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An empirically verified Passenger Route Selection Model based on the principle of least effort for monitoring and predicting passenger walking paths through congested rail station environments

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Abstract

Crowding at egress points and waiting areas in public transport environments during peak periods can potentially impede passenger movements, causing delays to scheduled services. Passenger modelling is a complex task. There are relatively few models able to simulate the complex behavioural characteristics of large volumes of people walking through confined public transport environments such as rail station concourse and platform areas. With the aid of robotic sensing technology however, rich data can be acquired to provide high quality inputs on which passenger behaviour models can be based.

This paper presents a methodology for predicting the preferred route selected by passengers during their egress. Proposed in this paper are a basic principle and a methodology for route choice based on the least effort that a passenger may consume during their travel between destinations. The methodology proposed takes into consideration the movement based passenger and congestion state. We employ the principle of least effort, formulated in terms of a metabolic energy, and congestion states. Our approach uses a new mathematical model for representing effort expended for each path, based on a formulation that minimizes the total amount of metabolic energy used when moving on a trajectory. Using results from an empirical study at Brisbane Central rail station, we show our approach collates well with real patterns of passenger egress. Our discussion concludes with an overview of how our approach could be used by rail service providers to optimise operations and improve customer experience.

Keywords – Route Choice, Least Effort, Congestion, Passenger Behaviour.

1. Introduction

Passengers using public transport systems are subjected to the problem of making pedestrian path or route choices within public transport environments like rail station precincts, concourse and platform areas. Such route choices become more complicated as public transport stations become more complex and integrated with retail and commercial land-use activities in addition to access to transport services often creating more options for egress. This increases the number of activities that may be performed in a station and can complicate the route choice process for individuals.

Predicting passenger flows and route choice is a requirement for the operation, planning and design of public transport facilities. How egress options are provided influences the emergence of congestion within facilities — if every passenger chooses the same route, pedestrian congestion will likely occur. In practice there are usually multiple egress options and so the question arises as to how people make decisions when choosing between them, for example between escalators and stairs, and further how the emergence of congestion may affect route choice.

Most of route choice models, available in the literature, are based on the shortest path, regardless of the route complexity (Verlander & Heydecker 1997) (Helbing D., P. Molnár 2001) (Ciolek 1978). More comprehensive models acknowledge that route choice is influenced by more than just simple aspects like time, distance and smoothness of the path. While only a few models include pedestrian densities — congestion — and passenger comfort (Seneviratne & Morrall 1985) (Daamen 2004), these models are inherently limited in that they focus on isolated variables where as in reality several *co-variants* are likely to exist.

There is a need for a more comprehensive model that can describe pedestrian route choice behaviour based on foreseeable co-variants such as congestion. Several approaches for principle of least effort (PLE) on route choice or path selection behaviour exist. However, it seems they have limited suitability for our real-world scenario. Our approach that is capable of the non-trivial task of path selection behaviour in real-world scenarios with robustness against different levels of congestion is needed. The contribution of this work — which uses real data from a field study — is to demonstrate that perceived congestion on path alternatives has a cost in the real world and so we have devised a formula capable of encapsulating this.

This research demonstrates some of the intricacies and influence of congestion driven real passenger route selection and proposes an expanded descriptive model that is then empirically validated using data collected from Brisbane Central rail station. The findings demonstrate that it is fruitful to incorporate congestion by relating route choice to 'effort', rather than looking at travel time or walking distance. The congestion dataset from the Brisbane Central field study is used to assess the influence of congestion. Route capacity is first assessed to determine whether different route choices can be observed for congested and non-congested conditions. Then, the influence of the severity of congestion on route choice is analysed to demonstrate the real world interplay between congestion and route choice.

The paper is organised as follows: Section 2 describes the factors influencing route choice; Section 3 elaborates on the theoretical framework used, the least effort principle and influence of congestion; in section 4, experimental results are presented and discussed, and; Section 5 concludes the present work and discusses future research plans.

2. Factors influencing route choice

Congestion is an important concept in transport analysis as its presence can change the behaviour of people's movement and travel choices. Over time, congestion levels can rise, this is particularly the case when the general demand for train travel rises. This can create significant challenges for public transport authorities and service providers by potentially causing delays to train movements, destabilising schedules and ultimately restricting the number of train paths that might be supported by a rail network (Palma & Lindsey 2001). Congestion during train travel can potentially create unpleasant experiences for passengers as well as safety concerns and in some cases has becomes a serious problem (Palma & Lindsey 2001) (Kirchner et al. 2014). Congestion imposes a wide range of problems for passengers like delay in travel time, rescheduling train timings, and potential way finding confusion during crowding and subsequent egress delays.

When assessing the design of transport facilities, it is important to be able to anticipate the routes that will most likely be taken by the passengers under different conditions, since it is one of the key factors affecting the occurrence of congestion. Route choice can be a complex process by which a passenger chooses amongst path alternatives given the infrastructure configuration within a station. Often stations are designed to limit and path alternatives and information systems guide way-finding in an attempt to consolidate egress. Nevertheless, personal preferences and perceptions can cause egress to fluctuate at key points among path alternatives, and lead to a subsequent cascade of effects. There is a need for a descriptive model for planning that captures this — a model that can describe real world route choice behaviour.

For this we need to improve our knowledge of factors influencing passenger route choice behaviour — both environmental and social — and focus on how these affect the route choice process. Typically, models and research in this area focus on factors that may affect the route choice of a passenger, which are related to the passenger, the environment, travel time, or a combination thereof and forego including social influencing factors, or treat them as an isolated variable rather than an integrated co-variant.

In this paper, the factors that involve congestion are studied and their influence on passenger route choice is analysed. The research motive is addressed by empirical observations of revealed choices of train users under varying conditions with respect to route choice factors, especially types of route alternatives (stairs, and escalators).

Factors identified from the literature that have been linked to influencing passenger route selection include:

- walking distance
- walking time
- effort
- pleasantness
- crowdedness

If congestion is included, it is typically considered as crowdedness and treated as an independent variable. However, the social influencing factor of congestion is a co-variant with these factors; and as such its potential effect is significant but perhaps not obvious — non-linear. The modelling complexity of treating congestion as a co-variant arises from interpersonal differences that drive perception — different passengers will potentially respond differently to different levels of congestion. For instance, the approach to a particular egress points around a rail station might be perceived in different ways. Crowdedness may have a relatively low influence on a typically busy concourse where passengers' perceive that 'of course it is busy here'. Conversely, it may have a more significant effect on typically quieter platform areas and perception may align more with 'why is everyone standing at the same spot?' Finally, the associated threshold level for behaviour change in relation to route selection due to congestion for each individual passenger is reliant on personal perception.

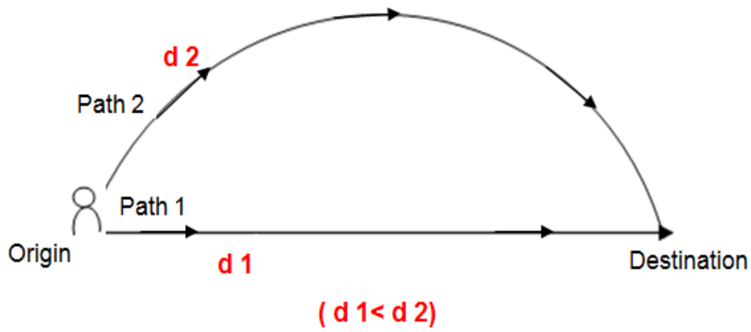
More specifically, walking distance passengers tend to choose the shortest route, although they are frequently aware that they are minimising distance as a primary strategy in route choice (Ciolek 1978) (Seneviratne & Morrall 1985a). Figure 1 shows two path options — Fig. 1a) without a point of congestion, and Fig.1b) with a point of congestion.

Referring to Fig1a), using the shortest distance approach, a passenger will probably choose path 1. However, if we consider Fig.1b) with congestion, it's reasonable that at least some passengers, due to their personal perceptions of congestion, will choose Path 2. In this case, congestion is a co-variant and this model would benefit from the inclusion of congestion at the time of decision. Likewise, walking time — where passengers choose the path with the shortest length — will foreseeably change given congestion levels. Again however, this effect of retarding passenger through rates is likely to vary considerably between passengers

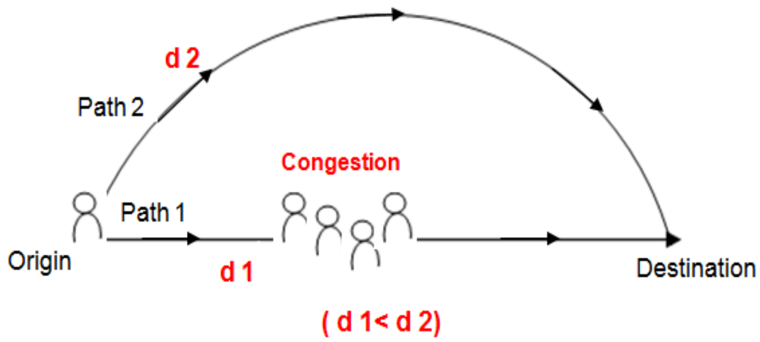
(Cheung & Lam 1998) (Daamen et al. 2005). For instance, a footballer may push straight through the congestion with no retardation where as an elderly person may be severely restricted by the crowd. The key point is that these individual passengers will have an awareness and appreciation of this prior to the route selection point. Or in other words, the perception of crowdedness will weigh differently for different people irrespective of whether their underlying route selection is based on the same factor such as shortest time.

Figure 1: Comparison between two paths (with and without congestion)

A. Route choice without congestion



B. Route choice with congestion



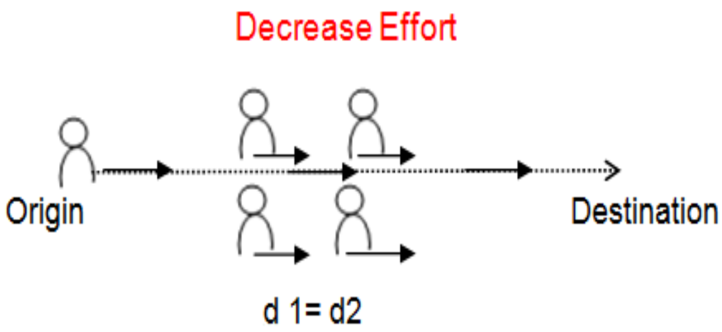
This interplay between typically modelled factors and an individual's perception of the repercussion of congestion on their decision is also apparent in effort driven models. Effort models tend to discriminate between paths using a measure of physical work required, such as the need to climb a gradient or steep terrain versus traversing flat terrain (Daamen & Hoogendoorn 2004) (Cheung & Lam 1998) (B. Givoni 1971).

Congestion again is a notable co-variant with the derivation of a path's effort. For example, referring to Fig.2, if a hundred people are walking faster than you but in the same direction as your intended travel — Fig.2a) — then this would result in you actually walking faster and effectively reduce the perceived effort required when walking along that path. Conversely, if the same hundred people were walking considerably slower than you or perhaps in a less uniform direction, then your perception of required effort to traverse this path would likely increase — Fig.2b). In both cases the crowdedness will be the same, a description of the crowd behaviour and the potential resulting perceptions and actions of individual passengers must be modelled to describe this.

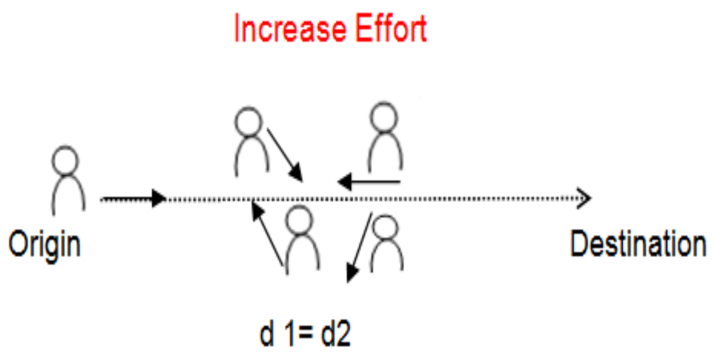
The same number of people doing the same behaviour entails two different forms of effort depending on how they interact with the actual behaviour we are interested in, which is effort, as shown in Figure 2. The distance from origin to destination does not change but the time will change. If these four passengers walk faster than you, you might walk faster. The distance will not change but the time will change, in both cases the congestion will be the same (four passenger same congestion), because we look to the congestion as an effort, not as a number of people. Congestion is not the number of people but the number of people doing a particular behaviour and how this behaviour interacts with the particular variable that we have entrusted in.

Figure 2: Illustration of key direction of moving congestion possibilities

A. Passengers walking same direction



B. Passengers walking opposite or random direction



Pleasantness or number of attractions along the route (Daamen & Hoogendoorn 2004) (Seneviratne & Morrall 1985a) as a route become more attractive, walking time and distance become less important factors; but here crowdedness may be perceived by a 'Tourist' as an indication of a route's potential saliency and in such actually increase its attractiveness. Again this is an example where the co-variant congestion has a nonlinear effect on its co-variant that is dependent on the individual passenger's perception (in this case route attractiveness).

Crowding is commonly experienced on buses, subways and waiting platforms (Palma & Lindsey 2001). Whilst congestion is touched upon in the crowdedness models (for example, Seneviratne & Morrall 1985) it is treated as an isolated variable and its notable influence as a co-variate is not captured. It is shown however to have value as an isolated variable. (Senevirantne & Morrall 1985) showed that even if the progress on a direct route is relatively slow until approximately ¾ of flow capacity is reached, the choice of a longer route still rarely happens and is considered to be outlier behaviour.

This result is indicative of the current trend in modelling towards group modelling. Clearly, congestion adds significantly to route choice selection as an individual specific co-variant for key group factors such as walking distance, walking time, or number of attractions along the route.

This paper does not argue against the usefulness of these models, rather we suggest that the outliers often discarded by group focused models can actually be modelled by introducing an individual specific co-variant to interplay with these crowd generalised variants.

3. Theoretical framework

Established models of route choice are available. However, many researchers have formulated the strategy for route choice considering one of the above factors whilst focusing their work on limited scenarios which do not test the underlying assumptions and/or focus their work on describing the 'bulk' of behaviour while treating the remaining expressions of behaviour as outliers. However, as discussed above, a proportion of this outlier behaviour is describable through the effect of individual's perception of congestion on their value system for route selection. Passengers consider several factors concurrently whilst making a decision. Here within we propose a formulation for encapsulating the known key route selection factors in such a form to enable our definition of congestion to be overlayed as a co-variant, and as such propose a more general route choice model which includes the varying level of influence due to individual perceptions.

3.1 The Principle of Least Effort (PLE)

It has often been observed that people choose routes that involve the least effort to get to their desired destinations. This observation has been known as the Principle of Least Effort (PLE) (Zipf 1951), which explains the individual behaviours of human beings. This phenomenon has been applied in many engineering applications that include pedestrian movements and route selection. It assumes a general law ingrained in human brains, often referred to as a psychological force that could be summarised as the "law of the least effort" where a person will choose the option that can perform the task with the least effort. This has been explored by several researchers when confronted between two choices (Silder et al. 2012) (Guo & Hall 2011) (Guy et al. 2010).

Some researchers have proposed the distance travelled as an indicator of the effort. However, this indicator does not account for walking speed. Few other researchers have proposed an *effort* metric — the time to reach the target destination. However, this approach does not consider the optimal one but it assumes individuals will walk at their maximum speed. Others have suggested *metabolic energy* as a metric for effort (Vieilledent et al. 2001).

Since we are looking for a general model, it will be better to take into account the fact that the data set consists of two different samples with corresponding different scale factors. Using escalators and stairs as an example, that's a number of people and the particular behaviour of going to and using an escalator, and how they interact with the decision to use the escalator or stairs, so this interaction can be captured using the principle of least effort.

The metabolic energy expenditure of walking may vary within the wide range of individual limits and also vary for a given individual depending on the factors that encompasses total weight, walking speed, type of surface and grade. Resorting to experimental data and practical evidence (B Givoni 1971) (Cotes & Meade 1960) (Zarrugh et al. 1974) (H.J.Ralston 1958), it is stated clearly that the relationship between the metabolic power $'\mathrm{P}'$ and the walking distance speed $'\mathrm{v}'$ takes the quadratic equation:

$$\mathrm{P} = \mathrm{A}\,\mathrm{v}^{2} + \mathrm{B}\,\mathrm{v} + \mathrm{C}$$

Where coefficients A, B and C are pre-individual parameters that are defined as,

$$A = 1.5\,\eta\,(W + L\,)$$
$$B = 0.35\,G\,\eta\,(W + L)$$
$$C = 1.5W + 2(W + L) + \left(\frac{L}{W}\right)2$$

With W being individual weight, L the load carried, η the terrain factor, and G the gradient.

Of the walker, at an instant $'t'$, can be described by a pair of coordinates $x(t)$ and $y(t)$, which define the trajectory sought, namely.

$$r(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Recalling that the instant velocity $v\,(t)$ of a moving body is defined as:

$$v\; = \dot{r}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix}$$

And hence, the magnitude of instance velocity can be expressed as:

$$v\,(t) = \|\dot{r}(t)\| = \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2}$$

Recalling that the power is the time rate of the energy, that is

$$P = \frac{dE}{dt}$$

Accordingly, we can write:

$$dE = P\,dt$$

$$\int_{t0}^{tf} (Av^{\,2} + b\,v + c)\,dt$$

The amount of energy spent by an individual on walking depends on several factors that include total weight, load carried, the grade and terrain. Individuals walking over different terrains consume greater energy resulting in a combination of greater muscle mass usage and a forward stooping posture.

To illustrate the PLE, comparison between two paths is shown in Figure 3 to differentiate between energy expenditure, distance and time of walking via each path. (Path 1: Straight line with ‘mud’ in the middle, Path 2: Piecewise with angle 30 degrees along x-axis. Where ‘mud’ represents some instance that causes effort — this could be mud, stairs, heat, congestion or noise for example.

For Path 1, the distance from origin to destination (OD) equals 100m, which includes three stages, namely Stage 1, Stage 2, and Stage 3. Stage 1 between points AA′ is 30m long with ($\eta = 1$), (v = 1.5 m/s). Stage 2 between points A′A′′ is 40m long 'mud' with ($\eta = 9$), (v = 1 m/s). Stage 3 between points A′′B is 30m long with ($\eta = 1$), (v = 1.5 m/s).

The subjects of equivalent weight were chosen for walking by carrying the same load with velocity 1.5m/s and terrain = 1 throughout their way. As a result for Path 2, the distance from (OD) is calculated to be 115.4m

As shown in Figure 3, referring to table 1 even though Path 1 is shorter than Path 2, the effort associated with Path 1 is greater than with Path 2.

As an assumption, the 'mud' on Path 1 leads to congestion mainly because of the decrease in the subjects walking velocity and increase in time delay and effort. If this assumption is taken into consideration, the PLE approach is satisfied. Now, this PLE approach can be extended to the Brisbane Central train station for real-time scenarios.

Figure 3: Comparison between two paths

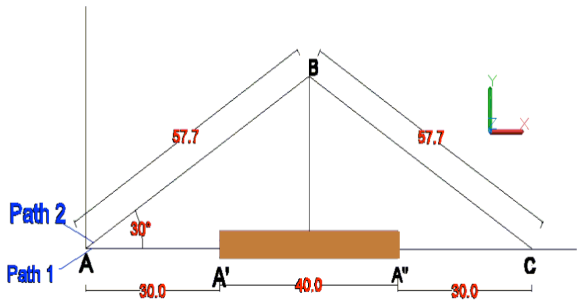


Table 1: Shows an overview of route usage

	Distance (m)	Time (s)	Effort (J)
Path 1 AA′A′′C	100m	80	1790
Path 2 ABC	115.4m	76.93	942.2

4. Experimental Results: Stairs or Escalator

This paper focuses on the influence of different types of walking infrastructure and congestion on passenger route choice. In this section, the congestion data is used to assess the influence of congestion. We apply the concept and method developed in this paper to investigate passengers' route choice between two alternative facilities — escalator or stairs in Brisbane Train Station as shown in Figure 4.

Apparently, resorting to the PLE with no congestion, the escalator route choice is associated with zero effort, as the passenger moves with zero speed. As a consequence the escalator route choice is preferable over the stair route choice. However, this is not valid in the presence of congestion, which is to be investigated in this section.

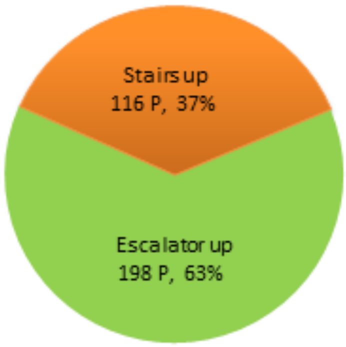
Empirical data were collected at Brisbane Central Train Station and used to perform various tests to validate the model developed in the previous section based on the Principle of Least Effort formulated by metabolic energy and congestion. Such model congestion states that passengers most likely prefer to choose escalators rather than stairs, as the former is associated with least effort. Our Sensing Hardware Platform (SHP), shown to the left of the image in Figure 4 and which has been demonstrated to be capable of robust person detection and tracking in situ in public train stations (Kirchner et al. 2014) was used to produce an empirical dataset of individual passengers movements. Data were collected to monitor the route choice behaviour of passenger from (9:05 am- 9:45am) on a weekday.

The congestion at the base of the escalator varied with each group of passengers during the time they passed through the area in front of the escalator. In all cases, congestion is only observed at the base of the escalator for a limited time. The level of congestion is assessed from the waiting times and the number of passengers on the escalator derived from our SHP data. Analysis of origin-destination relations shows that the congestion on the escalator has a significantly different influence on route choice. The frequency at which passengers used the stairs or escalator appeared to vary with the density of passengers at the base of the escalator. To facilitate a consistent method of measuring the congestion at the base of the escalator, a region measuring approximately 4m x 3m at the base of the stairs and escalator was defined by our SHP using depth images for the 49-minute video.

Figure 4: Our SHP located at Brisbane train station

The number of passengers in the region of Brisbane Central train station platform was counted manually to determine the level of congestion at the base of the escalator and stairs. During the peak hour period, the platforms at Brisbane Central train station are densely populated. When the trains arrive, the formation of queues at the escalators is a common occurrence. In total, 314 passengers movements were recorded from 09:05 – 09:45 where 63% used the escalator and 37% the stairs without considering the congestion for escalating. The percentage of passengers on the alternative route is larger in the case of congestion, As Shown in Figure 5.

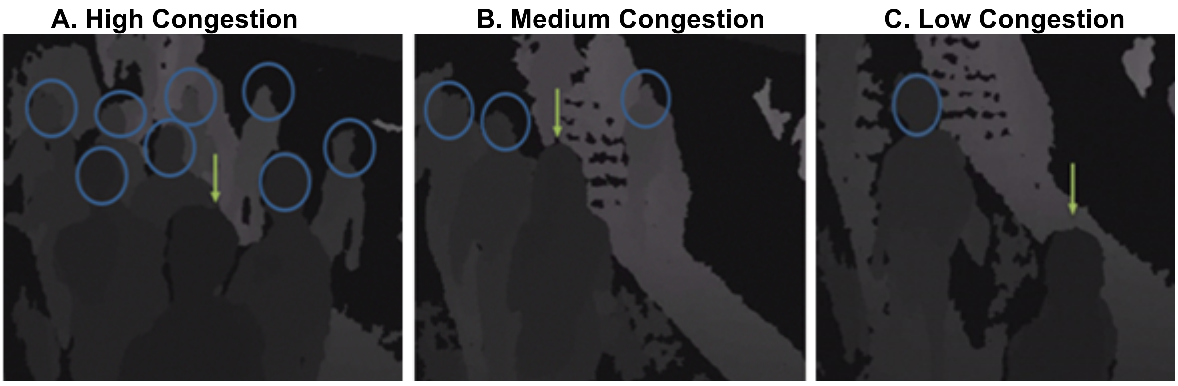
Figure 5: Numbers and percentages of passengers using the stairs or escalator without considering congestion



A comparison between Figure 5 and Figure 7 shows that the proportion using the stairs and escalator changes during different levels of congestion. We analysed the degree to which congestion levels affected people's route choice.

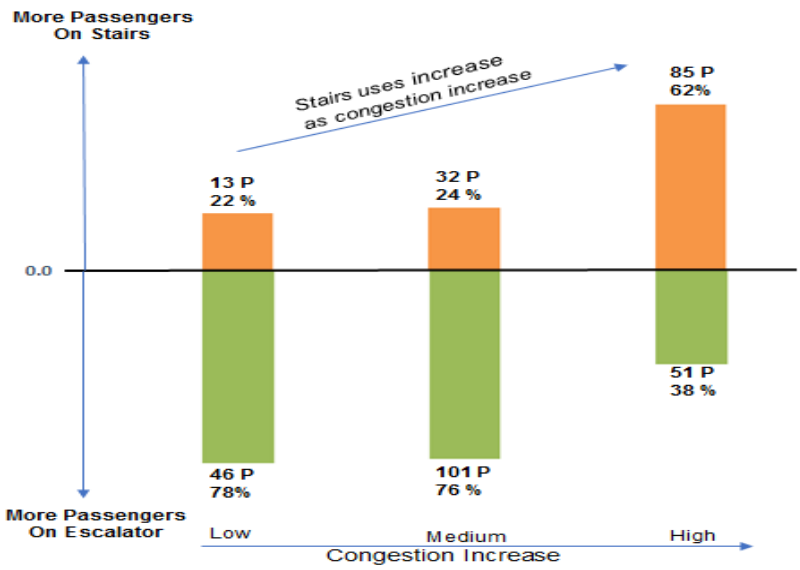
To do this we identified three different levels of congestion — high, medium and low — that are shown in Figure 6. To highlight passengers at the egress point, a blue circle has been placed to show the position of their heads. Low congestion levels at the egress point were classified as 0–2 passengers, medium congestion levels at 3–6 passengers and high congestion levels at 7–8 passengers. Significantly, high congestion levels represent the situation in which normal walking speeds are reduced due to queuing at the approach to the escalator. As shown in Figure 6, a significant difference in the level of congestion can be seen by comparing high congestion levels shown in Fig 6a) with low congestion levels, shown in Fig.6c).

Figure 6: Shows Route Choice behaviour Passengers' based on congestion levels



A sample of 328 passengers who were recorded ascending to the upper level of the rail station concourse chose to either use the escalator or stairs when moving from the train arrivals, 46 escalator users were recorded, 13 stair users were recorded from a total of 59 passengers during a low congestion state, 101 escalator users were recorded, 32 stairs users were recorded from total 133 passengers during medium congestion state. However, 85 people used the stairs and 51 used the escalator from a total of 136 during periods of high congestion levels. It's apparent from figure 7 that, during low congestion levels, approximately 78% of passengers preferred to use the escalator and 22% used the stairs. Based on the PLE approach, the escalator was the most preferred option. However during high congestion levels approximately 62% of passengers used the stairs and 38% used the escalator to avoid congestion.

Figure 7: Shows Congestion occurrence prediction with Numbers and Percentages of Passengers' who traveling over stairs and escalator



Apparently, when congestion levels are low passengers certainly choose escalator. When Congestion levels are medium, at 76% passengers are still likely to use the escalator and 24% stairs, and for high levels of congestion, at 62% passengers are more likely to choose the stairs to avoid congestion.

5. Conclusion and Future Work

This paper demonstrated that perceived congestion on route choice has a cost in real world (using real data from a field study) and that passenger path choice models can exploit this to describe behaviour. Furthermore, we have devised a formulation capable of encapsulating this complex interplay utilising the Principle of Least Effort and congestion occurrence. The predictions of this formulation were found to be consistent with the findings from a real world field study at Brisbane Central station.

In this contribution, we have discussed data collection for passenger route choice alternatives in Brisbane Central station. These data have been collected in public train stations by recording people's route choice as well as some personal characteristics. This research shows that pedestrians change their route choice with perceived congestion, and anticipate different route choice selection behaviour during congestion and without congestion. It is expected that station travellers in general will adapt their route choice behaviour given differences in levels of congestion. The analysis shows that the number of passengers who avoid using an escalator increases with congestion.

The implications of these findings will shape further model development. Specifically, this reconceptualization of the fundamental basis of the model to allow for co-variants contingent on passenger perceptions has increased the scope of the general model by incorporating outlier cases under the explanatory boundaries of the model. Simply put, this reconceptualization builds the foundations for a model that will enable improved operations, planning and design of public transport facilities.

In future research work, we intend to identify passenger categories based on walking behaviour and classify people under business, tourist and other causal passengers. We will apply a formulation of the PLE to these passenger groups in addition to levels of congestion.

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