

# **Strategic road pricing schemes accounting for demand uncertainty**

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## **Abstract**

Transport network pricing is a topic of interest to transport planners and users alike. Pricing schemes are a vital traffic management strategy which can be implemented to reduce congestion, among other network externalities. Previous research has explored pricing schemes to achieve a variety of network objectives, including social and environmental concerns. This work introduces a robust strategic system optimal (StrSO) pricing scheme which incorporates a novel user equilibrium-based behavioural model, specifically the strategic traffic equilibrium (StrUE) model. Prices are set on all links in a network to improve expected system performance by lowering expected system travel time, thus relieving congestion. The proposed solution method is scalable to large networks. The results show how the introduced measures of system performance can be used to investigate the relationship between congestion and reliability in the tolled network.

## 1. Introduction

Transport network road pricing is a topic of great interest to researchers and practitioners alike. It is one of the primary management tools available to road operators to improve network performance for the benefit of the system as a collective. Additionally, a well-planned tolling scheme will not only help relieve congestion, it can also produce a profit that will help operators expand and maintain infrastructure for a stronger, more reliable system.

Road network pricing research has a well-established foundation in the literature. One common research topic is marginal social cost (MSC) pricing, based on the economic ideas of Pigou (1920). This pricing scheme assumes users behave in a "selfish" manner, seeking to minimize their own travel costs. Prices are then set on each link such that a user is charged a toll equivalent to the marginal impact of her using a given link (i.e., the increase in travel cost to everyone on a link resulting from a single additional user). This is also referred to as first best pricing, in which all links in a network are priced. Second best pricing represents an extension of this problem, in which a subset of the network links are tolled.

While the first best pricing problem can be easily solved, a complexity is introduced when demand uncertainty is considered. In the short term, users face a varying day-to-day travel demand. For longer term planning, unpredictable changes in land use, technology, and many other factors make demand forecasts difficult. These inherent network uncertainties must be accounted for in pricing models to ensure they are robust to future changes in travel demand. The success of a particular project relies on accurately predicting tolling profits. Around the world, a surprising number of failed tollway projects have consistently relied on poorly forecasted demand for modelling, and suffered the consequences (Bain, 2009).

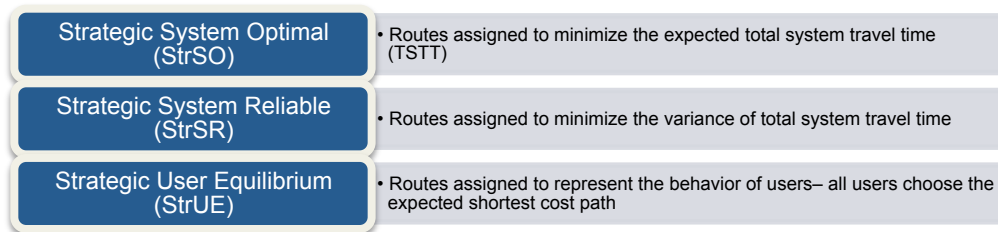
A particular example of this can be found in the well known case of the Sydney Cross City Tunnel (Phibbs, 2008). This was a public-private partnership project intended to connect the eastern suburbs to the western suburbs of Sydney that opened in 2005. Unfortunately, the operating company went into receivership less than two years after the tunnel opened, and it is estimated that \$220 million dollars of initial investment has been lost. While there are many complex factors that lead to the ultimate failure of any project, most agree that an important contributor to Cross City Tunnel case was the poorly forecasted demand values. It was estimated that a daily demand of 90,000 vehicles would use the tunnel, while the actualized number was closer to 30,000. Another complaint was that the toll was much too high and discouraged people from using the tunnel. While this is an extreme example, the importance of accounting for factors of uncertainty, particularly when it relates to the financing of an important public project, cannot be underestimated. A more detailed analysis of tollways in Australia and the impact of inaccurate demand forecasts can be found in Zheng et al (2011).

This work specifically explores a robust first best tolling framework in which demand uncertainty is accounted for. First best tolls are important both as an exploration of system behaviour and mathematically, even if they may not be feasible to implement at the current time for a large

network, such as Sydney. Second best tolls - in which optimal tolls are identified for a subset of links - is a challenging mathematical problem that will be explored in future work.

The contribution of this work lies in the novel behavioural model that is used to determine the user route choice under the assigned toll values. In the strategic equilibrium based assignment model implemented, referred to as StrUE, users determine route choice based on the expected shortest cost path for a known distribution of the demand. The assignment model output is a set of fixed link flow proportions which defines the link flow patterns. Then, on any given day, the actual link flow volumes will be a function of these fixed proportions and the actual realized demand. Furthermore, the link flows from any given demand realization will not necessarily conform to a state of equilibrium, representing the chaotic network behaviour observed in reality. This model therefore incorporates demand uncertainty in a novel way. The traditional first best tolling methodology will be extended to incorporate this new assignment model, and the network performance under the computed tolls will be explored. A summary of strategic modelling approaches is contained in Figure 1.

**Figure 1. An outline of the strategic traffic assignment modelling approach**



The research questions motivating this work address the relationship between uncertainty and pricing, and are as follows:

1. How do we derive marginal social cost prices for the StrUE model?
2. How do marginal social cost prices perform under the StrUE assignment model?
3. How does demand uncertainty impact network performance under the proposed pricing scheme?
4. What measures can we use to quantify the performance of this pricing scheme?

This paper begins with a literature review covering the topics of network uncertainty, first best MSC pricing, and the strategic traffic assignment approach. In Section 3, we describe the StrUE MSC (StrMSC) pricing problem and our measures of effectiveness of this model. Section 4 outlines the solution methodology, while Section 5 demonstrates the models on two example networks. A conclusion and discussion of future research directions concludes this work in Section 5.

## 2. Background

Marginal social cost pricing based on Pigouvian (Pigou, 1920) taxes has a rich history in the literature. This method aims to set tolls in such a way that a collective system optimal behaviour is induced, rather than drivers choosing routes unilaterally to minimize their own travel time

(selfish behaviour) (Yang et al, 1998); (Newbery, 1990). While maximizing social welfare by relieving congestion may be a common goal from public planning agencies, many other objectives have also been explored, among those aims that may represent the interests of private tolling agencies, such as: maximizing revenue, minimizing tolling locations, and minimizing the maximum toll collected (Hearn et al, 1997); (Hearn et al, 2001).

The tolling framework addressed in this work is classified as first best, which means that it is possible to toll every link in the network in order to achieve some objective.. Second best tolling scenarios, in which not all links in the network are available to be tolled because of political or social restrictions, have also been well-explored in the literature (Verhoef, 2002); (Lawphongpanich et al, 2004). However, in order to introduce the impact of the new strategic behavioural model on tolling, only pricing schemes in which all links in the network are priced are considered in this work.

While the pioneering works on pricing road networks assumed travel demand and other network characteristics (such as link capacity) to be fixed values, the impact of uncertainties on transport models has become another popular topic in the literature. This is particularly important for tolling scenarios, because optimal prices that are calculated for an unrealized level of demand could have an unpredictable impact on network conditions, a fact that is further discussed in by Lemp and Kockelman (2009). It is commonly agreed that the main sources of uncertainty in a transport network result from the demand (Clark et al, 2005, Duthie et al, 2011), supply (Lo et al, 2006), and behavioural choices from travellers (Damberg et al, 1996). Boyles et al (2010) examined first best pricing while accounting for uncertainty in road capacity and further looked at the impact of supplying users with information about the state of the network. This work highlights the difference between tolling schemes that respond to network conditions and tolls that are intended to address recurring, predictable congestion. Each of these sources could impact optimal toll design in different ways. Researchers begin by analysing difference sources in isolation (this work included), but more complicated models like Gardner et al (2011) that account for both uncertainty in demand and in supply may offer more realistic insights into the road network.

A number of works have approached the issue of demand uncertainty and its impact on tolling. Gardner et al (2008) examine the impact of long term demand uncertainty, such as that resulting from changes in land use, technology, and petrol prices, on robust tolling prices, and evaluate a number of approaches to solve this problem. They show that the marginal social cost tolls that are calculated using an expected demand can result in suboptimal system performance, especially when the actual system performance differs significantly from what was forecasted. Gardner et al (2010) further explore a number of solution methods for solving a similar problem, finding that using an inflated demand scenario gave the most consistently robust results. Li et al (2008) propose a bi-level mathematical programming formulation to solve for first best tolls aimed at increasing network reliability, where users' choices are determined using a multinomial logit model. Sumalee et al (2011) also examine the impact of stochastic demand by treating both network demand and link flows as random variables. This work addresses uncertainty in user behaviour by considering how different risk attitudes from users

might impact pricing results, which is additionally a method of incorporating users' value of travel time reliability. Li et al (2012) extend this model to find the optimal tolls with the objective of minimizing emissions.

This research differs from previous contributions in its novel behavioural model to capture the strategic decisions of users. Strategic traffic assignment was introduced by Dixit et al (2013), and finds equilibrium flows based on expected path costs. A strategic model in this context assumes that users in the network have *learned* about the distribution of the demand, implying that expected value and the variance of the demand distribution are known values. Users base their route decision on the expected shortest cost path, following this decision independent to the actual realization of the demand. This model results in link volumes that will vary from day-to-day, thus accounting for short term demand uncertainty that users face making day-to-day route choice decisions. Waller et al (2013) propose a linear formulation for a dynamic version of the strategic problem that finds optimal route flows across a discrete set of possible demand scenarios. Additionally, Duell et al (2013) introduced the strategic system optimal (StrSO) and strategic system reliable (StrSR) models that are applied in the current work, and furthermore examined the relationship between the three models on a number of test networks.

This work extends the strategic assignment model to a StrMSC first best pricing application. The novel approach to addressing demand uncertainty will result in different tolling schemes from previous work in the area. Additionally, the unique approach in this work allows us to introduce a new set of measures of system performance.

Finally, it is worth noting that the methods proposed here are only one small part of the ultimate decision-making process for any tollway project. Non-technical factors such as a bias toward optimism and politically and/or economically motivated misrepresentation, as well as social attitudes towards congestion pricing, are all important factors that play a role in toll prices that are actually used in practice (Flyvbjerg, 2008). In an interesting look at tollway projects in Australia, Davidson (2011) includes a case study for a potentially representative case for the Go Between Bridge in Brisbane. In this paper he noted the frequent changes to forecasted demand values used during the modelling process and described ways in which this value was misused. Such practices indicate that accounting for demand uncertainty when modelling tollway projects is a challenging and timely problem recognized by practitioners, and is indeed one of the recommendations to improve toll modelling made by Davidson in the conclusion of his paper.

### 3. Problem description

This section describes the proposed pricing model, including details about the underlying equilibrium model and assumptions. Additionally, the measures of effectiveness used to compare the model performance are defined.

The StrUE model provides the proportion of the total demand  $p_a$  that will choose to travel on each link  $a$ . Users choose their routes based on the *expected* cost of a path, which can be disaggregated to the link level. The expected link travel time,  $t_a$  is a function of the proportion travelling on that link and the expected total number of travellers. In a first best pricing model,

each link is subject to a toll,  $\tau_a$ . For a StrUE model with first best tolls, the expected cost to a user on link  $a$  is:

$$E(t_a(pT)) = t_a(pT) + VOTT\tau_a \quad [1]$$

where the VOTT (value of travel time) reflects the amount in dollars a traveller is willing to pay to reduce their trip by a unitary amount of time. This parameter has been empirically quantified for different networks. For simplicity, here we will assume that  $VOTT$  is such that users value travel time and money an equal amount, therefore  $VOTT = \$1/minute$ .

The first part of Equation [1] describes the travel time that a user would expect to experience when there is no toll. Often in traffic modelling approaches, this function is assumed to be some volume-delay function, in which the travel time on a link becomes greater as there are more travellers on that link. This work employs the well known BPR function (U.S. Department of Commerce, 1964), however using link proportions and a random variable  $T$  with probability distribution  $g(T)$  representing the total number of trips made. The link volume-delay function used in this research is displayed in Equation [2].

$$E(t_a(p_aT)) = t_f(1 + \alpha M_\beta \left(\frac{p_a}{c_a}\right)^\beta) \quad [2]$$

Where that  $t_f$  is the free flow travel time on link  $a$ ,  $c_a$  is the capacity of link  $a$ ,  $\alpha$  and  $\beta$  are shaping parameters based on link geometry.  $M_\beta$  is a term that represents the analytical  $\beta$ th moment of the demand distribution, and is not present in the traditional BPR function.

The second part of Equation [1] is the travel cost in units of time, based on the monetary toll value applied to the link. As previously noted, the concept of a marginal social cost is extended to the StrUE model to compute the link toll. In order to calculate optimal toll value  $\tau_a$  for link  $a$ , the proportion of flow on that link that results in the minimal expected total system travel time,  $\bar{p}_a$ , needs to be computed using the StrSO model. The toll for each link is equal to the product of the optimal link proportion and the gradient of the expected link cost function, evaluated at the optimal proportion of flow.

$$\tau_a = \bar{p}_a \frac{dt_a(\bar{p}_aT)}{dpT} \quad [3]$$

After deriving the gradient of the expected link cost function, the toll for each link can be computed using Equation [2]:

$$\tau_a = t_f \alpha \beta M_\beta \left(\frac{\bar{p}_a}{c}\right)^\beta \quad [4]$$

The expected travel time on the link is defined by [2], and the applied toll is defined by [4]. Thus, the total expected link cost experienced by the user will be the sum of [2] and [4] multiplied by VOTT. The total cost to users aggregated across the network and aggregated across all users represents one measure of performance for evaluating the network.

The inclusion of uncertainty in the proposed pricing model demands an additional measure of performance to quantify the variance in link travel time (dependent on the realized travel

demand). Link travel time variability can be quantified using the standard deviation  $\sigma(t_a)$  of the travel cost  $t_a$ . This measure was derived in Dixit et al (2013) and is defined in Equation [5].

$$\sigma(t_a) = \sqrt{\frac{2\beta t_{fa} \alpha^2}{c_a^{2\beta}} (M_8 - M_4^2) p_a^{2\beta-1}} \quad [5]$$

### 3.2 System level measures of performance

In order to evaluate the network level performance the expected total system travel time (E(TSTT)) is analytically computed as a measure of congestion, the standard deviation of TSTT ( $\sigma(TSTT)$ ) is computed as a measure of reliability, and the expected revenue, E(R) is computed as a measure of financial success.

The expected total system travel time E(TSTT) is a system performance measure derived by Duell et al (2013) for an untolled network. The *expected travel time* experienced by users will remain the same in a tolled network, although there will be an additional cost experienced by individuals that is not included in this measure of performance. The E(TSTT) is presented in Equation [6].

$$E(TSTT) = \tilde{z}(p) = \sum_a (t_f p_a M_1 + t_f \alpha \frac{p_a^{\beta+1}}{c^\beta} M_{\beta+1}) \quad [6]$$

Quantifying a system level variance is a basic measure of reliability. A system with a higher variance will see more fluctuation in individual link travel times, and thus will be less reliable. The system level measure is a reflection of the sum of the individual link variances ( $\sigma(TSTT)$ ) was derived in Duell et al (2013) for an untolled network. Equations [7] and [8] describe the variance of travel time.

$$\bar{\bar{z}}(p) = \sum_{a \in A} \left[ p_a^2 t_{af}^2 (M_2 - M_1^2) + 2 \frac{\alpha t_{af}^2}{c_a^\beta} p_a^{\beta+2} (M_{\beta+2} - M_{\beta+1} M_1) + \frac{\alpha^2 t_{af}^2}{c_a^{2\beta}} p_a^{2\beta+2} (M_{2+2\beta} - M_{\beta+1}^2) \right] \quad [7]$$

$$\sigma(TSTT) = \sqrt{\bar{\bar{z}}(p)} \quad [8]$$

Finally, the network operator is likely interested in the expected revenue from this tolling scheme. The revenue is equal to the proportion of the flow on each link which "pays" the toll multiplied by the first moment of the demand distribution, which is simply the total number of expected trips.

$$E(R) = \sum_a p_a M_1 \tau_a \quad [9]$$

Where  $M_1$  is the first analytical moment of the demand distribution, which is equal to the expected value of the demand.

### 3.3 System level measures of effectiveness

The motivation behind marginal social cost tolls is to improve social welfare, which can be accomplished by reducing network wide travel cost. Under uncertainty, this is mathematically

equivalent to reducing the  $E(TSTT)$ . The lowest  $E(TSTT)$  is achieved by StrSO, which is the assignment model used to set the tolls. When tolls are applied, the  $E(TSTT)$  can be computed using StrUE. The percentage difference between  $E(TSTT)$  from the StrUE model without tolls and the StrUE model with tolls can be used to define a measure of effectiveness,  $\Delta E$ , which is defined in Equation [10]. This represents the amount of congestion that has been relieved by the applied pricing scheme. A larger  $\Delta E$  equates to a less congested network under tolling, thus a successful pricing scheme. If  $\Delta E$  is negative the pricing scheme has actually caused congestion to increase relative to the un-tolled network.

$$\Delta E = 1 - \frac{E(TSTT)_{StrUE+tolls}}{E(TSTT)_{StrUE}} \quad [10]$$

Another important measure of effectiveness is variability in total system travel time. A system with less variability will be more reliable for users. In particular, the network operator will be interested in how much more reliable the network is under the proposed pricing scheme. Similar to equation [11],  $\Delta\sigma$  reflects the percentage decrease in standard deviation of of network TSTT resulting from tolling.

$$\Delta\sigma = 1 - \frac{\sigma(TSTT)_{StrUE+tolls}}{\sigma(TSTT)_{StrUE}} \quad [11]$$

A summary of the system performance measures and their interpretations can be found in Figure 2.

**Figure 2. System performance measures for the strategic approach**

$E(TSTT)$	• The total amount of travel time expected in the network (minutes)
$\sigma(TSTT)$	• The standard deviation of total system travel time – a measure of reliability and robustness (minutes).
$\Delta E$	• The reduction in travel time due to the tolls (%)
$\Delta\sigma$	• The reduction in standard deviation due to the tolls (%)

## 4. Solution Methodology

The previous section described the pricing model and performance metrics that will be used to evaluate the proposed model. This section briefly summarizes the solution methodology and model assumptions. The following assumptions are necessary in order to solve the StrUE with MSC tolls model:

- The demand for each origin-destination pair is proportional to the expected demand; that is, the proportion of the total demand that travels between each OD pair does not change. The change in observed link volumes results from the range of actualized demand values.



- Link travel times are independent, which implies that the  $\sigma(TSTT)$  does not include any covariance terms.
- There is no error in user perception. This implies that all users know the expected shortest cost path and do not make errors in their route choice. This is a common assumption in traffic assignment models, although using logit choice models is another option which may be investigated further in the future.
- The VOTT of all network users is uniform and equal to \$1/minute, implying that users equally value travel time and money. Lacking network specific data, this assumption is employed for model tractability. However, assuming that all network users have the same VOTT, then this assumption affects only the E(R) by a factor. It does not affect other performance measures.
- The demand fits a lognormal distribution (with known mean and variance). Therefore the higher level moments may be solved analytically as:

$$M_s = \exp(s\mu + 0.5s^2\sigma^2) \quad [12]$$

- The link volume-delay relationships are represented using the BPR function.

The following describes the steps used to solve the proposed pricing model:

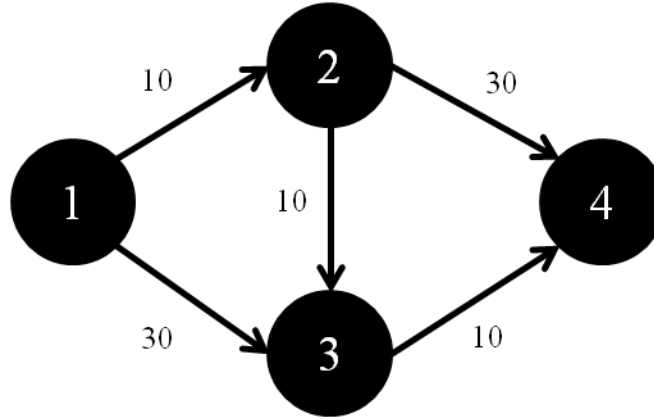
1. Solve the StrUE model and determine base case  $E(TSTT)$  and  $\sigma(TSTT)$ ;
2. Solve the StrSO model to determine the link flows  $\bar{p}$ ,  $E(TSTT')$  and  $\sigma(TSTT')$ ;
3. Determine the marginal cost tolls using Equation [3];
4. Solve the StrUE model with the applied tolls and compute the  $E(TSTT)$  and  $\sigma(TSTT)$ ;
5. Calculate the performance measures listed in Section 3.3.

## 5. Results and Discussion

This section contains a demonstration of the StrMSC model on two example networks and discusses the implications of varying levels of demand volatility, defined by the coefficient of variation, on network performance.

### 5.1 Example network

Figure 3 shows the simple example network used to illustrate the approach to solving the StrMSC model. This is similar to the Braess's paradox network, consisting of four nodes and five links. It is well known that due to the equilibrium behaviour of users, the  $E(TSTT)$  of the network in Figure 3 is greater than it would be if the link connecting nodes two and three were to be removed. In the case of strategic marginal social cost tolling, a network manager is able to determine optimal toll values to reroute users to achieve a less congested system performance. The demand between (1,4) is lognormally distributed with a mean of 100 and with a standard deviation of 5.

**Figure 3. Demonstration network**

The StrUE model results in an  $E(TSTT)$  of 1,465 minutes with a  $\sigma(TSTT)$  of 559 minutes. When the tolls are applied, the  $E(TSTT)$  is reduced by 30% and the  $\sigma(TSTT)$  by 70%.

**Table 1. Network performance measures for the demonstration network**

	<i>StrUE</i>	<i>StrMSC</i>	<i>StrSO</i>
$E(TSTT)$ (min)	1,465	1,021	1,014
$STD(TSTT)$ (min)	559	170	146

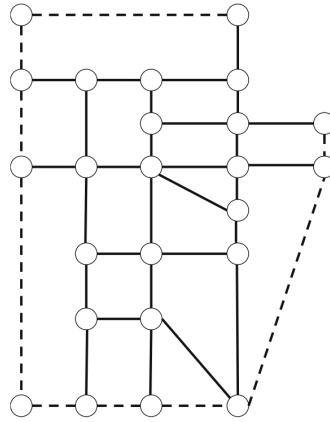
Table 2 displays the proportions of the aggregate demand on each flow that are the output of the StrUE, StrSO, and StrMSC models, as well as the toll “price” that was charged on each link. In a deterministic model, the flow patterns resulting from the equilibrium model and the MSC model would be exactly the same; however, with the added uncertainty inherent in the StrMSC approach, the resultant flow patterns are not exactly equal to the StrSO model.

**Table 2. Link proportions and MSC tolls from the demonstration network**

<i>Link</i>	<i>StrUE</i>	<i>StrSO</i>	<i>StrMSC</i>	<i>Tolls</i>
1-2	0.899	0.646	0.680	22.88
1-3	0.101	0.355	0.321	6.26
2-3	0.798	0.291	0.360	0.09
2-4	0.101	0.355	0.319	6.26
3-4	0.899	0.646	0.681	22.89

## 5.2. Example grid network

A larger test network was chosen to demonstrate the scalability of the strategic MSC tolling approach. Figure 4 illustrates this network. This is a common test network in transport literature based on Sioux Falls, South Dakota, consisting of 24 nodes and 76 links. All nodes are both origins and destinations. Network and demand information was obtained from Bar-Gera (2011). The demand for the example grid network followed a lognormal distribution with a mean of 360,600.

**Figure 4. Example grid network**

The volatility of the demand is defined by the coefficient of variation of a demand scenario. In highly volatile systems, the aggregate demand has a higher standard deviation, implying that the demand actualization is fluctuating between a greater range of values. This section examines the impact of demand volatility on the StrMSC model by performing an experiment in which the coefficient of variation is varied from 0 – 1.0 in increments of 0.05 and the StrMSC pricing model is solved for each level of demand volatility.

The results of this experiment on total system travel time are displayed in Figure 5. The horizontal axis of Figure 5 shows increasing coefficient of variation. Additionally, Figure 5 contains two system performance measures; the left vertical axis contains the value for  $E(TSTT)$ , corresponding to the dashed blue line. The right-hand vertical axis shows the percentage decrease in the system congestion achieved by the volatility scenario. These results are represented by the red line,  $\Delta E$ .

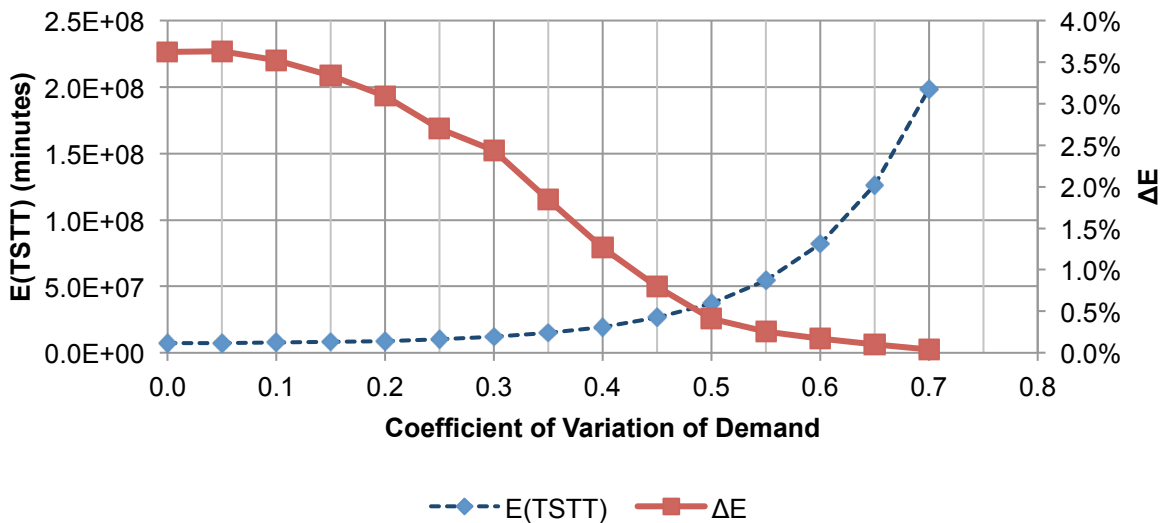
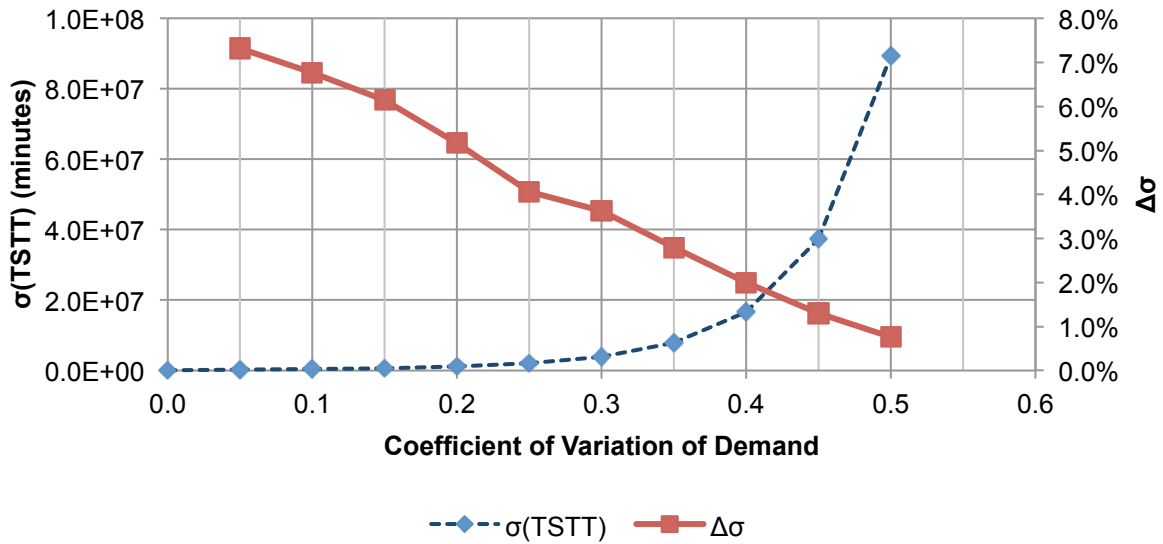
**Figure 5. Impact of demand volatility on  $E(TSTT)$  in example grid network**

Figure 5 showed that as demand uncertainty increased, so did  $E(TSTT)$ . However, the opposite was true for  $\Delta E$ . This implied that the reduction in  $E(TSTT)$  that was possible decreased significantly in highly volatile systems. Figure 5 only displays results when the coefficient of variation is between 0 - 0.7, although the sharp increase in  $E(TSTT)$  continues. However, such volatile systems were considered unrealistic and not shown here. The  $\Delta E$  was the greatest in systems with less volatility, which implies that planners can achieve a greater system improvement when the demand actualizations do not vary as much. With a coefficient of variation greater than 0.7, both StrUE and StrSO identified the same solution, meaning that no improvement in system performance was possible.

The  $\sigma(TSTT)$  and its corresponding performance metric  $\Delta\sigma$  reflect the reliability of the system to its users. A system with less variation in total system travel time will experience less fluctuation in individual link travel times. Thus, it is also important to examine the effects of demand uncertainty on the  $\sigma(TSTT)$ . These results are shown in Figure 6. Again, the horizontal axis reflects an increasing level of volatility in the system, while the left-hand vertical axis shows the  $\sigma(TSTT)$  and corresponds to the blue line, and the right-hand vertical axis shows the  $\Delta\sigma$ , corresponding to the red line.

**Figure 6. Impact of demand volatility on  $\sigma(TSTT)$  for the example grid network**



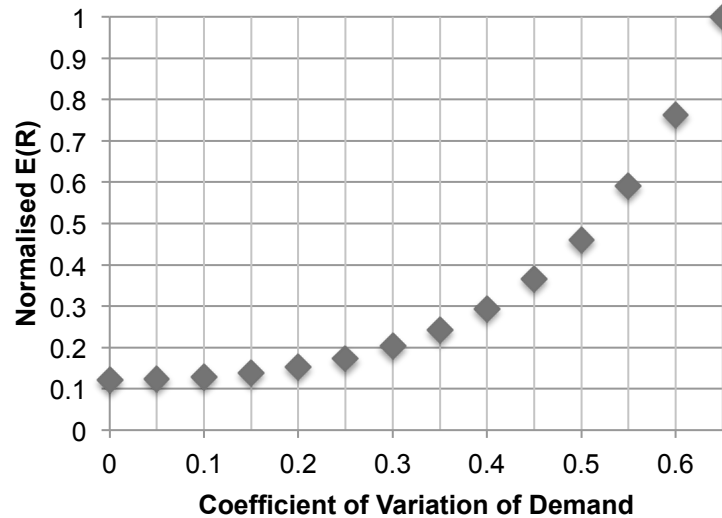
The  $\sigma(TSTT)$  increased as the volatility associated with the demand increased in a way that looked to be similar to the increase in  $E(TSTT)$ . However, the  $\Delta\sigma$  is the greatest in less volatile systems, and decreases in a linear manner to almost zero when the coefficient of variation was greater than 0.6. When  $\Delta\sigma$  is near zero, no pricing scheme is able to improve system conditions. For the example grid network, the greatest improvements in reliability were possible in the less volatile systems.

Another factor of interest in the StrMSC pricing model is the amount that users pay to tolling operators. However, this measure will be dependent on the VOTT utilized in the tolling framework. This work assumes a value of 1, meaning that people equally value time and money,

which is not realistic. However, if we assume that all people in the network have the same monetary value of time, then the  $E(R)$  value calculated from the StrMSC model will be inflated by some factor, and the relative change in  $E(R)$  due to varying levels of system volatility is still a measure of interest.

Figure 7 contains the  $E(R)$  with respect to different levels of demand volatility. Similar to previous experiments, the  $E(R)$  value was found as an output to the StrMSC model while varying the coefficient of variation between 0-0.65 in increments of 0.05. However, in order to display the relative behaviour, the  $E(R)$  has been "normalised" such that the greatest  $E(R)$  was set equal to one, and the other  $E(R)$  values were adjusted accordingly; therefore, Figure 7 contains no information about the magnitude of  $E(R)$ . Figure 7 show that the  $E(R)$  increases nonlinearly as the system volatility increases. This observation suggested two things; first, the scenarios with a high volatility of demand require users to pay a higher cost, but they experience less improvement in the system performance (which was observed in Figures 5 and 6). However, when users are paying a higher cost, the  $E(R)$  is also higher, meaning this may be a more desirable scenario for a toll operator.

**Figure 7. The impact of varying volatility of demand on  $E(R)$**



## 6. Conclusion

This work proposed a novel method to determine marginal social cost tolls accounting for the inherent uncertainty in demand that users face when making individual route choice decisions on a day-to-day time frame. Determining optimal tolls is an important practical problem. We describe the StrMSC tolling framework, as well as a number of system performance measures that planners can use to examine the performance and reliability of the tolled network. One example network suggested that as there is a higher amount of volatility associated with the demand, the amount of system improvement that can be achieved in the network decreases.

In the future, this work will examine a pricing scheme that is based on the Strategic System Reliable formulation (Duell et al, 2013), that will instead charge users for the additional system travel time variability that is added due to their individual route choice. Furthermore, this approach accounts only for a short term uncertainty that users face while choosing routes, not the long term planning uncertainty that is also a prevalent factor in transport models. Future work will need to explore methods of accounting for additional sources of uncertainty, both long term travel demand and that resulting from the supply. Additionally, the StrMSC pricing model should be compared to pricing schemes that account for uncertainty using alternative approaches.

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