Empirical Evaluation of Brisbane Macroscopic Fundamental Diagram

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Abstract

The Macroscopic Fundamental Diagram (MFD) relates space-mean density and flow, and the existence with dynamic features was confirmed in congested urban network with real data set from loop detectors and taxi probes. Since the MFD represents the area-wide network traffic performances, it gives foundations for perimeter control strategies and an area traffic state estimation enabling area-based network control. However, limited works have been reported on real world example from signalised arterial network.

This paper fuses data from multiple sources (Bluetooth, Loops and Signals) and develops a framework for the development of the MFD for Brisbane. Existence of the MFD in Brisbane network is confirmed. Different MFDs (from whole network and several sub regions) are evaluated to discover the spatial partitioning in network performance representation.

1. Introduction

This research aims to demonstrate empirical examples of Macroscopic Fundamental Diagram (MFD) of Brisbane network. Recently, the existence of the MFD with dynamic features was verified in congested urban network in Yokohama city by (Geroliminis and Daganzo 2008, 759-770). Similarly to the conventional link-based fundamental diagram, the MFD represents an area traffic states by defining the traffic throughput of an area at given density levels, and describes the dynamics of area-wide traffic conditions. Thus, the MFD can help in comprehensive understanding of network traffic conditions, and many studies have been reported on traffic control strategies and network state estimations based on the MFD's concept (Daganzo 2007, 49-62, Geroliminis et al. 2012, Haddad and Geroliminis 2012, 1159-1176, Horiguchi et al. 2010, 700-705, Keyvan-Ekbatani et al. 2012, 1393-1403, Knoop et al. 2012, Yoshii et al. 2010). However, most of the previous works have been conducted in highway networks with real world data set or simulation environments. Thus, real world example from signalised arterial network is limited.

In order to apply the MFD to real world, reliable variables' estimations are essential. The variables include section traffic flow and density. Urban signalised network is normally equipped with stop line detectors and/or mid-link detectors. These detectors count the number of vehicles and measure the occupancy that can be converted into traffic density. Although density is a key variable in the MFD analysis, the signalised networks' density estimation from the point measurements cannot be straightforward. Unlike the case of freeway networks, where uninterrupted flow is expected, signalized arterials are characterized with stop-and-go behaviour, and the density from the point measurement cannot be a representative of the whole section. In particular, the occupancy (or the density) obtained from the stop line detectors are highly affected by the stopping vehicles during red time phases (Wu et al. 2011, 255-266). Therefore, the development of sound estimation methods is essential.

This study proposes a framework for vehicle density (or vehicle accumulation) estimation in signalised arterial network, and then, shows empirical examples of the MFD with real world data set. Firstly, a density estimation method is formulated with stop line detector counts, signal phase and probe vehicle samples. Then, the method is applied to Brisbane network. Here, network is partitioned into several regions in order to confirm the difference in the MFD

shapes between whole network and regions. Based on these, future research needs and practical applications are discussed.

2. Overview of Macroscopic Fundamental Diagram

The idea of the MFD was first introduced by Godfrey (1969, 323-327.), and re-initiated later by Mahmassani et al. (1987) and Daganzo (2007, 49-62). Recently, the MFD observed with dynamic features in congested urban network in Yokohama by Geroliminis and Daganzo (2008, 759-770). The MFD was defined as the relationship between area 'production', the weighted average of flow of all links, and 'accumulation', the weighted average of density. The analysis results, from a microscopic simulation of San Francisco Business District (SFBD) and a field observation in downtown Yokohama, showed that well-defined MFD exists for homogeneously congested areas, while conventional flow-density relationships for individual links displayed highly scattered plots. Such a crisp shape MFD, which relates the area-wide network traffic performances (throughput) and the area density, well describes network performance. Thus, a number of studies on perimeter control strategies and an area traffic state estimation utilizing the MFD concept has been reported (Knoop et al. 2012, Horiguchi et al. 2010, 700-705, Geroliminis 2007, Daganzo 2007, 49-62, Yoshii et al. 2010).

The key requirement for the well-defined MFD with less scatters is the homogeneity of the area-wide traffic condition, which is not universally expected in the real world. Buisson and Ladier (2009, 127-136) further investigated the MFD shape in heterogeneous environments. Based on the analyses, carried out using the real data set from a medium-sized city network in France, they figured out that network types and unusual events such as incidents have a strong impact on the MFD shapes. In order to further clarify the necessary condition for well-defined MFD, Mazloumian et al. (2010, 4627-4647) and Geroliminis and Sun (2011, 605-617) have identified that the spatial distribution of link densities is the key factor for defining the MFD shape. The findings suggest that MFD can be applied for unevenly congested network if the network can be partitioned in homogeneous zones. Based on these finding, Ji and Geroliminis (2012, 1639-1656) investigated the methodology of the network partitioning into compact shape zones, where well-defined MFD was expected and perimeter control can be applied based on the MFD concept.

The previous works have given theoretical insights for the MFD study and its applications. However, limited works have been done with real world dataset. Thus, this work aims to show empirical examples of the MFD from the signalised arterial network of Brisbane. Also, the Brisbane road network consists of radial routes, while previous works mostly focused on grid networks. Therefore this work will be the first attempt to show the MFD of radial-type road network with less circular routes, where drivers have less chance of route choices.

3. Density estimation on signalised arterials

3.1 Density estimation techniques

A number of literatures on density estimation models exist for signalized arterial sections. The models include classical input-and-output procedures (Sharma et al. 2007, 69-80, Webster 1958), the use of Kalman filtering with occupancy as a measurement (Papageorgiou and Vigos 2008, 1-17, Qian et al. 2012, 50-56, Vigos and Papageorgiou 2010, 312-321), the model based on kinematic wave theory (Liu et al. 2009, 412-427), and the use of mobile sensor to estimate intersection delay and queue length (Ban et al. 2011, 1133-1156). This previous research has shown promising results with possible direction for further improvements. However, the study sites have been limited to the sections without mid-link sink and/or source points, where conservation of vehicles is expected within the section.

When the vehicles' conservation is assumed, density is available simply from vehicles' cumulative counts at upstream and downstream of the section, i.e., the subtraction of cumulative counts at downstream from the one at upstream gives the number of vehicles

existing in the link. However, this method is subject to the counting errors of detectors and mid-link sources and sinks, which violate the vehicles' conservation assumption and cause significant errors in estimation. In order to overcome this issue, CUmulative plots and Probe Integration for Travel timE estimation (CUPRITE) (BHASKAR et al. 2009, 41-54, BHASKAR et al. 2010, 151-163, BHASKAR et al. 2011, 433-450) model has been proposed, which integrates probe vehicle data with cumulative counts. Although it is originally designed to estimate link travel time, CUPRITE can calculate the link density at the same time.

3.2 CUPRITE for Density Estimation

CUPRITE integrates probe vehicle samples with cumulative plots. By introducing probe samples that traverse the whole section, the cumulative plots are modified and the counting inconsistencies due to counting errors and/or mid-link sinks and sources can be reduced or cancelled. This method draws reliable cumulative plots at upstream and downstream intersections. This research employs the vertical distance of the plots, the number of vehicles in the section, for the section density estimation.

Here, probe vehicles are the vehicles equipped with vehicle tracking equipments. We assume that the times when the probe vehicle is at upstream (t_u) and downstream (t_d) locations are accurately obtained. The travel time of this vehicle is $t_d - t_u$. We define the rank of the probe vehicle in the cumulative plots as $D(t_d)$ (given that downstream counts are correct and we fix probe's rank with downstream cumulative plots D(t) and define the point through which upstream plots U(t) should pass.

Figure 1 and Figure 2 summarise the CUPRITE architecture for density estimation assuming a mid-link sink case, where upstream detector is overcounting (for detail, refer to BHASKAR et al. (2011, 433-450)).

Step 1: Cumulative plots are defined with upstream and downstream detector counts. If the detector data is individual vehicle counts (pulse data) or the aggregation period is small enough (e.g. 20 seconds), then the cumulative plots are obtained by simply accumulating the detector counts. However, if the detector counts are aggregated (for instance every 5 minutes), then the cumulative counts do not reflect the actual traffic fluctuations within the aggregation intervals. These fluctuations can be captured by integrating the detector counts with signal phase data (Refer to BHASKAR et al. (2011, 433-450) for details on how to integrate loop and signal data for estimation of cumulative plots).

Step 2: Probe data (the list of $([t_u] \text{ and } [t_d])$ is fixed with downstream cumulative plots and the rank for each probe vehicle is defined $[D(t_d)]$.

Step 3: Points through which U(t) should pass are defined from the list of $[t_u]$ and $[D(t_d)]$.

Step 4: U(t) is redefined by vertical scaling and shifting the plots so that it passes through the points defined in Step 3 (Refer to BHASKAR et al. (2011, 433-450) for details about Step 2 to Step 4).

Step 5: Finally, average density (or number of vehicles in the section) is defined as the vertical distance between the plots.

Figure 1 Illustration of CUPRITE for density estimation







4. Data and study site descriptions

4.1 Stop line detector and signal phase data

The signal controls in Brisbane surface streets are equipped with Sydney Coordinated Adaptive Traffic System (SCATS). The signalised intersections are centrally controlled and the data from the controller is logged and stored by Brisbane City Council. The detector counts and signal phase for this study are collected from SCATS traffic reporter and history reader.

The vehicle counts are measured at stop lines for each lane and aggregated every 5 minutes. The aggregated counts are integrated with the signal phase data, where the counts during the signal red phase are assigned to be zero, and counts are uniformly assigned only during the signal green phase within the detection interval. Refer to BHASKAR et al. (2010, 151-163) for the methodology for integration of signal timings with aggregated traffic counts from detector data for accurate representation of cumulative plots.

4.2 Bluetooth records as probe vehicle samples

Bluetooth scanners have been installed at major intersections by Brisbane City Council for traffic monitoring purposes. The Bluetooth scanners record has a particular range of coverage area (e.g. 100 meter radius), within which the scanner detects any Bluetooth equipped devices such as mobile phones, laptops and other electronic devices, and records their Media Access Control (MAC) ID and Timestamp. MAC ID is a unique anonymous identifier allocated to each device. By matching MAC IDs at two different locations, the travel time can be calculated as the difference of the timestamps. If vehicles with active Bluetooth device are recorded at two successive intersections, the difference of the upstream and downstream passing time gives the individual vehicles' space-mean travel time. Thus, the Bluetooth record can be used as probe vehicle samples.

4.3 Bluetooth data cleansing

As discussed in Kieu et al. (2012) and Tsubota et al. (2011), Bluetooth data includes various transportation modes (such as pedestrians, bicycles and atypical vehicles such as couriers). This causes significant scatter in Bluetooth travel time profile. The segregation of the records from different modes is out of the scope of this research. However, outlier filtering is essential before applying to density estimation. Here, the filtering method is briefly explained.

The Median Absolute Deviation technique is applied for removing outliers in travel time profile. The method defines Upper Bound and Lower Bound Values (UBV and LBV, respectively) from sample median and mean absolute deviation from the median (MAD). The samples larger than the UBV or smaller than LBV are considered as outliers.

$$UBV = median + \sigma f \tag{1}$$

$$LBV = median - \sigma f \tag{2}$$

Where σ is the standard deviation from the MAD, which is approximated as $\sigma = 1.4826 \times MAD$, assuming the data being normally distributed. MAD is defined as

$$MAD = median(|X_i - median(X_i)|).$$
(3)

The value of σf defines the scatter of the sample. The parameter *f* is a scale factor that determines the gap between UBV and LBV. For this study, *f* = 2 is chosen as suggested in Kieu et al. (2012). Figure 3 shows the example of filtering results. The grey dots show the outliers, whereas the red dots are considered as valid plots and used for CUPRITE application for density estimation.





5. Brisbane network MFD

5.1 Definition of MFD

The MFD is defined as the relationship between network 'production', the weighted sum of flows of all links, and 'accumulation', the weighted sum of link densities (both quantities are weighted with the link lengths – units of lane-kilometres). In this study, the sections are defined with two consecutive intersections equipped with Bluetooth scanners. The section flows are measured at the downstream stop line of the sections and aggregated every 5 minutes. The average section densities are estimated with the method outlined in 3.2. Figure 4 illustrates the section variables in this study.

Then, the area average flow Q and density K are calculated according to the following definitions for every 5 minutes (time t is omitted from the equations).

$$\boldsymbol{Q} = \frac{\sum_{i} \boldsymbol{q}_{i} \boldsymbol{l}_{i}}{\sum_{i} \boldsymbol{l}_{i}},\tag{4}$$

$$K = \frac{\sum_{i} k_{i} l_{i}}{\sum_{i} l_{i}},\tag{5}$$

Where q_i and k_i are the flow and the density of section *i*, respectively. l_i is the length of section *i*.

Figure 4 Conceptual illustration



5.2 Study site

Figure 5 shows the Brisbane network. The yellow dots are intersections equipped with Bluetooth scanners in October 2012. White lines show the major arterials. Brisbane network mostly consists of radial routes heading to the CBD area with few connections between radial routes. Study sections are defined as consecutive dots along the routes. Figure 5 shows all the 184 intersections equipped with Bluetooth scanners in October 2012. However, some scanners are not available depending on the days and times. Also, stop line detector counts can include many error records due to unexpected incidents such as road works and signal malfunction. Such records cause significant errors in cumulative plots and the density estimation. In this study, the records from the intersections with malfunctioning Bluetooth scanners or loops are removed. This has resulted in 155 intersections' data instead of Bluetooth equipped 184 intersections.

Figure 5 Bluetooth scanner locations



5.3 Results

Figure 6 illustrates the whole Brisbane MFD for 22nd October 2012. The plots consist of 155 intersections (301 sections). The plots show well defined relationship with very less scatter. This result suggests the existence of the whole network MFD in Brisbane network. However, the diagram only shows free flow regime, i.e. the area average flow increases as the density increases.

Figure 7 summarises the MFD for different time of the day to see how the MFD captures the congestion dynamics. In the morning, the system starts getting filled after 6AM and the both density and flow take the maximum around 8AM. Then, as the accumulation decreases, the flow also decreases. After 9AM, the morning peak finishes and the plots stay at a point during off peak hours (shown as a red circle). In the afternoon, the network starts getting congested again after 3PM and shows the peak around 5:20PM. Then, the MFD goes back toward the origin point.

Figure 6 Whole Brisbane MFD (22 Oct 2012)



Figure 7 Whole Brisbane MFD for different times of the day (22 Oct 2012)



In order to further investigate the network characteristics, the network is divided into 4 regions considering physical barriers (i.e. river) and the network shape. Figure 8 shows the 4 regions used in the following analysis. Figure 9 summarises the MFDs from different regions. When comparing the MFDs' shapes of different regions, one can find a peak in the MFD of region 2. When the density reaches 50 (veh/km), the flow takes maximum, and then, starts slightly decreasing, where the system becomes the critical condition. Such shape is not found in the whole network MFD (Figure 6). This confirms that network partitioning helps better understanding of network performance as suggested in Tsubota et al. (2013) based on simulation experiments.

Figure 8 Brisbane regions



Figure 9 MFDs for different regions (22 Oct 2012)



Figure 10 illustrates the whole Brisbane MFD for 23nd October 2012. The same sections as used on the 22nd are served for the analysis. The plots show less scattered diagram as obtained on 22nd (Figure 6). However, when we separate the morning and afternoon plots, different maximum flows are observed. In the morning, the maximum flow stays around 100 (veh/5min), whereas, in the afternoon, the peak reaches 120 (veh/5min).



Figure 10 Whole Brisbane MFD (23 Oct 2012)

■ AM (12AM-12PM) □ PM (12PM-12AM)

By dividing the MFD into regions, significant scatters are found in the morning in region 3 (Figure 11). In this morning, extremely long travel time is observed in some sections in region 3 due to a major accident. At 1:00AM in the morning on 23rd, a semi-trailer crashed into a house along Ipswich Road at Annerley, a major arterial connecting southern suburbs and the CBD. Due to this, the southbound (outbound) of the Ipswich Road had to be closed for rescue until 8:40AM (Calligeros 2012).

Figure 12 shows Bluetooth travel time plots of some of heavily congested sections. The plots show that some vehicles experienced 10-20 minutes to travel the 1-1.5 km sections around 8:30AM. This is 4 to 5 times more than usual. The southbound of Ipswich Road was seriously congested (see the graph A in Figure 12) due to the road closure. Northbound was also affected (see the graph B in Figure 12), although northbound was not subject to the closure. This is thought to be an impact of drivers' rubbernecking behaviours around the accident area.

The scatters in the MFDs of region 3 reflect these local congestions (Figure 11). In region 3, the AM plots deviate from the normal plots at 6:00AM, and the density increases with taking less flow values. At 8:30AM, the density takes the maximum, and then, the plots go back to the normal line at 9:00AM following the anti-clockwise loop. This way, the MFD captures the impact of local congestion to the regional traffic condition.





Figure 12 Local congestions on 23 Oct 2012



6. Discussion and Conclusion

This work first proposed traffic density estimation method for signalised arterial sections, and then showed some empirical examples of the MFD of Brisbane network with real data set. The results confirmed that the MFD with well defined shape exists in Brisbane network. Although the whole network diagram shows only the free flow regime, the plots well captures the congestion dynamics. Also, by partitioning the network into several regions, the MFD exhibits different shape in region 2 with critical regime (Figure 9). This suggests the importance of the network partitioning for appropriate representation of the network performance. The analysis conducted on the other day showed the impact of local congestions on the regional MFDs. Significant scatters are found in region 3 (Figure 11) due to abnormal congestion in some sections, which was not well captured in the whole network MFD (Figure 10). This also confirmed the importance of network partitioning for better understanding and monitoring of the network conditions.

Above findings support further research needs on the MFD for network surveillance and control. For instance, different partitioning strategies should be tested for better representation of the network performance. Brisbane network consists of typical radial routes with few connections with each other. The routes head to the CBD area, and normally, the congestion builds up from the centre toward the outer area. For this type of network, the partitioning based on the distance from the CBD might be more appropriate, such as concentric circles like public transport fare zones. Appropriate network zoning helps to understand how congestion develops from zones to zones, and to identify the areas that work as bottlenecks. Then, in order to evaluate the network performance, appropriate indicators based on the MFD need to be developed. Traditional performance indicators have mainly focused on single sections' evaluation. Further investigation is essential for network performance index.

This work will contribute to zone based network monitoring and control. Traffic congestion has spatially wider impact, whereas most conventional performance indicator represents the local congestion profile, which could prevent traffic operators from having the whole picture of the congestion. Monitoring based on the properly partitioned map would help in assessing the impact of local congestion to area-wide performance, and also in implementing area-wide traffic management schemes such as inflow regulation and road pricing.

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