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## Abstract

Traditionally traffic demand models require as input the impedance of a demand with respect to the network supply; mode choice or departure choice for example, take into account the travel time for each option. Bearing this in mind, the main criticism of using static models to evaluate travel times is that the estimated travel time could diverge considerably because these models have no capacity constraints. On the other hand, dynamic models, such as mesoscopic models, have a level of detail that is sometimes unnecessarily high for the final requirements. The Quasi-dynamic model developed in Aimsun could contribute to a more realistic estimate of the travel time while avoiding the need for a full dynamic model.

This paper presents the integration of a Quasi-dynamic model inside the integrated framework of Aimsun and evaluates a comparison of all models in terms of travel time estimation. The evaluation is performed using real networks validated with real data sets.

## 1. Introduction

The strategic transport models are tools for predicting travel demand and traffic conditions for future scenarios. They exist for every major metropolitan area and serve as a support tool for making decisions concerning transport systems.

The traditional way to structure strategic transport model is the four-stage model (Ortúzar and Willumsen, 20001). It has two main components: the demand model and the supply model.

The demand model includes the first three stages:

- Trip generation (or trip frequency) estimates the total amount of daily trips
- Distribution model distributes the trips of previous stages between origin and destination zones.
- Modal split determines the origin-destination matrix, divided into different transport modes.

The traffic assignment or supply model is the process of determining how the traffic demand, usually defined in terms of an origin-destination matrix, is loaded into a road network, providing for the computation of traffic flows through the network links. The underlying hypothesis is that vehicles travel from origin to destination in the network along the available routes connecting them. The characteristics of a traffic assignment procedure are determined by the hypothesis on how vehicles use the routes. The main modelling hypothesis is the concept of user equilibrium, which states that vehicles try to minimize their individual travel times, that is, drivers choose the routes that they perceive as the quickest under the prevailing traffic conditions. This modelling hypothesis is formulated in terms of Wardrop's first principle: *The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.*

Traffic assignment models based on this principle are known as user equilibrium models as opposed to models in which the objective is to optimise the total system travel time independently of individual preferences - for details see either (Sheffi 1985), or (Florian and Hearn 1995)

The supply model explicitly describes the interaction of the travel demand with the infrastructure and transport services in terms of supply impedance or cost, such as travel time, delay time, etc. Traffic simulation is an example of a supply model where the traffic demand and infrastructure is an input, and the traffic simulation computes flow, speed, travel time on the level of links, paths and network.

General simulation-based approaches (Tong and Wong 2000), (Lo and Szeto 2002), (Varia and Dingra 2004), (Liu et al. 2005) explicitly or implicitly split the process into two parts: a route-choice mechanism determining how the time-dependent flows are assigned to the available paths for each time step and the method to determine how these flows propagate in the network. A systematic approach based on these two components was proposed by (Florian et al. 2001) within the scope of dynamic traffic assignment.

Focussing on supply models or traffic assignment models, the classification could be by i) time representation, ii) resolution and iii) uncertainty.

The classification of the traffic assignment models in terms of time representation could be:

- Static model: there is no time representation, and all processes are instantaneous. This model could be interpreted as representing a specific time interval (peak hour, non peak hour) where stationary conditions are taken into account.
- Dynamic model: there is an explicit representation of the time and the temporal relationship between all processes included in the transport system. The outcome is a time-dependant evolution of congestions and delays.
- Quasi-dynamic: it represents models between static and dynamic models that uses a set of static models that are temporally linked. There is a simplification of the time representation.

The classification regarding their resolution could be:

- Microscopic: the dynamic of network loading process is modelled by an explicit representation of all vehicles (particles) and physical elements of the supply (lane based).
- Macroscopic: the dynamic of network loading process is represented by aggregated traffic variables: flow, density and speed
- Mesoscopic: represents models between microscopic and macroscopic which are characterized by a kind of aggregation of the microscopic model.

And finally, the classification of transport models according to their uncertainty:

- Deterministic: the model does not consider uncertainty and attempts to represent average conditions.
- Stochastic: the model considers the uncertainty (as imperfect modelling) using simulation with a pseudo-random number generation process for evaluating input distributions.

This paper compares traffic assignment models for strategic transport planning analysis, focussing on the network loading process. The current state is the use of static assignment models, whose main drawback is the unrealistic outcomes; dynamic models are more realistic but suffer from potential computation time limitations, especially when they are involve in an iterative process with a travel demand model.

This paper compares different network loading models implemented in Aimsun: a) Static assignment, a macroscopic and deterministic model where the travel time of the link is computed by a volume-delay function without capacity constraints; b) the quasi-dynamic model is a macroscopic model capacity constrained with explicit representation of vertical queues and c) the mesoscopic model, a dynamic and stochastic model.

Finally, the aim of this paper is to describe the added value of the quasi-dynamic model compared to a traditional static model without capacity constraints

## 2. Model Description

### 2.1 Static model

Traditional static assignment models adopt the original formulation of (Beckmann et al, 1956) and the key component in this process are the volume-delay, or link cost functions,  $s_a(v_a)$  that model the travel time on the link as a function of the traffic volume on the link, modeling in this way the congestion effects. Many alternative forms have been proposed for the volume delay functions, from the BPR (Bureau of Public Roads, 1964) seminal one:

$$s_a(v_a) = t_0 \left[ 1 + \alpha_a \left( \frac{v_a}{c_a} \right)^{\beta_a} \right]$$

where  $v_a$  is the volume on link  $a$ ,  $t_0$  is the free-flow time, and  $\alpha_a$  and  $\beta_a$  are calibration parameters of the function for link  $a$ , to more sophisticated, like those proposed by Florian (Florian and Nguyen, 1976):

$$s_a(v_a) = d_a \left\{ \delta + \alpha \left( \frac{v_a}{l_a} - \gamma \right) + \left( \alpha^2 \left( \frac{v_a}{l_a} - \gamma \right)^2 + \beta \right)^{1/2} \right\}$$

where  $d_a$  is the length of the link,  $l_a$  the number of lanes of the link and  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are constants whose values are determined by the calibration of the model.

### 2.2 Quasi-dynamic model

Quasi-dynamic simulation takes ingredients from static macroscopic simulation and the fundamental diagram to improve the accuracy of static macroscopic simulation. The main improvements are:

- Quasi-dynamic simulation is link capacity constrained.
- Travel times are calculated using the fundamental diagram, so they are more realistic than those from static macroscopic simulations.
- Quasi-dynamic simulation is based on directly observable link parameters: maximum speed, maximum capacity and traffic jam density.

In a first stage, flow assignment is calculated independently of travel times, that is, the static macroscopic vehicles are assumed to travel instantaneously. Then, capacity reduction from downstream links or nodes, determines congested links.

Quasi-dynamic simulation has two main components: the nodal model solution and the flow assignment. Consider a particular node: the solution of the nodal model depends on the upstream flows that arrive at that node. On the other hand, flows leaving the node affect other downstream nodes.

To solve this dependency, quasi-dynamic simulation proceeds in the following iterative manner to find equilibrium:

- Calculate assigned flows using turn proportions from the nodal models.
- Calculate turn proportions, i.e. solve the nodal model, with the incoming flows.

The first iteration is just an assignment that doesn't take into account link capacities. After the first iteration, nodal models ensure that link capacities are respected in outgoing links. In (Bliemer et al, 2014) it is shown that this algorithm converges to equilibrium: a situation where neither the turning proportions neither the assigned flows change.

Once the algorithm has converged, travel times are calculated. These are based on the notion of vertical queues, a physical simplification: we suppose that when cars arrive at the end of the link, if they cannot enter in the downstream node or link, they arrange themselves in a queue with no physical space (meaning vertical), waiting to enter.

Travel times are then broken down into two parts: free flow travel time plus time in the congested queue. Free flow travel times are the length of the link divided by the speed corresponding to the inflows in the uncongested part of the fundamental diagram, or in the case of triangular fundamental diagram, just free flow speed. Waiting times are calculated in a simple queue theory fashion, that is, the expected leaving time is the waiting flows divided by outgoing flows. The percentage of outflows divided by inflows will be called the reduction factor.

Here is a simple example:

$$F = 4000$$

$$C = 3000$$

In this example, 4000 veh/h try to enter in the downstream link which can only receive 3000 veh/h. In this case, 3000 enter, and a queue of 1000 veh/h is built. Then the last vehicle needs  $1000/3000 = 1/3$  h to leave the link. It follows that the mean waiting time for a simulation of duration  $T$  is:

$$\frac{1}{2} * T * 1000/3000 = T/6$$

It is clear from this example that congestion only builds up when there is a capacity drop downstream, either in the section or the node. So the way capacities are calibrated plays a central role in the quasi-dynamic simulation.

Origin-destination travel times are based on the calculations above. However, to give reliable estimates and keep consistency with the model, they don't just add link travel times. Given a particular origin-destination path, they get the accumulated reduction factor, multiplying all the link reduction factors of the path. The total travel time is the sum of all link free flow travel times plus the waiting time calculated as above with the accumulated reduction factor. Notice that in this way, during a simulation of duration  $T$ , we are assuming that all the cars sent from the origin during time  $T$  arrive at the destination. Thus, when comparing with mesoscopic simulation, we have to wait for all the cars to leave to give reliable travel time comparisons.

It is worth noting that one of the implicit assumptions in this model (reflected mainly in the nodal model) is that flows are homogeneous, so there is no overtaking and cars obey the FIFO rule. This implies that if a turn of a link is congested, the other turns of the same link are congested too. We will see the practical implications of this in the tests and results below.

## 2.3 Mesoscopic model

The mesoscopic model in Aimsun works with individual vehicles but adopts a discrete-event simulation (Law and Kelton, 1991) approach, in which the simulation clock moves between events and there is no fixed time-slice. An event is defined as an instantaneous occurrence

that may change the state of the traffic network, i.e. the number of vehicles in sections and lanes, the status of the traffic signals etc. Events can be scheduled (known in advance to occur at a particular time in the simulation); or conditional (added to the event list dynamically during the simulation whenever some logical condition is satisfied). Specifically, a mesoscopic simulation includes the following types of events: Vehicle generation (Vehicle Entrance), Vehicle system entrance (Virtual Queue), Vehicle node movement (Vehicle Dynamics), Change in traffic light status (Control), Statistic collection (Outputs), Matrix change (Traffic Demand).

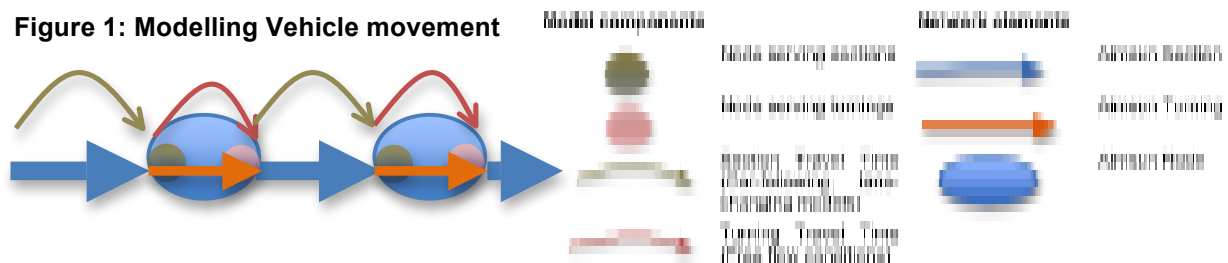
These events model the vehicle movements through sections and lanes by using a simplification of the car-following, lane changing and gap-acceptance models used in the microscopic simulator. Nodes, on the other hand, are modelled as queue serves. All events have an associated time and a priority. Both these attributes are used to sort the event list. For example, events related to a change to the status of a traffic light or a new vehicle arrival are going to be treated before events that relate to statistic collection or vehicle movements inside a node.

Mesoscopic vehicle movement in Aimsun is modelled depending on the location of a vehicle:

- Modelling vehicle movement in sections: car-following, lane-changing
- Modelling vehicle movement in nodes (node model):
- Modelling vehicle movement in turnings
- Modelling vehicle movement from sections to turnings: Apply gap-acceptance model
- Modelling vehicle movement from turnings to sections: Apply lane selection model

Figure 2 illustrates mesoscopic vehicle movements in Aimsun.

**Figure 1: Modelling Vehicle movement**



In Aimsun vehicles are assumed to move through sections and turnings so section and turnings are vehicle containers. The section capacity, in terms of number of vehicles that can stay at the same time in a section is calculated by using the jam density (user-defined parameter) multiplied by the section length and the number of lanes. By contrast, the turning capacity is calculated in a similar way, but using the feasible connections in the node instead of the number of lanes.

Car-following and lane-changing models are applied to calculate the section travel time. This is the earliest time a vehicle can reach the end of the section, taking into account the current status of the section (number of vehicles in section).

Aimsun mesoscopic simulator is based on this node model that moves vehicles from one section to its next section of its path. This model contains two actions that take place in all nodes: a) **Serving sections**, this server calculates the next vehicle to enter the node. This is done by applying the gap-acceptance model and then using the exit times calculated by the car-following and lane-changing models mentioned above; b) **Serving turnings**. This server calculates the next vehicle to leave the node. The selection is done by applying the car-

following and lane changing models to calculate the travel time and getting the earliest time when a vehicle can enter in its downstream section.

See (Casas et al, 2010) and (Aimsun 2014) for more detailed description about the different behavioural models implemented.

### 3. Computational results

#### 3.1 General Setting

We are going to present differences between static macro, quasi dynamic and meso simulation through evaluation of simulations in three toy networks and one real case network. The quantities of interest are link volumes and origin destination travel times. The main point is to ensure that the models are as comparable as possible, through the following points:

- Meso jam densities have been kept fixed and reaction times have been modified to match macro and quasi dynamic link capacities.
- Quasi dynamic simulations last one hour. Since the quasi dynamic calculates mean travel times for all the demand, an extra empty demand has been simulated in the meso so that all the cars leave the network.
- We have used no control plans.
- Since the quasi dynamic works with constant demand, we have used it in the meso too.
- Meso travel times by origin destination pairs have been calculated adding travel time with waiting virtual time.
- Flows comparison between meso and macro have used the maximum flow of the meso during the first hour.

#### 3.2 Networks

In this section we show some sketches of the toy networks used. Capacities are shown in such sketches. The first example is a corridor reflecting the simplest case to compare. After, we have simulated a merge tree and a diverge tree. In both cases the result of the nodal modal characterizes the solutions. Recall that, in the currently used nodal model, if a certain link has a congested turn, due to the FIFO rule, the other has to be also congested. Travel times have been expressed in minutes, and flows and capacities in vehicles per hour. In maroon the ids of the sections and centroids, and in black their capacities.

##### 3.2.1 Corridor

This network (see Figure 2) is composed by a set of links (identified in red) with its associated capacity (label in black). For example link 331 has a capacity of 1000 v/h.

**Figure 2: Corridor network**



The demand from origin O to destination D in this network is 3000 v/h.

**Figure 3: Corridor quasi-dynamic simulation outputs**

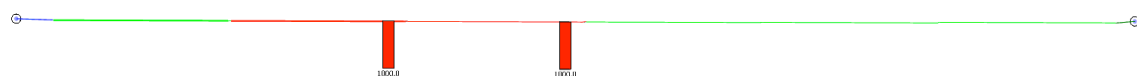


Figure 3 shows the simulation outputs where the vertical bars represent v/h in queue and the colours of links represents flow-capacity ratio.

The travel time comparison between meso and quasi-dynamic are equal, where the mesoscopic model is 73 minutes and quasi-dynamic model is 72 minutes. The travel time in static model is calculated using the default volume-delay function, and is around 3000 minutes. This big difference could be reduced calibrating explicitly the volume delay function in order to reduce this difference.

Concerning to the flow comparison, next table shows the flow comparison for each link taking three models of network loading: quasi-dynamic(QD), mesoscopic and static.

**Table 1: Flow comparison in Corridor network**

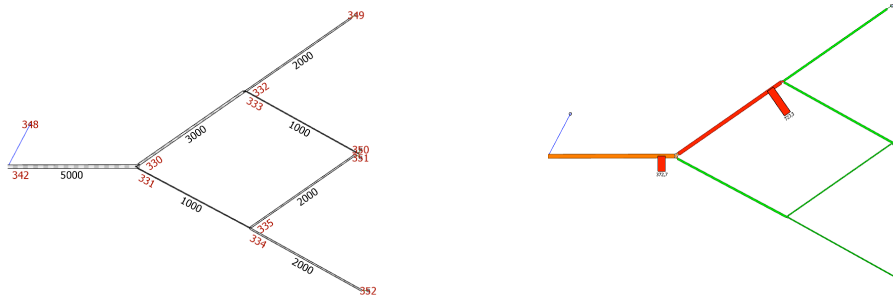
Link Id	QD	Meso	Static
328	3000	2530,8	3000
329	2000	1763,4	3000
330	1000	999,6	3000
331	1000	1000,2	3000
332	1000	1000,8	3000
333	1000	1001,4	3000

In this table the static model without any capacity constraint, by definition all demand goes through the link, meanwhile the quasi-dynamic model the flow corresponds directly with the minimum between the demand (3000 v/h) and the link capacity. The difference between quasi-dynamic and mesoscopic model is due to the explicit representation of queues in terms of space, so the capacity is reduced due of queue propagation upstream.

### 3.2.2 Diverging tree

The network structure, shown in Figure 6a, permits analyse the diverging nodal model. The red labels represent identifiers of links or centroids and black labels represent link capacities.

**Figure 4: a) Diverging network and b) Quasi-dynamic simulation outputs**



**Table 2: Travel time comparison in Diverging network**

	Mesoscopic (min)	Quasi-dynamic (min)
O348 – D349	17	16
O348 – D350	18	16
O348 – D351	7	4
O348 – D352	7	4

The demand matrix defines from origin 348 to destination 349 and 350 1500 v/h to each one and to destination 351 and 352 550 v/h to each one.

Comparing travel times, Table 2 shows OD pairs travel time where the mesosocopic model is slightly high travel time.

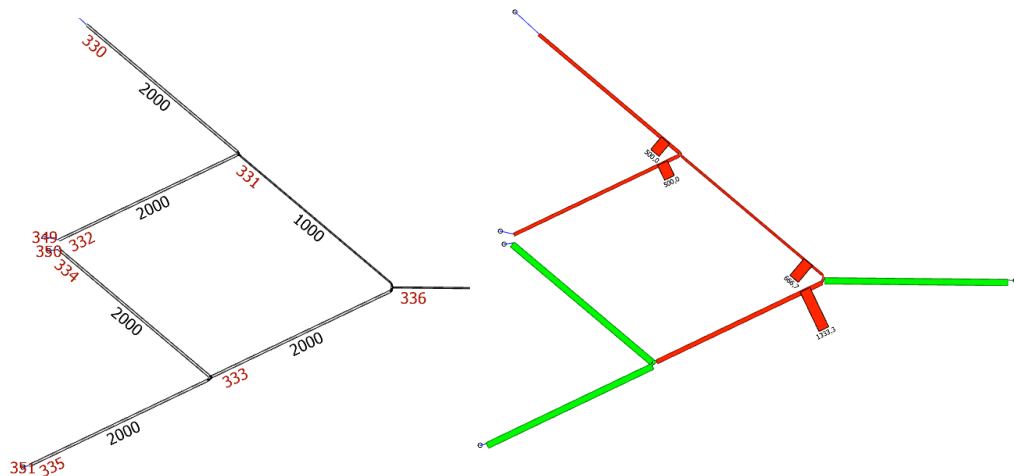
**Table 3: Flow comparison in Diverging network**

<b>Id</b>	<b>Quasi-dynamic flow</b>	<b>Mesoscopic flow</b>	<b>Static flow</b>
330	2000	2013,6	3000
331	1000	999,6	1100
332	1000	1027,8	1500
333	1000	999,6	1500
334	500	519	550
335	500	514,8	550
342	3727,27	3406,2	4100

### 3.2.3 Merging tree

The network structure, shown in Figure 10a, permits analyse the merging nodal model.

**Figure 5: a) Merging network b) Quasi-dynamic simulation outputs**



The origin-destination matrix defines from each origin to destination 352, 1000 v/h. The result of quasi-dynamic model is shown in Figure 10b.

**Table 4: Travel time comparison in Merging network**

	Mesoscopic (min)	Quasi-Dynamic (min)
O348 – D352	169	151
O349 – D352	111	151
O350 – D352	24	61
O351 – D352	62	61



Table 5 shows the flow comparison between three models.

**Table 5: Flow comparison in Merging network**

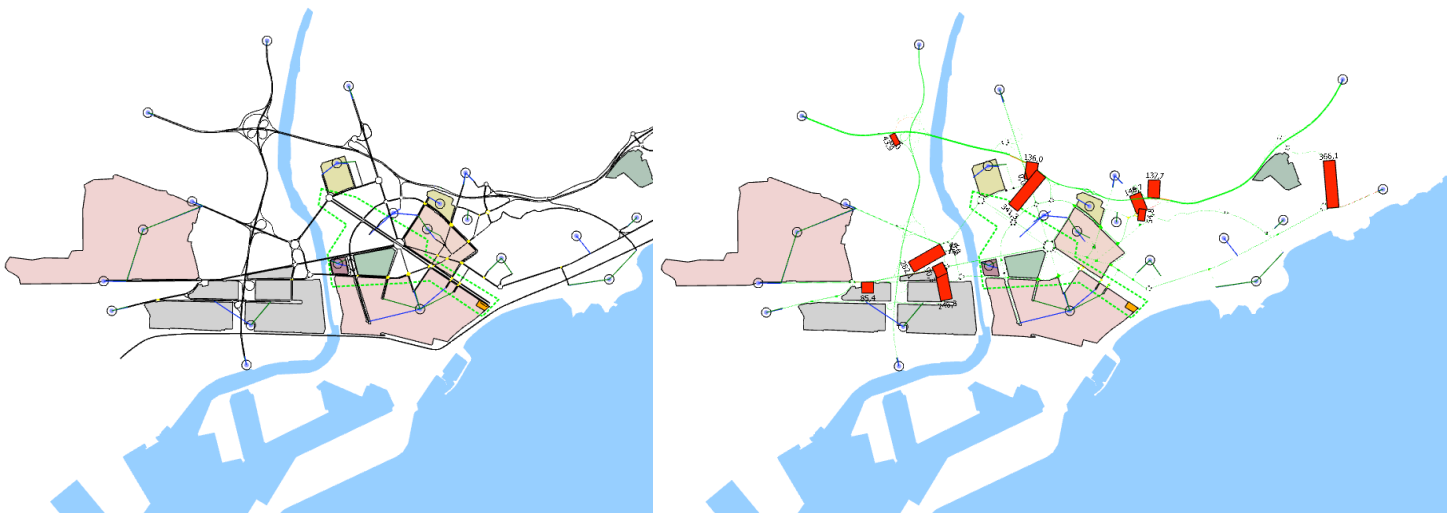
ID	Quasi-Dynamic flow	Mesoscopic flow	Static flow
330	500	80,4	1000
331	333,333	114,6	2000
332	500	637,8	1000
333	666,667	900,6	2000
334	1000	900	1000
335	1000	518,4	1000
336	1000	1000,8	4000

The simulation outputs reflects a significance increase of in terms of travel time when we compare quasi-dynamic and mesoscopic model (see Table 4). We interpret this difference due to asymmetry of the network in terms of capacity that affects the nodal mode in the mesoscopic model, for instance the different travel time from origin 350 to destination 352 is due to the FIFO rule applied for satisfying 1000 v/h capacity in link 336 (exit link) is not reproduced in meso.

### 3.2.4 Tarragona network

Tarragona is a city located in northeast of Spain, and the network used represents an aggregated network of the city (see Figure 6a). Some values that describes this network are: total section length 97 km, 476 sections, 132 intersections, 23 centroids and the total number of trips is 18654.

**Figure 6: a) Tarragona network b) Quasi-dynamic simulation outputs**

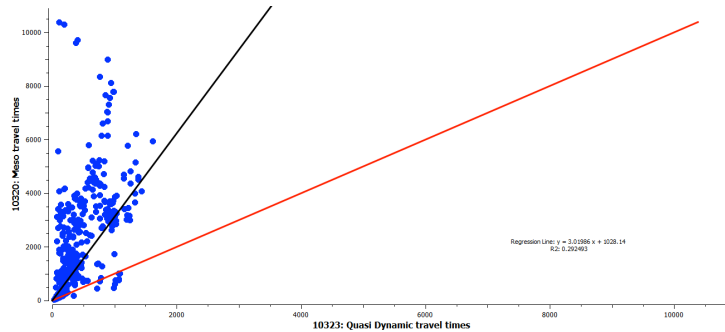


Considering travel time, Figure 7 shows travel times (in seconds) per origin destination of the quasi dynamic (horizontal axis) versus meso (vertical axis). Generally speaking, meso travel times are considerably higher and the main reason is the effect of the traffic control and the

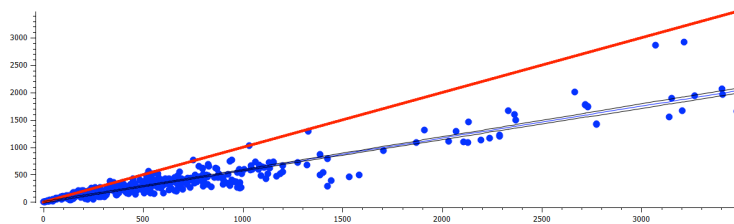
propagation of queues upstream. This is supported comparing mesoscopic and quasi-dynamic flows (see

Figure 8) where clearly the mesoscopic flows are lower than quasi-dynamic. On the other hand, as already noticed in previous examples, macro flows are higher than quasi-dynamic ones, as seen in Figure 9)

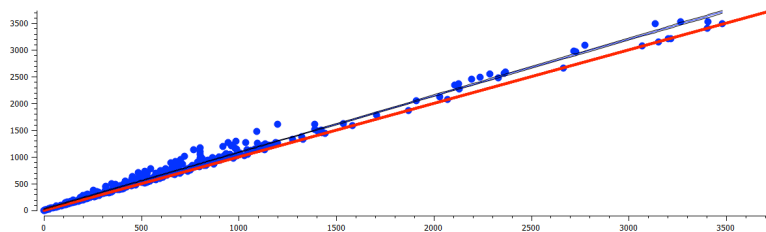
**Figure 7: Tarragona's travel time comparison**



**Figure 8: Flow comparison Quasi-dynamic and Mesoscopic. X QD flows, Y mesoscopic flows**



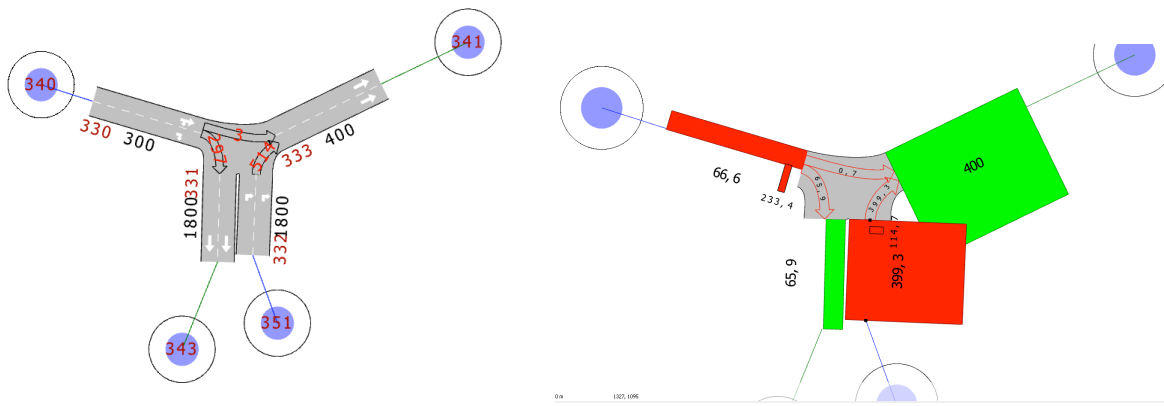
**Figure 9: Flow comparison Quasi-dynamic and Static. X QD flows, Y static flows**



### 3.2.5 Roundabout sketch

Finally we want to mention some particularities associated to Tampere's nodal model (Tampere et al 2011). We came through working with roundabouts, where the capacities of the links inside the roundabout were much smaller than the links leading to the roundabout. The following toy network (see Figure 10) was a piece of roundabout. In maroon we have the ids, in black link capacities and for simplicity in red, placed in the turns, we have written the demands: for instance, there are 3 vehicles going from centroid 340 to centroid 341.

**Figure 10: a) Network description b) Quasi-dynamic simulation outputs**

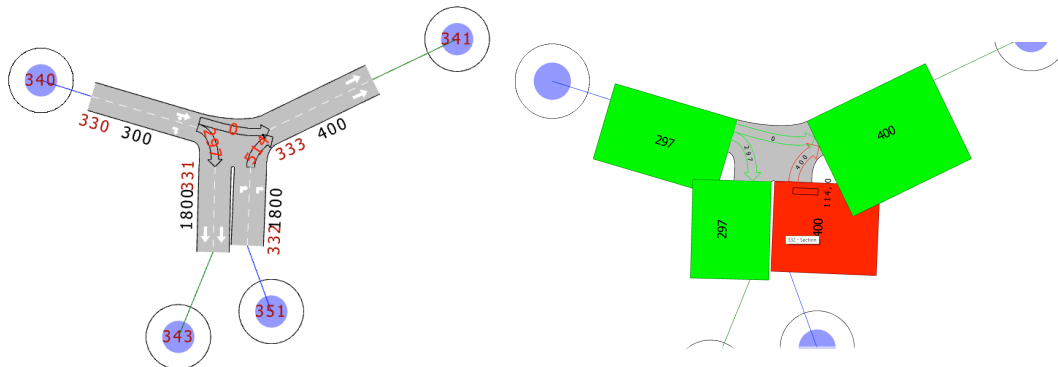


The solution of the quasi dynamic is shown Figure 10 b.

Observe that flows going from centroid 340 to 343 are constrained by the capacity of the receiving link and only 0.7 from the 3 veh/h can enter in the link, nearly a 22%. By the FIFO rule, flows from link 330 to link 331 have to be also constrained in a 22%, thus only entering 66 veh/h.

Consider now the following situation, where no cars go from centroid 340 to centroid 341 (see Figure 11a). The solution of the quasi-dynamic simulation (more precisely the nodal model) is shown below in Figure 11b.

**Figure 11: a) Network modified b) Quasi-dynamic simulation outputs of network modified**



Notice that now, in this case, the 297 veh/h car enter in link 331. This produces a situation where small perturbations have a huge effect on the results.

We want to remark though, that this is due to the fact that link 332 with capacity of 1800 leads to link 333 with significantly smaller capacity, i.e. 400veh/h. In general, we cannot expect to find this situation very often, except maybe in roundabouts.

### 3.2.6 M4-Sydney network

The comparison has been done with a part of Sydney network (see Figure 12).

**Figure 12: Sydney M4 network**



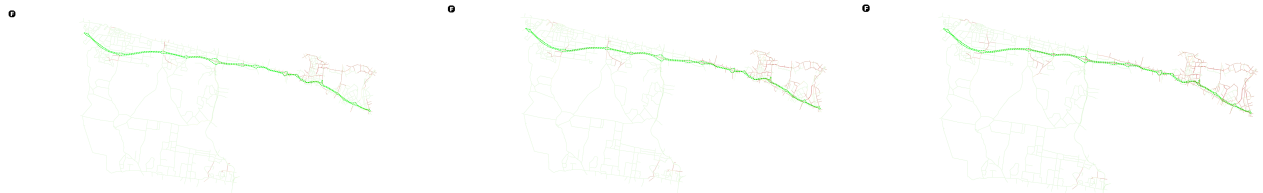
The quasi-dynamic results in Figure 13, the congestion is focussed on top right part of the network. The location of the congestion has been simulated using mesoscopic model for comparing the presence of congestion and its evolution over space.

Figure 14 shows the density at 8:00 am, 9:00 am and 10:00 am, where is consistent the initial location of all congestions estimated by quasi-dynamic model and its propagation upstream.

**Figure 13: Quasi-dynamic simulation outputs a) global b) congested area**



**Figure 14: Mesoscopic outputs: a) 8:00 am b) 9:00 am c) 10:00 am**



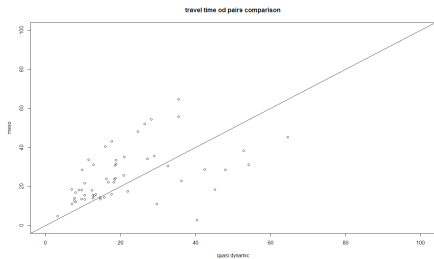
In terms of travel times, the comparison of the main od pairs in terms of travel time confirms the previous results where the mesoscopic model trends to have a higher travel time (see Figure 15). In this network the exception are the points that reflect a high travel time in quasi-dynamic model respect to mesoscopic model. The explanation of this difference is because all points corresponds a set of origins which path crosses a link with a vertical queues that increase artificially the travel time in quasi-dynamic model for FIFO constraint.

The validation comparing quasi-dynamic model and static model could be summarised as:

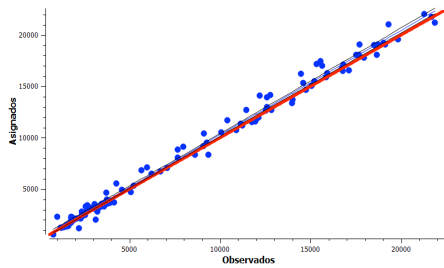
- The flow validation between static model and the real data set gives a R2 equal to 0.99 (Figure 16).
- The flow validation between quasi-dynamic model and the real data set gives a R2 equal to 0.988 (Figure 17).
- The flow validation between quasi-dynamic model and static model gives a R2 equal to 0.992 (Figure 18).

- The travel time validation between quasi-dynamic model and static model shows the main difference and contribution of the quasi-dynamic model, taking into account the similarities of travel time with respect to mesoscopic model (Figure 19)

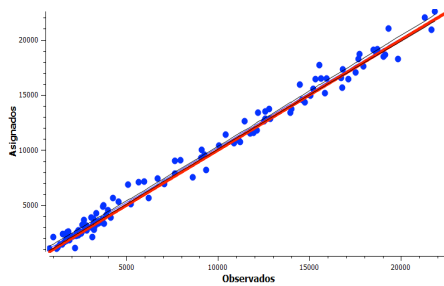
**Figure 15: Travel Time comparison Quasi-dynamic and Mesoscopic. X-QD travel time, Y mesoscopic travel time**



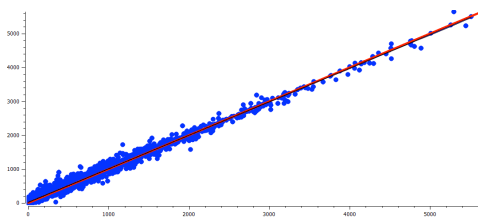
**Figure 16: Regression plot static vs rds (flows)**



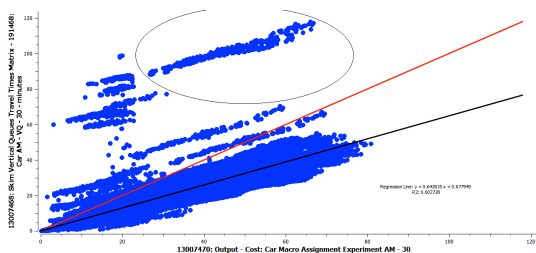
**Figure 17: Regression plot quasi-dynamic vs rds (flows)**



**Figure 18: Regression plot static vs. quasi-dynamic (flows)**



**Figure 19: Regression plot static vs. quasi-dynamic (travel time)**



### 3.2.7 Significant od travel time differences

We can see in Figure 19 (in circle) some paths with a large differences between static macro travel time and quasi dynamic. The selected set of points have in common two different origins. Moreover, such paths have in common the link shown in Figure 20. This link has nearly 30% of its flow willing to take the off ramp, which has a significantly smaller capacity. This creates a congestion letting only up to 67% of the flow leave the highway, and, due to FIFO restrictions of the model, only a 67% can continue in the highway. This creates a queue of approximately 15 minutes mean delay per vehicle per simulation hour in such link.

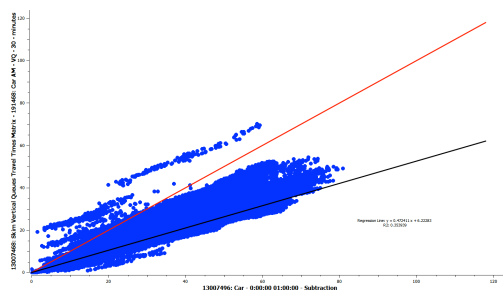
In Figure 21 we show the travel time comparison between static macro and quasi dynamic, once we removed all the paths containing the links 51352 and 5877, the two most congested ones.

With further exploration of the rest of the paths in the selected area in Figure 22 we conclude that such paths correspond to ones containing links with high congestion. Thus we can visually cluster the paths into two groups (approximately, depending on the level of congestion): the ones in free flow (below the red bisecting line) and the congested ones (above the bisecting line). Of course, this analysis is subject to the selected VDFs of the static macro.

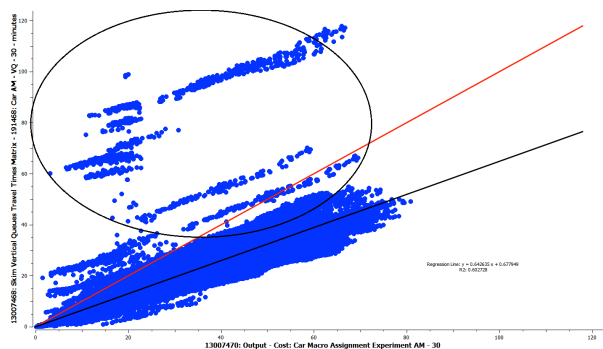
**Figure 20 Quasi-dynamic result of section 51352**



**Figure 21 Congested links removed**



**Figure 22 Congested paths in quasi dynamic**



The same analysis has been performed with the paths shown in Figure 15 that have greater quasi-dynamic travel times than meso travel times. Namely, such paths contain congested links.

## 4. Conclusions

From the simulations above we have seen that the static flows are always greater than quasi-dynamic flows. This happens always by the way both methods are build.

We also have seen that generally speaking, meso travel times are higher than quasi-dynamic travel times. We interpret that this is due interactions of vehicles in nodes, that the quasi-dynamic (the nodal model) simulation doesn't take into account.

As we have seen in many examples, how link capacities are calibrated is crucial to the results of the quasi-dynamic simulation both for the quasi-dynamic solution and the nodal model. In particular, in the nodal model, when links with large capacities lead to links with small capacities, discontinuities in flows can appear.

## References

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