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Modelling Traffic Assignment Objectives with Emission Cost Functions

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Abstract

Vehicle emissions make up a significant proportion of greenhouse gases, which has provoked interest in capturing emissions in traffic models. This paper provides methods of modelling vehicle emissions for specific emission types in the context of traffic assignment to provide traffic flow patterns with minimal vehicle emissions. Speed limits are incorporated to enable the identification of the minimum total vehicle emissions for a traffic network with given fixed demand. Using a method of emission costing, such as CO2-equivalent or health risk values, similar approaches can be applied to an objective function with a weighted combination of emissions, given a general link cost function that captures a range of emissions. These methods are applied to an example network to indicate the differences in emissions for different flow patterns. Bounds on the lowest emissions possible, for given traffic demand, for each emission type are stated.

1. Introduction

The Traffic Assignment (TA) Problem is commonly applied in the context of modelling the flow of road users across a network with respect to travel time or so-called generalised cost on each link. Recently, there has been increased awareness concerning greenhouse gases and associated climate change, with emissions from transport in New Zealand making up approximately 20% of the country's total greenhouse gas (GHG) emissions and over 40% of the emissions from the national energy sector (Ministry of Transport, 2017).

High transport emissions have provoked interest in considering networks with emissions as a link cost to identify the extent to which emissions can be reduced, especially in the context of the Vehicle Routing Problem (VRP) (Demir, Bektas, Laporte 2014a, Zhang et al 2015). Often solely fuel consumption is used as a link cost due to its correlation with various emissions and direct cost to vehicle users in the form of generalised cost functions. In this respect the freight transport industry has motivation to investigate fuel consumption minimisation to reduce company costs, and is seen as a focus in several papers (Kellner, 2016, Demir, Bektas, Laporte 2014b).

This paper investigates the implementation of specific vehicle emission functions as generalised costs over a given network. This is done to provide insight into emission reduction potential in relation to existing traffic flow patterns for given fixed demand.

Two TA solution classes are of interest; the user equilibrium (UE), where each network 'user' selfishly minimises their personal objective to minimise their travel cost. In a UE solution all used paths for one origin-destination pair have equal cost (and there are no unused paths with lower cost) according to the final UE traffic flow. The system-optimal (SO) solution seeks to minimise the overall cost of travel across the network. These TA solution classes follow Wardrop's first and second principles, respectively (Wardrop, 1952).

TA models assume that all network users have perfect knowledge of their available choices, in particular generalised link costs. Therefore, a UE with emission costs assumes that all users have complete understanding of the emissions cost on each link, which would require some form of route planning aid to provide necessary information, and depend heavily on the users' environmental concerns. The application of tolls on links in a road network as a means to shift a UE to a SO flow solution is well known (e.g. Lindsey and Verhoef 2001), and has been considered by Raith, Thielen, and Tidswell, (2016), where fuel-optimised SO solutions are the main focus.

We use emission functions as proposed in Song et al (2013). Different emission types have the same base function, but with different parameters for each type of emission, and additionally for fuel consumption. This paper extends our initial research which considered fuel consumption only (Raith, Thielen, and Tidswell, 2016).

Link cost functions modelling different emission types of Hydrocarbons (HC), Nitrous Oxides (NOx), Carbon Monoxide (CO), and Carbon Dioxide (CO2) will be examined in this paper, in addition to Fuel Consumption (FC) (Song et al 2013).

The emission link cost functions are not strictly increasing functions of traffic flow – an assumption necessary to ensure uniqueness of the UE/SO solution (Patriksson, 1994). Incorporating speed limits allows the computation of a SO solution with respect to emissions, where each emission type has its individual optimal speed producing minimal amounts of the respective emission per kilometre travelled. However, these optimal speeds may not be practical, taking values that may be too low or uncommon for regular speed limits imposed.

2. Model formulation

2.1. Traffic assignment model

2.1.1 Emissions model classification

There are various methods of modelling vehicle emissions, with a range of complexities involved due to the different factors that affect emissions to consider. The models can generally be split into macroscopic or microscopic models. Several models are subsequently mentioned, however, Demir, Bektas, and Laporte (2014b), and Zhou, Jin, and Wang (2016) provide an overview of specific emissions models and references.

Macroscopic traffic models, such as the TA model used as a basis for this paper, use the average speed of a vehicle to estimate the system-wide cost. Often these models are based on measurements and on-road experiments for a range of vehicles, with the aim of creating a speed-dependent regression function. Regression functions can be created for specific vehicle classes (often categorised by weight), and vary by the parameters considered, such as road gradient, vehicle load, and cold starts. Notable models are derived from COPERT, MOBILE, and HBEFA, where differences are generally due to regional differences in vehicle fleets, year of creation, and specific regression function structures. The Vehicle Emissions Prediction Model (VEPM) is a New Zealand (NZ) model, based on the European COPERT and UK NAEI, calibrated to the NZ vehicle fleet (New Zealand Transport Agency, 2013).

Microscopic models use variables such as instantaneous speed and acceleration to estimate emissions, along with parameters of air density, drag coefficients, and vehicle weight. These models require much more detailed information than the macroscopic models, and can be more complex as a result.

The link cost functions in the context of TA, as in this paper, can only rely on the average speed of the vehicle, with predefined cost function parameters tuned specifically for the respective cost, such as fuel consumption or CO2 emissions. The link functions as defined by Song et al (2013), are derived from emission data collected by Portable Emission Measurement System (PEMS). Parameters exist for emissions of Hydrocarbons (HC), Nitrous Oxides (NOx), Carbon Monoxide (CO), and Carbon Dioxide (CO2), as well as for Fuel Consumption (FC).

Other emission link cost functions similar to Song et al (2013) exist, which also solely rely on average vehicle speed and have the same general shape. TRANSYT-7F (Wallace et al, 1998, Benedek and Rilett, 1998) is an exponential link cost function, and increases at a much slower rate for high average speeds when compared with the Song et al (2013) link cost function. In Sugawara and Niemeier (2002) the link cost function is an exponential of a polynomial, with a sharp increase in emission cost past average speeds of 100km/hr. The Song et al (2013) link cost function was chosen due to its realistic costs at high average speeds, as well as a more recent publication. The analysis presented in the following can be adapted to use any emission link cost function, which is a convex function of average speed.

2.1.2. Limitations

The Song et al (2013) model for estimating emissions does not account for a range of potentially significant factors, examples being road grade, driver behaviour, and notably congestion effects. Although congestion effects are described partially by the travel time function proposed by the Bureau of Public Roads (BPR) (1964), the start-stop behaviour common in congested flow is not entirely captured. This congested flow was shown to have a large effect on emissions when compared to free flow traffic (Greenwood, Dunn, and Raine, 2007). Ideally, a separate measure of congestion would be incorporated into an emissions function, such as the volume over capacity (VOC) ratio, or speed over speed limit as in Borge et al (2010), generally requiring link-specific discrete classifications of congestion intensity.

Despite these issues, TA link cost functions rely on the use of average speed, with the implemented emission link functions providing an acceptable estimate on vehicle emissions given the degree of information available for general networks.

2.2. Traffic assignment

2.2.1. Background

The road transport network is represented by a directed graph $G = (V, A)$ for nodes V and links A. Also required are origin-destination (OD) pairs K, with $\{(s_k, t_k, d_k)\}_{k=1}^{K}$ where $s_k, t_k \in V$ are the origin and destination nodes of OD pair k respectively, and $d_k \in \mathbb{R}^+$ is the demand of the OD pair k. Let f_i be the flow, or number of vehicles, on link $i \in A$ and h_r be the path flow for OD pair s_k, t_k for path $r \in R_k$, where R_k is the set of all simple paths for OD pair s_k, t_k, such that

$$f_i = \sum_{k \in K} \sum_{r \in R_k} h_r \delta_i^r \qquad \forall\, i \in A,$$

where $\delta_i^r = 1$ if link i belongs to path r, and $\delta_i^r = 0$ otherwise. Each link flow f_i is the sum of all path flows for all OD pairs traversing link i.

This gives rise to the following optimisation problem with link cost function c_i, a solution which satisfies UE:

$$\min \sum_{i \in A} \int_0^{f_i} c_i(x)dx$$

$$s.t. \sum_{r \in R_k} h_r = d_k, \qquad \forall\, k \in K,$$

$$h_r \geq 0, \qquad \forall\, k \in K, r \in R_k,$$

$$f_i = \sum_{k \in K} \sum_{r \in R_k} h_r \delta_i^r\,, \qquad \forall\, i \in A,$$

which includes constraints to meet demand, and that path flows are non-negative. It is assumed that link cost functions are positive and continuous to ensure the existence of a TA solution (Partiksson, 1994).

The SO minimisation problem can be defined by changing the objective function to:

$$\min \sum_{i \in A} f_i c_i(f_i).$$

The link cost function may take the form of a flow-dependent travel time function to obtain a travel time solution for the network, or of a vehicle emission function, as introduced in the next section, which is the primary focus of this paper.

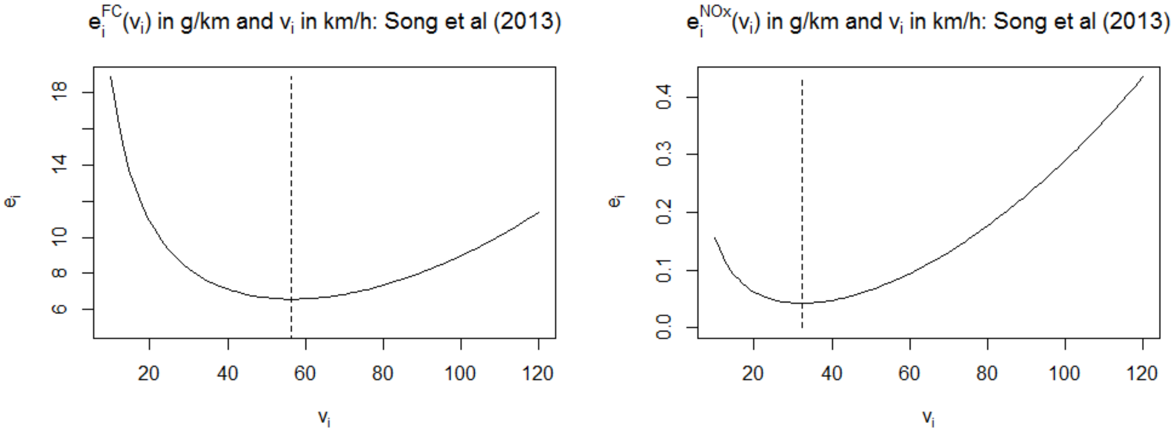
2.2.2. Link cost functions

Emissions per kilometre on each link i are calculated using an emissions model based on average speed developed by Song et al (2013), which take the form of $e_i^j(f_i)$ in g/km for emission j:

$$e_i^j(f_i) = e_i^j(v_i(f_i)) = \frac{a^j}{v_i(f_i)} + b^j + c^j v_i(f_i) + d^j (v_i(f_i))^2\,,$$

where $v_i(f_i)$ is the average speed in km/hr on link i and is a function of f_i, flow on link i, and $v_i \equiv v_i(f_i)$ for ease of notation. Parameters a^j, b^j, c^j, d^j are calibrated for FC, HC, NOx, CO, and CO2 emissions, as well as fuel consumption, with $a^j, d^j > 0$ for $j \in E = \{FC, HC, NOx, CO, CO2\}$. Under this assumption, the emission link functions with respect to speed are continuous, non-negative, and strictly convex, but notably not convex with respect to flow f_i. Additionally, there exists a unique speed $v^{j,opt} > 0$ which for all links i minimises emissions per kilometre e_i^j on the respective link, as $e_i^j(v_i) \rightarrow +\infty$ for $v_i \rightarrow 0$ and for $v_i \rightarrow +\infty$.

Figure 1: Functions $e_i^{FC}(v_i)$ and $e_i^{NOx}(v_i)$ with different minima $v^{FC,opt}, v^{NOx,opt}$ (dotted) corresponding to different parameters a^j, b^j, c^j, d^j.



Example costs with respect to average speed are depicted in Figure 1, for fuel consumption (FC) and emission type NOx.

The calibration of parameters a^j, b^j, c^j, d^j accounts for whether the vehicle is heavy or light. It is assumed that the majority of traffic consists of light passenger vehicles, and that the light vehicle emissions models, as described by Song et al (2013), are appropriate in this case.

The average speed v_i is derived from the length s_i of link i and its average travel time t_i, as defined by the travel time function proposed by the Bureau of Public Roads (1964):

$$t_i(f_i) = t_i(0) \cdot \left(1 + \alpha_i \left(\frac{f_i}{k_i}\right)^{\beta_i}\right),$$

$$v_i(t_i(f_i)) = \frac{s_i}{t_i(f_i)},$$

$$v_i(f_i) = \frac{s_i}{t_i(0) \cdot \left(1 + \alpha_i \left(\frac{f_i}{k_i}\right)^{\beta_i}\right)},$$

where $t_i(0)$ is the free flow time, f_i is the number of vehicles, or flow, k_i is the practical capacity, and α_i, β_i are positive parameters defining the intensity of congestion effects on the travel time, all with respect to link i.

It is assumed that and α_i, β_i are positive and non-zero, $t_i(f_i)$ is a continuous, monotonic, strictly increasing function for all $f_i \geq 0$. This ensures that the standard TA problem with respect to travel time can be solved to obtain a unique solution with respect to link flows (Sheffi, 1985).

Emission functions as a function of average speed are not monotonic, with low and high speeds producing high emissions (see Figure 1), and also non-monotonic with respect to flow, resulting in no guarantee in being able to uniquely identify UE or SO traffic patterns when solving TA with standard emission link functions.

2.2.3. Link cost functions with speed limit

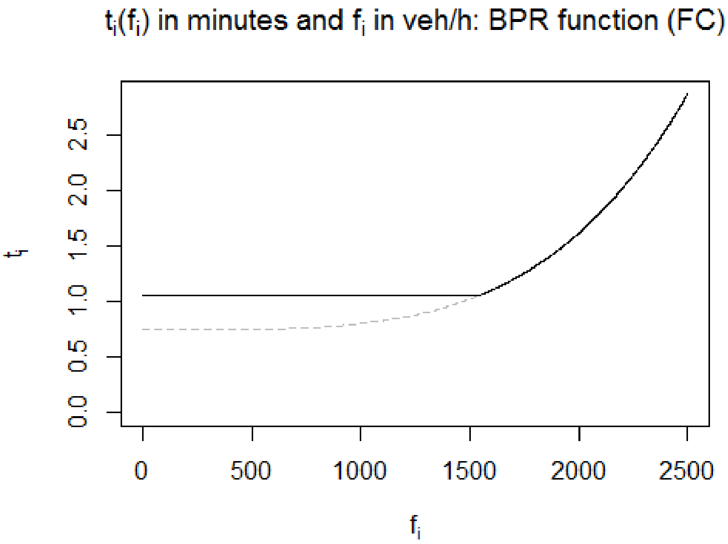
If the speed limit on link i is defined as v_i^{max}, where $v_i^{max} > 0$ can be chosen arbitrarily, then the limited average speed on the link becomes

$$\tilde{v}_i(f_i) = \min\{v_i(f_i), v_i^{max}\},$$

with $\tilde{v}_i \equiv \tilde{v}_i(f_i)$, and the corresponding limited travel time, as shown in Figure 2, is:

$$\tilde{t}_i(f_i) = \max\left\{\frac{s_a}{v_i^{max}}, t_i(f_a)\right\}.$$

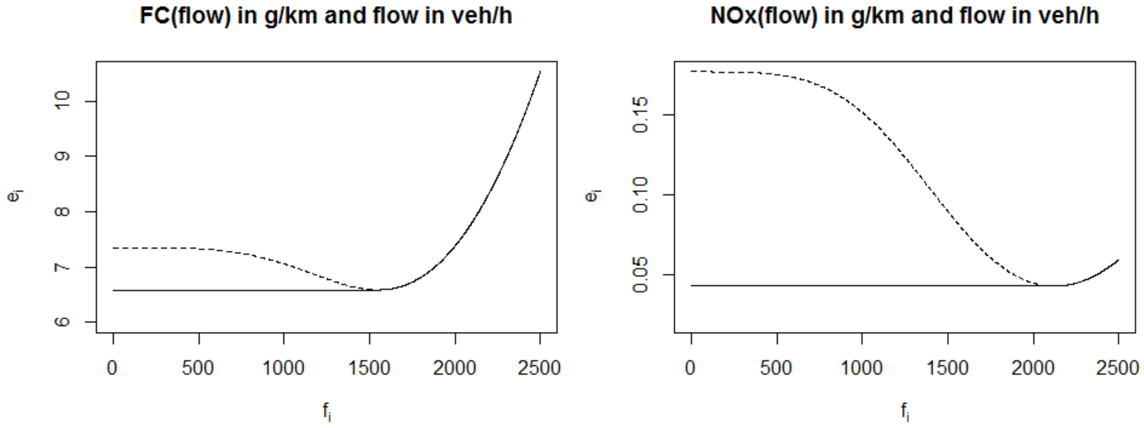
Figure 2: Function $t_i(f_i)$ (dotted), with optimal speed limit for FC, $\tilde{t}_i(f_i)$ (solid)



If v_i^{max} is set to the optimal speed $v^{j,opt}$, where $v^{j,opt}$ is the unique speed that minimises the corresponding emissions function e_i^j given a set of parameters a^j, b^j, c^j, d^j, let

$$\hat{e}_i^j(f_i) = e_i^j(\min\{v_i(f_i), v^{j,opt}\})$$

Figure 3: Functions $e_i^{FC}(f_i)$ and $e_i^{NOx}(f_i)$ (dotted), and $\hat{e}_i^{FC}(f_i)$ and $\hat{e}_i^{NOx}(f_i)$ with $v^{max} = v^{j,opt}$ (solid) for $j \in \{FC, NOx\}$



Where $\hat{e}_i^j(f_i)$ represents emissions per kilometre of emission type j on link i with an optimal speed limit for the emission of $v^{j,opt}$, and is an increasing function with respect to flow (although not strictly increasing). Examples are the solid lines in Figure 3.

This allows the definition of a total emissions function with speed limits for emission type j, for the network instance, denoted C_T:

$$C_T = \sum_{i \in A} f_i\, s_i \hat{e}_i^j(f_i).$$

Solving TA with objective C_T identifies the SO solution of TA with respect to emission type j under the assumption that travel speed on link i can be limited to a maximal speed $v_i^{j,opt}$. The solution found will have an optimal objective function value, but it will not necessarily be a unique solution in terms of link flow due to the plateau in the link cost function shown in Figure and Figure. If ${v_i}^{max} > \frac{s_i}{t_i(0)} = v_i(0)$, where $v_i(0)$ is the free flow speed, then the speed limit constraint will not be active, as average speed is already bounded by the free flow speed as a maximum. Yang et al (2012) study the impact of speed limits in TA, discuss solution properties, and comment on the ability to enforce traffic patterns, and the effects of speed limits may have on total emissions and travel times, although without attempting to minimise emissions.

2.2.4. Combination of emissions

The formulation in Section 2.2.3 allows the identification of a SO solution with respect to individual types of emissions. In practice, all types of emissions are generated by travelling vehicles, and hence need to be considered simultaneously.

Given the link cost function of the form:

$$e_i^j(f_i) = e_i^j(v_i(f_i)) = \frac{a^j}{v_i} + b^j + c^j v_i + d^j v_i^2,$$

with $v_i \equiv v_i(f_i)$, a weighted emissions function can be defined as follows.

Let $\bar{a} = \sum_j w^j\, a^j$, and similar for parameters b, c, d with non-negative weights w^j for $j \in E$

Then let

$$\bar{e}_i(v_i(f_i)) = \sum_{j \in E} w^j\, e_i^j(v_i(f_i)) = \frac{\bar{a}}{v_i} + \bar{b} + \bar{c} v_i + \bar{d} v_i^2.$$

The overall emissions function $\bar{e}_i(f_i)$ is convex as it is a (non-negative) weighted sum of the individual emissions functions, which are convex. Therefore, $\bar{e}_i(v_i)$ has a unique optimal speed $\bar{v}^{opt}$ which allows the definition of a total emissions function with speed limit for a weighted combination of emissions:

$$\bar{C}_T = f_i s_i \sum_{i \in A} \hat{\bar{e}}_i\,(f_i),$$

with

$$\hat{\bar{e}}_i(f_i) = \bar{e}_i(\min\{v_i(f_i), \bar{v}^{opt}\}).$$

Solving TA with objective $\bar{C}_T$ identifies the SO solution of TA with respect to overall emissions. Therefore, it identifies the minimum total system-wide emissions for the network instance, if travel speed can be controlled by speed limits.

The formulation of $\bar{C}_T$ requires suitable weightings of each emission type, for example via their Global Warming Potential values (GWP) (IPCC, 2007) or corresponding CO2-equivalent values. However, emissions such as CO and NOx do not have well defined GWP due to their indirect global warming effects (Gillenwater, Van Pelt, Peterson, 2002). A further method is to use estimations of health effects from each emission type, and assign costs through these estimations, such as the ones stated in the last column of Table 1 from Bigazzi and Figliozzi (2013). Table 1 also states the values of parameters a^j, b^j, c^j, d^j for each emission type j as well as the emissions per kilometre at the respective optimal speed to provide a sense of scale for the emission type.

Table 1: Parameters a^j, b^j, c^j, d^j *for each emission type (Song et al, 2013), with optimal speed for minimal respective emission. Minimum emission* $e(v^{j,opt})$ *indicates the scale of each emission for a single vehicle, and costing per kg of emission is provided (Bigazzi and Figliozzi, 2013), in US$/kg, using the stated 'medium' cost. Note: FC parameters have been adjusted from values by Song et al (2013) by a factor of 10 to obtain more realistic FC values.*

Objective (g/km)	a	b	c	d	$v^{j,opt}$ (km/hr)	$e(v^{j,opt})$ (g)	Cost ($/kg)
Fuel consumption (FC)	1.56×10^{3}	3.54×10^{1}	-3.88×10^{-1}	7.76×10^{-3}	56.494	65.86	-
Hydrocarbons (HC)	1.08×10^{1}	-7.11×10^{-3}	3.76×10^{-4}	3.63×10^{-5}	51.315	0.32	12.91
Nitrous Oxides (NOx)	2.00×10^{0}	-4.49×10^{-2}	-3.36×10^{-4}	3.49×10^{-5}	32.292	0.04	14.54
Carbon Monoxide (CO)	8.08×10^{1}	1.16×10^{0}	5.03×10^{-3}	5.35×10^{-4}	40.757	4.24	0.37
Carbon Dioxide (CO2)	4.78×10^{3}	1.11×10^{2}	-1.24×10^{0}	2.37×10^{-2}	57.095	201.10	0.02

3. Case study

The following case study addresses the transport network of Birmingham, England, one of the TA instances available at the web-site https://github.com/bstabler/TransportationNetworks along with a number of other networks in similar format. This network was chosen due to the relatively large size and clearly identified units. The instance has 14,639 nodes, 33,937 links, and 898 zones (origins/destinations).

The TA optimisation problems are solved using the Traffic Assignment frameworK (TAsK), implemented by Perederieieva et al (2015). The solver was configured to use the so-called 'Algorithm B' and A* shortest path algorithm out of the available options, and was adapted to allow modelling of emission link cost functions and the ability to compute SO solutions. It should be noted that both UE and SO models can be stated as equilibrium problems, where the UE has generalised costs as link cost functions, and the SO has link cost functions made up of generalised costs and its derivative (the marginal cost to reflect how each additional road user affects all other link users' travel cost) (Sheffi, 1985).

The network was solved to optimality for each emission type (and fuel consumption) to a precision of 10^{-6} (relative gap measure of convergence) with a corresponding optimal speed limit in place. Results for the UE and SO solutions with respect to travel time are also included as a reference point, where the UE solution has more probable real-life flows and costs, as opposed to the idealistic other solutions. The problem with each emission type considered individually was solved using its respective optimal speed limit, with the exception of the travel time (TT) objectives (no speed limit).

Table 2: Results for objectives using parameters from Table 1, where the notation TT corresponds to a travel time link cost (such that UE-TT is the user equilibrium solution for minimising travel time, SO-TT is the system-optimal solution for minimising travel time). Each total objective cost is stated relative to the minimal objective (highlighted) in each column (where the SO-CO total travel time is 166.62% the total travel time of the SO-TT solution). The final column, EM, represents the weighted emissions, with weightings from Table 1.

Objective	$v^{j,opt}$ **(km/hr)**	**Total objective cost (%)**						
		Travel time	**Fuel cons.**	**HC**	**NOx**	**CO**	**CO2**	**EM**
UE-TT	-	100.99	120.79	132.54	443.47	151.37	120.05	142.75
SO-TT	-	100.00	120.58	132.35	445.40	151.33	119.83	142.69
SO-FC	56.494	130.17	100.00	100.59	173.05	106.16	100.01	100.10
SO-HC	51.315	138.05	100.46	100.00	151.00	103.32	100.57	100.06
SO-NOx	32.292	205.43	117.53	116.59	100.00	103.19	118.13	111.00
SO-CO	40.757	166.62	106.00	104.21	112.32	100.00	106.36	101.84
SO-CO2	57.095	129.32	100.01	100.72	175.85	106.55	100.00	100.12
SO-EM	47.504	146.29	101.58	100.45	135.12	101.49	101.77	100.00

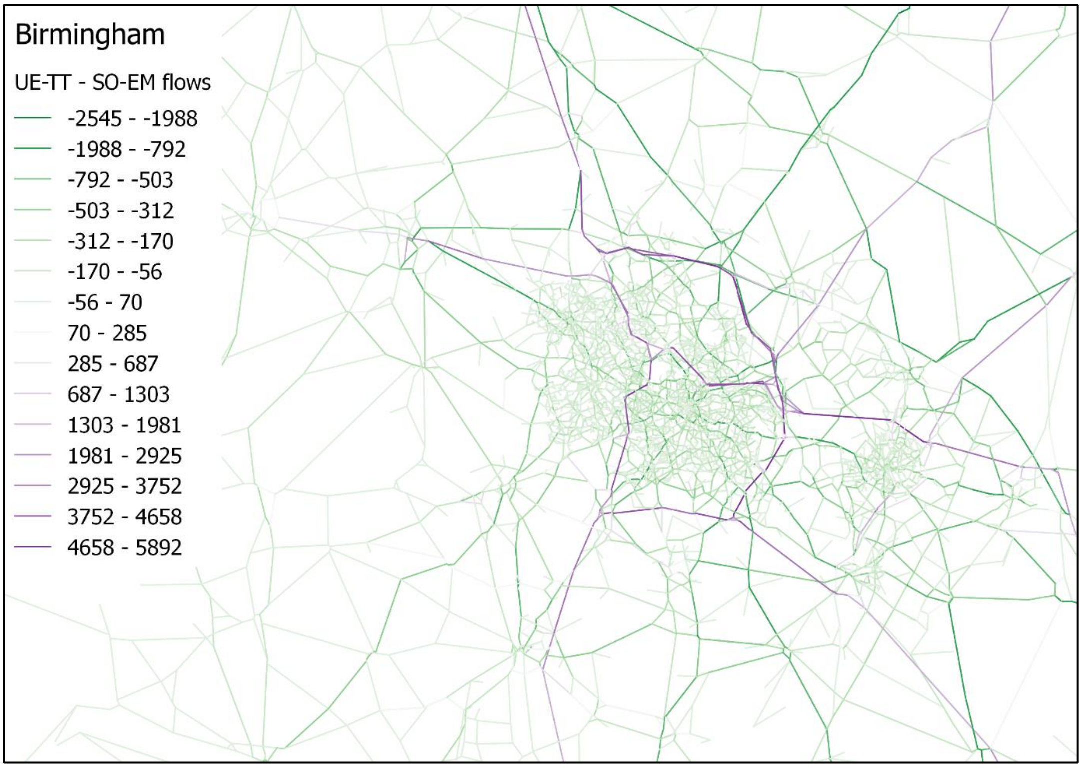
Summary results can be seen in Table 2, which are relative to the best total objective, the highlighted cell, in each column. UE-TT denotes the user equilibrium solution for travel time, SO-FC the system-optimal solution for fuel consumption, and so on, with each objective cost calculated from the resulting flows. The final row, SO-EM, represents the system-optimal solution for emissions with weightings from Table 1, with a speed limit calculated from the weighted parameters $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ as previously stated. Table 3 represents the same data as in Table 2, but relative to the UE-TT solution rather than the best total objective.

All emission objectives were found to have a much higher travel time than the UE-TT flow solution (seen as the base case in row 1), which can be partially attributed to the original high speeds in the UE-TT solution being restricted by the speed limit in the emission minimisation solutions, as well as a general increase in congestion due to re-routing of traffic flow. Considering the SO-EM solution, all emissions are found to be kept low relative to their optimal as in Table 2, except for NOx emissions. This can be largely attributed to the significantly different optimal speed limits for each emission type, where the optimal speed for NOx is much lower than the other emissions. It is also worth noting the similarity in total EM objective cost between the SO-FC and SO-EM solutions, despite FC not being included in the objective.

Table 3: Similar to Table 2, displaying objectives for each solution, instead relative to the user equilibrium solution.

Objective	$v^{j,opt}$ (km/hr)	Total objective cost (%)						
		Travel time	Fuel cons.	HC	NOx	CO	CO2	EM
UE-TT	-	100.00	100.00	100.00	100.00	100.00	100.00	100.00
SO-TT	-	99.02	99.83	99.86	100.44	99.97	99.82	99.96
SO-FC	56.494	128.89	82.79	75.89	39.02	70.13	83.31	70.13
SO-HC	51.315	136.69	83.17	75.45	34.05	68.26	83.77	70.09
SO-NOx	32.292	203.41	97.31	87.97	22.55	68.17	98.40	77.76
SO-CO	40.757	164.98	87.75	78.63	25.33	66.06	88.60	71.34
SO-CO2	57.095	128.05	82.79	76.00	39.65	70.39	83.30	70.14
SO-EM	47.504	144.85	84.10	75.79	30.47	67.05	84.78	70.05

Figure 2: Birmingham, England network displaying flow differences between the UE-TT and SO-EM solutions.



An example of flow difference between UE-TT and SO-EM solutions is shown in Figure 4, where the purple links carry higher flow in the user equilibrium solution for travel time (UE-TT), and the green links higher flow in the system-optimal solution (SO-EM) for a combination of weighted emissions, with weightings in Table 1. Overall the flow in the UE-TT has been shifted from some major links, such as those in purple, to the remaining roads, in green, in the SO-EM solutions to obtain a lower overall emissions cost.

4. Conclusions

A TA model with emission objectives for light vehicles has been examined and applied to a network. Different objectives when solving the TA problem have been shown to produce different flow patterns that minimise corresponding emissions, with an increase in other costs (other emissions types or travel time). A weighted sum of emissions is an alternative objective which captures all emission types, and provides a minimal cost solution with respect to the weightings. The implementation of speed limits allows the calculation of a SO flow pattern which can be interpreted as a lower bound on the corresponding objective, whether that is a single emission or a weighted combination of emissions. Given fixed traffic demand, a traffic pattern, usually assumed to follow the UE principle, cannot obtain better emissions than the obtained SO solutions for respective or weighted emissions.

Further investigation could be made into the effects of rounding speed limits to more realistic values, and the selection of a subset of links to apply a speed limit to, rather than all links. The use of alternative emissions functions, for instance derived from the NZ VEPM database, or functions that better capture congestion effects, could be examined. Accounting for multiple vehicle classes with differing link emission functions is a further possibility, where especially heavy vehicles should be modelled as their emissions are relatively high when compared with light vehicles. Finally, being able to solve the TA problems optimising emissions without speed limits would be of interest, which means solving problems with link costs functions that are not increasing. In this case it would be necessary to develop a heuristic approach to identify a good solution of the non-convex SO problem.

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