

This dynamic OD demand solution approach relies on fixed assignment matrix during gradient computation, and it can be thought as an approximation of the traffic simulator based on the last simulated assignment matrix. The main criticism of this approach lies in fact that it uses linear mapping between demand and counts, and that it only uses information given by the last simulated demand. It is obvious that such solution approach will result in bias approximation of mapping function \mathcal{A} and consequently of expected traffic counts, that has to be overcome.

In this work we present metamodels that, by using the same amount of simulations as the classic approach, use information of all the obtained assignment matrices to provide a more robust metamodel. In addition, this metamodel is non-linear, in the sense that the derivative of the traffic counts with respect to the demand is not constant.

2.2 Consistent metamodel formulation

Metamodel is defined as a function that maps the demand to traffic counts and approximates the output of the traffic simulation at lower computational costs. Even the classic OD demand solution approach given by Algorithm 1 can be interpreted as a metamodel, that corresponds to approximation of OD demand by a linear mapping with the last simulated assignment matrix, $y = Ad$, where y represent the counts obtained by the metamodel.

Prior to design of framework to create metamodels, it is convenient to define consistency. A metamodel is said to be consistent with respect to a demand if the metamodel evaluated on that demand vector gives the same counts as the traffic simulation model. When using the classic solution approach, the metamodel provided by the constant assignment matrix is only consistent with the demand that has generated that assignment matrix; that is, the last simulated demand. Thus, we aim to develop the framework that derives metamodels consistent with all the estimated demands through iteration steps.

2.3 Framework to obtain metamodels

To define a metamodel, a mapping between the demand space to the traffic counts space, $\mathbf{d} \rightarrow \mathbf{y}^{\text{rds}} : \mathcal{R}^n \rightarrow \mathcal{R}^m$, needs to be defined. The consistent metamodel uses information from all the simulated demands in iteration steps, that will be denoted as d_1, \dots, d_k . These demands will have corresponding assignment matrices A_1, \dots, A_k and simulated traffic counts y_1, \dots, y_k . This metamodel computes, for a given input demand, a weighted average of the simulated demands upto current iteration step¹. The weights should be higher for the assignment matrices corresponding to demands that are closer to the input demand.

The general way to derive an average metamodel is: given a demand d , the weights are computed as:

$$\omega_l = \Omega(\|d - d_l\|), \quad l = 1, \dots, k$$

Where $\Omega(x)$ is a positive and non-increasing function that satisfies $\lim_{x \rightarrow 0} \Omega(x) = +\infty$. Different metamodels can be derived using different weighting functions Ω . The averaged assignment matrix is computed as:

$$A = \frac{\sum_{l=1}^k \omega_l A_l}{\sum_{l=1}^k \omega_l}$$

And the counts given by the metamodel are:

$$y = Ad$$

Please note that the assignment matrix is demand dependent, as the weights are demand dependent, leading to nonlinear model. In order to use this metamodel for demand estimation, we use the average assignment matrix to compute the gradient of the cost function, as in Equation 5, and perform inner gradient steps with the average assignment matrix. Note that the demand is updated in every inner step, and therefore the assignment matrix itself.

This metamodel is consistent with all the demands simulated: If the input demand is a demand that has been simulated, d_r , the weight associated to that demand is $\omega_r = +\infty$, and the others are $\omega_l = \Omega(\|d - d_l\|) < +\infty$. For this reason:

$$A = \frac{\sum_{l=1}^k \omega_l A_l}{\sum_{l=1}^k \omega_l} = \frac{\omega_r A_r}{\omega_r} = A_r$$

And then $y = Ad_r = y_r$.

¹For this reason, this metamodel will be referred either as consistent or average metamodel.

Such generic formulation of the metamodel framework, allows definition and implementation of various weighting functions. Here, we propose some examples of different weighting functions:

- Inverse: $\Omega(x) = \frac{1}{x}$
- Softmax function: $\Omega(x) = e^{-x}$
- Inverse softmax: $\Omega(x) = e^{1/x}$
- Nearest neighbors: Take the p closest demands to the given demand, and set the weights of the others to 0. Compute the weights of the p closest demands using one of the weighting functions above.

We have selected the inverse function for the experiments in this paper. This is due to more robust dynamics of the inverse function compared to the other ones. The softmax function penalizes too much the demands that are not the last one, keeping in practice only the last simulated demand (equivalent to the classic approach). The inverse softmax function, on the other hand, gives, approximately, the same weights to every demand, so the average computed is not actually weighted.

Although this metamodel has been defined for dynamic OD matrix estimation, this metamodel can be used in static OD demand estimation. In the static OD matrix estimation that requires an assignment matrix to define mapping function \mathcal{A} , the same proposed operations with metamodel can be applied to the assignment matrices and demand vectors.

2.4 Solution algorithm: Stochastic gradient method

In addition to the proposed metamodel, a stochastic gradient method has also been derived. As explained in Gardner (1984), the aim of the gradient method in machine learning optimization problems is to compute approximations of the gradient that are less expensive to evaluate. These approximations are computed by evaluating the cost function in a datum. As explained in Li et al. (2014), mini-batch gradient descent allows to update the weights by computing the cost function on some of the data, thus allowing for a trade-off between cost of the evaluation and accuracy of the gradient approximation.

In the OD matrix estimation problem, an analogue of these methods would be to update the values of some OD pairs in every iteration. Unlike the machine learning scenario, this cannot be used to save time on evaluating the cost function, as the cost function is a simulation and it is not feasible to carry out the simulation on just some of the OD pairs. However, as Gardner (1984) and Li et al. (2014) claim, the advantages of using stochastic and mini-batch gradient methods are not only computational. As the usual gradient descent method computes the best direction to follow in a greedy way, it is very likely to fall into local minima. Both stochastic and mini-batch methods are a bit more exploratory, as they follow an approximation of the gradient, thus allowing to escape from local minima. This highly desired property has led to selection of a mini-batch gradient method for dynamic OD matrix estimation.

The stochastic gradient method for dynamic OD matrix estimation has been implemented as follows: every time the cost function is evaluated, the subset of OD pairs is selected. In the inner

iterations, the demand is updated only for the defined subset of the OD pairs. On the other hand, the demand values of non-selected OD pairs are kept same as in the last iteration step of the cost function evaluation.

Algorithm 2 Stochastic gradient method in OD matrix estimation

- 1: set $d = d^{hist}$
 - 2: **while** stopping criteria are not met **do**:
 - 3: Simulate the network with d as input, set to A the simulated assignment matrix.
 - 4: Choose at random set of the OD pairs, d_{r_1}, \dots, d_{r_k} . Keep the demand values of the other OD pairs, $d_i, i \notin r_1, \dots, r_k$.
 - 5: **for** i from 1 to $internal_steps$ **do**:
 - 6: $d \leftarrow d - stepSize \cdot \nabla cost_A(d)$
 - 7: Set the values of $d_i, i \notin r_1, \dots, r_k$ as they were before step 5.
-

As stated, one of the reasons to apply this method is due to more exploratory nature than the classic gradient method. Another reason is that it is more robust to creating over-congested situations resulting in a spill-back flow propagation over the entire network. A common issue that is faced when estimating demand is that, by changing all the OD pairs at the same time, the linear approximation given by the assignment matrix becomes very bias easily, as some of them create congestion in the network and therefore the linearity does not hold anymore. A way to make this less likely is by not updating all the OD pairs.

The stochastic gradient method does not depend on the metamodel that is being used, and it can be used regardless of the one chosen. They can be seen as two separate processes: the metamodel is a framework that gives an approximation of the cost function, and the stochastic gradient method is a solution approach of the cost function to estimate demand.

3 Case study

In this section, we will first describe the input data used by method, i.e., historical OD demand generation and DTA traffic assignment procedure. We consider four assessment scenarios in terms of mapping function and solution approaches:

1. **Inverse metamodel**: metamodel mapping function solved with Gradient algorithm;
2. **Inverse stochastic metamodel**: metamodel mapping function solved with Stochastic Gradient algorithm;
3. **Stochastic gradient method**: linear mapping function solved with Stochastic Gradient algorithm; and
4. **Classic gradient method**: method widely used in practice based on linear mapping function solved with Gradient algorithm.

While approaches 1, 2 and 3 represent methods developed in this paper, approach 4 has been selected as a complementary method, widely used in practice, whose limitations and assumptions we aimed to solve. Numerical experiments are performed on large-scale network, (Vitoria, Basque Country, Spain) with real data to evaluate the performance of the proposed solution approach. Furthermore, two historical OD patterns have been designed, one close to real demand and other one biased with higher demand in order to assess the performance of the proposed approaches for dynamic OD matrix estimation.

3.1 Network setup

The Vitoria network has been selected for case study, consisting of 57 centroids, 3249 OD pairs with a 600km road network, 2800 intersections and 389 detectors, presented as black dots in Figure 1. This network is available in the mesoscopic version of the Aimsun traffic simulation model for the reproduction of traffic propagation over the network. The true OD demand is available for this network, which allows analysts to assess the performance of the proposed method. The true assignment matrix and traffic counts on detectors are derived from the assignment of true OD matrix in Aimsun for the evening period from 19:00 to 20:00 reflecting a congested state of the network. The simulation period is divided into 15 minute time intervals with an additional warm-up time interval. The link flows resulting from the assignment of the true OD demand are used to obtain the traffic count data per observation time interval.

Figure 1: Vitoria network graph. Black dots: location of the loop detectors.



The trips between some of the OD pairs are not completed within one time interval due to congestion of the network or the distance between OD pairs. In this way, a vehicle entering the network during a particular departure time interval might need more than one time interval to reach a traffic detector, where the departure time interval and detection time are different. In the chosen study network, the maximum travel time between OD pairs observed on the network takes three time intervals, which leads to very sparse assignment matrices, and the number of lagged time intervals to be 3.

3.2 DTA with mesoscopic simulation model and metamodel

In the experiments, we use the mesoscopic event-based demand and supply models in Aimsun Next (Aimsun (2017)), each synthesizing microscopic and macroscopic modelling concept. They couple the detailed behaviour of individual drivers' route choice behaviours with more macroscopic models of traffic dynamics. The travel demand in Aimsun Next is represented by dynamic OD demand matrices. Vehicle generation is done for each OD pair separately with arrival times that follow an exponential distribution. The iterative interaction between demand and supply models allows the system to update the set of routes and the travel times after each iteration leading to robust estimation and prediction of traffic conditions in the network. For this study, a route choice set will be pre-computed in Aimsun Next and used as fixed for all the simulation runs in the experiments. In this way, dependence of re-routing effects on the changes in the OD demand is ignored. Here we focus to investigate effects of travel time variation and congestion spill-back on traffic observations in the network.

It is important to note that, for every gradient step evaluation we execute metamodel instead of DTA traffic simulation; i.e in every inner loop iteration, the metamodel is evaluated instead of running DTA traffic simulation. Thus, in this case study, 200 inner loop iterations have been executed per outer iteration, resulting in 200 metamodel evaluations instead of traffic simulations.

3.3 Prior OD matrix scenarios

To estimate the dynamic OD matrix for a specific day and time period t , the prior matrix turns out to be an important source of information. The prior OD matrices are derived by adding bias and noise to the real OD matrices to create two different scenarios on prior knowledge on traffic demand, that reflects both structural and stochastic daily fluctuations present in within-day travel demands.

The following two scenarios have been designed:

1. **Unbiased scenario:** This scenario is based on the assumption that the prior OD matrix is the best estimate of the mean of the dynamic OD matrices. The prior OD matrices are generated for 100% of the "true" OD demand level and varied by adding uniformly random components in range of +/- 20%, that is:

$$\tilde{d}_{i,j,t} = d_{i,j,t} \times [0.8 + 0.4 \times \alpha_{i,j,t}] \quad \alpha_{i,j,t} \sim U(0, 1) \quad (6)$$

2. **Congested scenario:** This scenario addresses situations where the prior OD matrices reflects travel demand in peak-hours, when congestion occurs in the network. The prior OD matrices are generated for 120% of the "true" OD demand level and varied by adding uniformly random components in range of +/- 20%, that is:

$$\tilde{d}_{i,j,t} = \times d_{i,j,t} \times [0.96 + 0.48 \times \alpha_{i,j,t}] \quad \alpha_{i,j,t} \sim U(0, 1) \quad (7)$$

4 Results

The performance of the selected OD estimation methods corresponding to 2 historical OD demand scenarios is presented in Figures 2 and 3. For the purpose of this study, convergence was defined

as reaching a cost function value within 30 iterations, for all the methods. The first 5 iteration steps have not been shown in order to make the figures more readable. The cost function value is very high at the beginning of the estimation process and does not allow clear visualization of the method's performance differences.

Figure 2: Cost function as a function of the outer iterations in the unbiased scenario.

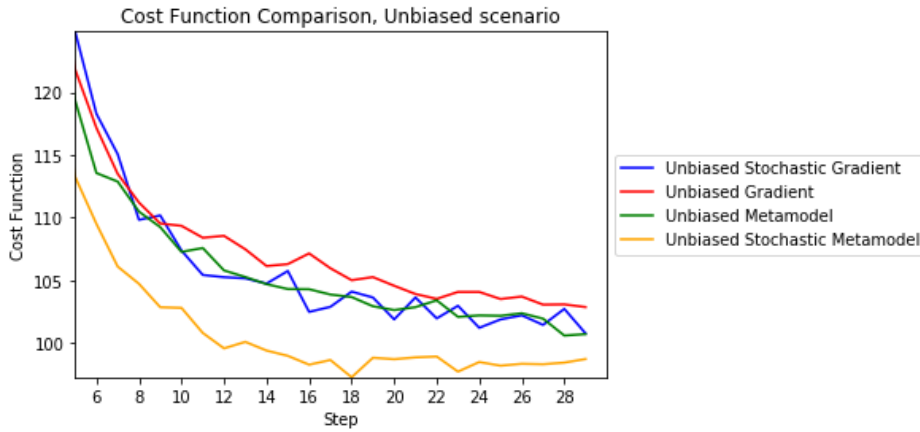
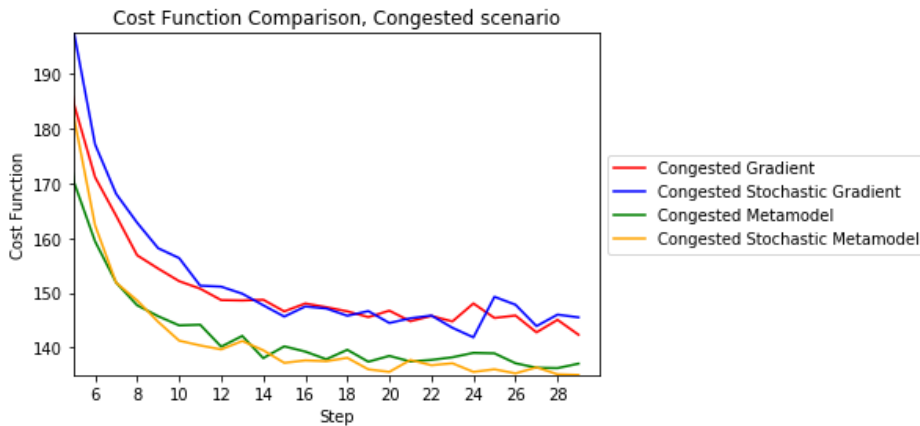


Figure 3: Cost function as a function of the outer iterations in the congested scenario.



The proposed stochastic metamodel method demonstrates better performance, since it is able to maintain decrease of the cost function with the lowest value through iteration steps for both historical demand scenarios. Although, all four methods show similar trend line of the cost function convergence, the classic gradient method is the one that performs the worst. However, the differences in terms of cost function performance are not extraordinary. These methods were not expected to work much better, as they all fundamentally use the assignment matrix to optimize the cost function. However, one may observe that the cost function values of the methods based on metamodel decreased for around 10%. This effect is more evident in the congested demand scenario, where metamodel was designed as a nonlinear function dependent on changes in OD demand to capture the effect of the congestion in the network on traffic counts.

Next, it was important to investigate how estimated OD demands once assigned to network can produce traffic counts close to their real observations. Figures 4 and 5 provides a performance overview of classic gradient solution approach and proposed stochastic metamodel approach in terms of relationship between simulated and observed traffic counts. Black lines show regression fits to the estimated counts and the initial counts. The red line shows the $y = x$ line, where all the points would lay in a perfect estimation.

Figure 4: Real counts vs simulated counts in unbiased scenario

(a) Classic gradient method

(b) Stochastic metamodel method

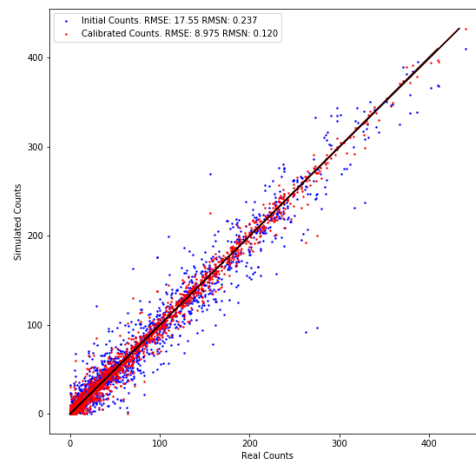
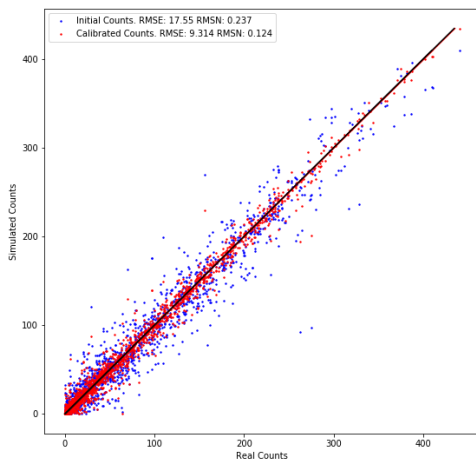
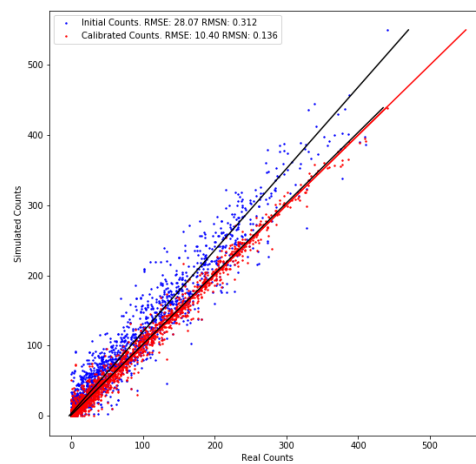
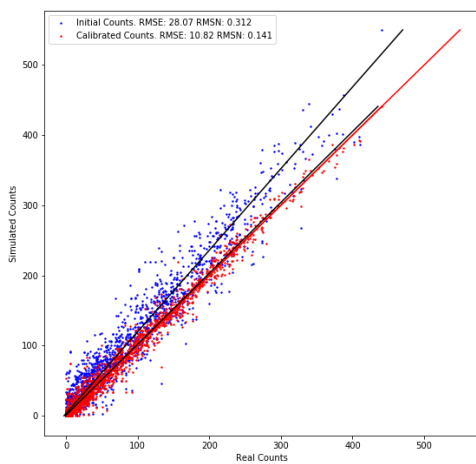


Figure 5: Real counts vs simulated counts in congested scenario

(a) Classic gradient method

(b) Stochastic metamodel method



It appears clearly from both figures that both methods show R-squared value increase and reductions in RMSE and RMSN error with the improvement in range of 50 – 55% compared to initial value before OD demand estimation. But relevant performance differences between methods is hard to observe. This is due to the fact that the cost functions achieved improvement of only 5 – 10% which resulted in slight improvement in traffic counts. To get a full performance overview of analyzed methods in terms of RMSN in estimated counts, the results are summarized in the Table 1. Again, both metamodel and stochastic metamodel methods show the best choice regarding RMSN counts.

Table 1: Counts RMSN of different methods in the two scenarios.

	Classic	Stochastic	Metamodel	Metamodel & Stochastic
Unbiased	12.4 %	12.3 %	12.2 %	12.0 %
Congested	14.1 %	14.0 %	13.7 %	13.6 %

5 Conclusions

This paper proposes a framework to derive metamodel to solve dynamic OD matrix estimation. By choosing different functions, different metamodels can be obtained. The motivation to build this framework is to have metamodels that are consistent with respect to all the simulated demands during the estimation process while maintaining low computational costs. Compared to the classic bi-level iterative methods, the metamodel uses information from all the simulated demands. This feature allows us to build a non-linear metamodel without the need of running additional traffic simulations for function evaluation. This is a relevant consideration, since in dynamic OD matrix estimation the number of iterations and traffic simulation runs is the main constraint.

In addition, a stochastic gradient method has been proposed to update the demand in the gradient iteration steps. The metamodel and stochastic gradient method can be used at the same time, as they are independent. The metamodel with the inverse function and the stochastic method have been compared with the classic gradient approach. When using the metamodel and the stochastic method at the same time, the quality of the solutions improves around a 5% in terms of cost function. The experiments indicate that the proposed method based on metamodel yields to robust performance for both quality of the initial OD matrices, especially during the congested state in the network. An improvement of the method presented in this thesis can be seen in two directions: 1) adaptation of the model when additional data (i.e., speeds, density, demand derived from floating car data) can be considered to improve the quality of the estimated OD demand; and 2) explore alternative gradient solution approaches to avoid convergence in local minima.

Acknowledgements

This research has been performed as part of the SETA project funded by the European Union’s Horizon 2020 Research and Innovation program under the grant agreement No 688082.

References

- Aimsun (2017), *Aimsun Next 8.2 User's Manual*, Aimsun SL, Barcelona, Spain.
- Antoniou, C., Ben-Akiva, M. & Koutsopoulos, H. N. (2006), 'Dynamic traffic demand prediction using conventional and emerging data sources', *Intelligent Transport Systems, IEE Proceedings* **153**(1), 97–104.
- Ashok, K. & Ben-Akiva, M. E. (2000), 'Alternative approaches for real-time estimation and prediction of time-dependent origin-destination flows', *Transportation Science* **34**(1), 21–36.
- Bell, M. G. H. (1991), 'The estimation of origin-destination matrices by constrained generalised least squares', *Transportation Research Part B: Methodological* **25**(1), 13–22.
- Cascetta, E. (1984), 'Estimation of trip matrices from traffic counts and survey data: A generalized least squares estimator', *Transportation Research Part B* **18**(4-5), 289–299.
- Cascetta, E., Papola, A., Marzano, V., Simonelli, F. & Vitiello, I. (2013), 'Quasi-dynamic estimation of o-d flows from traffic counts: formulation, statistical validation and performance analysis on real data', *Transportation Research Part B Methodological* **55**, 171–187.
- Cremer, M. & Keller, H. (1987), 'A new class of dynamic methods for the identification of origin-destination flows', *Transportation Research Part B: Methodological* **21**(2), 117–132.
- Djukic, T., Masip, D., Breen, M., Perarnau, J. & Casas, J. (2018), Heuristic-based framework for dynamic od demand estimation in the congested networks, in 'Transportation Research Board 97th Annual Meeting, Washington DC, United States', pp. 1–15.
- Frederix, R., Viti, F. & Tampère, C. M. J. (2013), 'Dynamic origin-destination estimation in congested networks: theoretical findings and implications in practice', *Transportmetrica A: Transport Science* **9**(6), 494–513.
- Gardner, W. A. (1984), 'Learning characteristics of stochastic-gradient-descent algorithms: A general study, analysis, and critique', *Signal Processing* .
- Li, M., Zhang, T., Chen, Y. & Smola, A. J. (2014), Efficient mini-batch training for stochastic optimization, in 'Proceedings of the 20th ACM SIGKDD international conference on Knowledge discovery and data mining - KDD '14', pp. 661–670.
URL: <http://dl.acm.org/citation.cfm?doid=2623330.2623612>
- Osorio, C. (2017), 'High-dimensional offline OD calibration for stochastic traffic simulators of large-scale urban networks'.
- Shafiei, S., Saberi, M., Zockaie, A. & Sarvi, M. (2017), 'A Sensitivity-Based Linear Approximation Method to Estimate Time-Dependent Origin-Destination Demand in Congested Networks', *Proceedings of Transportation Research Board - 96th Annual Meeting* (Washington D.C.), 1–16.
- Toledo, T. & Kolehkina, T. (2013), 'Estimation of dynamic origin-destination matrices using linear assignment matrix approximations', *IEEE Transactions on Intelligent Transportation Systems* .

- Van Zuylen, H. J. & Willumsen, L. G. (1980), 'The most likely trip matrix estimated from traffic counts', *Transportation Research Part B: Methodological* .
- Yang, H., Sasaki, T., Iida, Y. & Asakura, Y. (1992), 'Estimation of origin-destination matrices from link traffic counts on congested networks', *Transportation Research Part B: Methodological* **26**(6), 417–434.
- Zhou, X. & Mahmassani, H. S. (2007), 'A structural state space model for real-time traffic origin-destination demand estimation and prediction in a day-to-day learning framework', *Transportation Research Part B: Methodological* **41**(8), 823–840.