Car Following Model Improvement for Traffic Safety Metrics Reproduction

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Abstract

Car Following models have a critical role in all microscopic traffic simulation models. Current microscopic simulation models are unable to mimic the unsafe behaviour of drivers as most are based on presumptions about the safe behaviour of drivers. Gipps model is a widely used car following model embedded in different micro-simulation models. This paper examines the Gipps car following model to investigate ways of improving the model for safety studies application. The paper puts forward some suggestions to modify the Gipps model to improve its capabilities to simulate unsafe vehicle movements (vehicles with safety indicators below critical thresholds). The result of the paper is one step forward to facilitate assessing and predicting safety at motorways using microscopic simulation. NGSIM as a rich source of vehicle trajectory data for a motorway is used to extract its relatively risky events. Short following headways and Time To Collision are used to assess critical safety event within traffic flow. The result shows that the modified proposed car following to a certain extent predicts the unsafe trajectories with smaller error values than the generic Gipps model.

1. Introduction

In recent years, traffic researchers have attempted to make current Car Following (CF) models more realistic through enhancement in a range of simulation software and through suggestions for new models. In more than 60 years since the first CF model was introduced, diverse models using different techniques have been proposed. CF is a primary sub-model for each microscopic simulation model. Early models, Gazis–Herman–Rothery (GHR) (Gazis et al., 1959, Herman et al., 1959, Rothery, 1997) and linear models (Pipes, 1967) have only following ability. Later and for the first time, safety distance or collision avoidance (CA) models set up a safety distance. The Gipps (1981) model, the most successful CA model, can switch between free flow and following situations. Among all the CF models, psychophysical models are completely dissimilar. Driver's performance is modeled as sequential control, reacting against few measures (thresholds).

Since CF entails the interaction of nearby vehicles in the same lane, it has a significant role in traffic safety studies. Moreover in new technologies, such as Advance Vehicle Control Systems, CF has been used to mimic driver actions (Brackstone and McDonald, 1999). The potential of micro-simulation to evaluate safety related factors has been recognized by some research (Archer and Kosonen, 2000, Barceló et al., 2003, Bonsall et al., 2005). Even though there has been slight advancement in applying these models to analyze traffic safety, some safety studies using microscopic simulation have been undertaken particularly at intersections. Archer (2005)

used micro-simulation within signalized and unsignalized urban intersections, calculating some safety indicators which are based on the concept of "conflicts". He compared these indicators in both simulated and observed situations. Unrealistic microscopic behavioral sub-models of car-following, lane changing and gap acceptance are the main concern for safety measurement. These models need to reflect human behavior more realistically.

This paper chose the Gipps CF model to investigate its ability for safety study purposes. Gipps CF model is widely used in several microscopic software packages. Lee and Peng (2005) report that Gipps CF model has the best presentation of human driver behavior in their research.

Two most widespread sorts of crashes on motorways are rear-end crashes and sideswipe crashes. Rear-end crashes are the attention of this paper, and consequently CF should be under investigation rather than Lane changing models. Accordingly, in order to evaluate safety within microscopic simulation models, mainly Time to Collision (TTC) and Short following headways are selected, which directly show the following metrics. It should be noted that in this work, the anticipation ability of follower drivers on cars farther downstream is ignored and only immediate leader is considered as the main source of stimulus. The authors are going to include the anticipation ability of drivers in future works.

The main focus of this research is the motorway safety problem. This paper, in section 2, explains the real data that has been used. The Gipps CF model structure is briefly introduced and the calibration process of the simulation models is also briefly illustrated. Later in section 3, the Gipps model performance results are demonstrated. Headway and TTC reproduction in the Gipps model is specifically analyzed and the role of the "*simulation step*" is examined. In section 4 some modifications within the Gipps model are proposed based on the observation of Gipps model's results. The modified CF models performance indicator is compared with generic Gipps model. An improvement within safety indicators is also shown in the modified models. Finally the conclusions are presented in section 5.

2. Real data and modelling section

2.1 Safety indicators

(TTC) is a proximal safety indicator initially introduced by Hayward (1972). These measures are the most accepted quantitative metrics in conflict studies (Chin and Quek, 1997). TTC is broadly used and the latest work by Oh and Kim (2010) employed it as a metric of rear-end collision potential. Hayward (1972) described TTC as "the time required for two vehicles to collide if they continue at their present speed and on the same path". The Time-to-Collision of a vehicle-driver combination i at instant t with respect to a leading vehicle i - 1 can be calculated with:

Equation 1:

$$TTC_{i} = \frac{X_{i-1}(t) - X_{i}(t) - l_{i}}{\dot{X}_{i}(t) - \dot{X}_{i-1}(t)} \qquad \forall \dot{X}_{i}(t) > \dot{X}_{i-1}(t)$$

Where: \dot{X}_i : Speed, X_i : position l: Vehicle length.

Several other studies employed TTC, to evaluate traffic safety. Minderhoud and Bovy (2001) reported from the literature several values for critical time to collision, namely 4 second or 5 or 3 or 3.5 seconds. Time Integrated TTC (TIT) (Bonte et al., 2007) is similar, but this time it is an integral value of the TTC profile, once TTC is below the threshold. TIT can present an index of severity. So this indicator is chosen to choose the trajectories which are critical.

Headway is also considered in this paper. "*Time headway is one of the indicators that is used to estimate the criticality of a certain traffic situation*" (Vogel, 2003). In most of these situations,

headway is a measure of driver risk. Evan and Wasielewski's research on a large number of vehicles within the traffic stream shows that accident-involved and offensive drivers have a "higher level of risk in every day driving" that means they have close following distances (Evans and Wasielewski, 1982). Evans and Wasielewski (1983) found again "offensive drivers, young drivers, male drivers, drivers with no passenger, drivers who did not wear a seatbelt" have shorter headways in their driving habits. A similar study conducted later in Finland by Rajalin et al. (1997) confirms the Evan research results, but this time drivers with close following distances (recorded in one spot upstream in a two lane highway) were asked the reason for their short headways. The main justifications from the drivers were being in hurry or a "desire to overtake". As a result short headway is in consideration.

2.2. The trajectory data (NGSIM)

Trajectories of vehicles within traffic flow are a rich source of information in investigating any microscopic behavioral sub-model. The chosen safety indicators can only be calculated by trajectories' data. Fortunately, vehicle trajectories on a motorway are freely available from Next Generation Simulation, (US Department of Transportation FHWA, 2009). Researchers for the NGSIM program collected comprehensive vehicle trajectory information on southbound US 101, Hollywood Motorway, in Los Angeles, CA, on June 15th, 2005. The area covered was roughly 640 meters in length and included five mainline lanes. In this paper 15 minutes of data are used: from 7:50 to 8:05 a.m. including around 3000 vehicle. The trajectory data includes quantified measures such as speed, position, headways, acceleration and so forth.

Many studies used NGSIM data to support their models, calibration and validation process, (Zhang and Bham, 2007, Shengnan and Ghulam, 2007, Punzo et al., Kan and Bham, 2007, Ghods and Saccomanno, Brockfeld and Wagner, 2006). Punzo (2011) in a comprehensive review on the trajectory based research stated that only in 2007 and 2008 more than 30 studies used NGSIM data. However very few raised the issue of the NGSIM accuracy, among them are (Hamdar and Mahmassani, 2008, Thiemann et al., 2008, Punzo et al., 2011). Although the way that Cambridge Systematic, Inc in NGSIM calculates speed and acceleration from space trajectories and how they reduced the error is not publicly known, the mentioned studies report errors and noises existence within NGSIM data. This paper assumes that NGSIM data is satisfactorily accurate as any probable error within NGSIM data would be the same for either part of the model comparison.

2.2.1. Selecting the critical trajectories

The major focus of this research is CF behavior, among the available trajectories, those in which either themselves or their leader experience lane changes was omitted. The focus is also on safety thus the worst trajectories are chosen in terms of two safety criteria: shortest TTCs and shortest headways. 251 trajectories remain and the rest of study is applied to these trajectories. The procedure to choose the critical trajectories used the following criteria. The safety indicators are quantified using each specific safety indicator and the vehicle measure from NGSIM data.

- The trajectories should not have LC experience.
- All vehicles should have at least one TTC below 3 seconds.
- Vehicles should have headways less than 1.50 seconds.
- The filtered trajectories from the above, are sorted from large to short TIT values (5 to 0.1 existed in 251 trajectories records)

2.3. The Gipps CF Model

The base CF model in this paper is the Gipps CF model and therefore the Gipps model is introduced here. Basically the Gipps model proposed speed according to Equation 2. Gipps (1981) assumes that the follower car can estimate all the parameters with the exception of b_{n-1} . He proposed to use \hat{b} instead of b_{n-1} . He didn't explain what this actual amount is.

Equation 2:
$$v_n(t + \tau) = \min \left\{ v_n(t) + 2.5a_n\tau(1 - v_n(t)/V_n) \sqrt{\frac{0.025 + v_n(t)}{V_n}} \right\}$$

 $b_n\tau + \sqrt{b_n^2\tau^2 - b_n[2[x_{n-1}(t) - s_{n-1} - x_n(t)] - v_n(t)\tau - v_{n-1}(t)^2/\hat{b}]}$

 a_n : The maximum acceleration which the driver of vehicle n wishes to undertake; b_n : the most severe braking that the driver of vehicle n wishes to undertake; s_n : the effective size of vehicle n. that is. The physical length plus a margin into which the following vehicle is not willing to intrude, even when at rest; V_n : the speed at which the driver of vehicle n wishes to travel; $x_n(t)$: the location of the front of vehicle n at time t; $v_n(t)$: the speed of vehicle n at time t; τ : the apparent reaction time, a constant for all vehicles; \hat{b} : The driver of vehicle n estimation about b_{n-1}

2.4. Model calibration

The sum of Theil's Inequality Coefficient which is defined as Equation 3 and is more sensitive and accurate than other errors is selected as the objective function to be minimized. It should be noted that a combination of speed and space terms are used to be minimized: y = (0.5 *v) +x. This combination is reported to provide a better calibration results. Five parameters are selected as the variables, namely: a_n , T, b_n , B and θ . All of these variables are defined in the previous section, except θ which is "a further safety margin by supposing that the driver makes allowance for a possible additional delay of θ "(Gipps, 1981).

Equation 3:

$$\varepsilon_{model} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - x_i)^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}y_i^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}} \qquad x_i: \text{ simulated, } y_i: \text{ real value at time } i, n: \text{ number of observation}$$

A Genetic Algorithm (GA) is implemented to search for global solutions for minimizing the objective variables. Since each driver has his/her own behavior characteristics, each individual trajectory is optimized specifically. In this way simulation models can get the best performance of each tested model. The feasible limits according to reported values in the literature for each parameter are defined as a function's constraints.

2.5. Model evaluation

To evaluate either the generic or modified Gipps CF models the following measures are used.

• The RMSE which is the square root of the variance of the residuals.

Equation 4:
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\frac{x_i - y_i}{y_i})^2}$$

• The ME: Mean Error

Equation 5: $ME = \frac{1}{N} \sum_{i=1}^{n} \frac{|x_i - y_i|}{x_i}$ Where x_i : simulated value y_i : real value at time i, n: number of observation

3. The Gipps CF model performance to reproduce safety indicators

3.1. TTC and headway exploration in the Gipps CF model

To examine the safety indicators of TTC or short headway created by the Gipps CF model, individual real trajectories and their simulated profiles need to be looked at precisely. For this reason a pair of vehicles as a sample is modeled. The Gipps model is written in Matlab codes. To examine real behavior of the Gipps model, the simulation is implemented for two different simulation steps: 0.1 and 1 second. Figure 1 and Figure 2 show that once simulation step is 0.1 and the Gipps is updating more often, the smallest simulated TTC is about 7 seconds. However once simulation step is increased to 1 second, lower TTCs like 3 seconds can also be created. Moreover speed profile at Figure 1 shows that speed profile also varies more in simulation step of 1 second. This indicates that any time the Gipps model is applied, the safe distance is preserved. Short TTCs actually happen at the time that Gipps does not update. In this situation the real leader might sharply decelerate and the follower does not react until the next simulation step. It should be noted that the number of simulated TTC points in Figure 1 and Figure 2 should be compared to themselves only, because in simulations step of 0.1 second, the number of studied points is higher. The question is what the smallest TTC is once the Gipps model is applied? Or if the Gipps CF is applied more often, what TTC values will occur?



Figure 1: Headway, TTC and speed profile at simulation step 0.1 second





To explore how the Gipps model creates TTCs a leader and a follower are artificially modeled. Different scenarios are tested. Leader speed is 13 m/s, while in each time follower speed is changed from 16 to 9 m/s. Figure 3 indicates one example of these tests in which the follower a velocity of 16 m/s is showed. What actually happens is to calculate simply speed from Equation 2 by changing spacing. It shows that the Gipps model until spacing of 20 meters, always predicts a lower speed for follower than leader and for more than 20 meters spacing, it gives a higher speed for the follower than leader which causes TTC. Hence the minimum TTC is 6, even when the initial speed of follower is high like 3 m/s higher than the leader in the graph. The assumption here is that leader does not change its speed, and it is constant. Apart from the value of TTC it is expected that the frequency of TTCs should be much less than observed. This is in a situation where the Gipps CF model is applied. In other words if the Gipps model is applied, the minimum feasible TTC in simulation is about 6 seconds which is not a dangerous event.





In NGSIM in which almost all of the spacing is below 20 meters, if apply the Gipps model with very short update interval it can never get small TTC except in the initial points. To examine the real distribution of TTC in the NGSIM data refer to Figure 4. To assign a distribution function to TTC distribution at NGSIM data, a general extreme value properly suits NGSIM TTC values in Figure 4.

Figure 4: Observed NGSIM TTC data (0.1 second observation), left figure: TTC frequency bar chart right figure: TTC below 100 seconds distribution and a fitted General extreme value distribution



In the left figure the Density in y-axis refers to frequency divide by total number of the observed data

3.2. Simulation step role in the Gipps mode to reproduce headways and TTCs

Individual trajectory study shows that simulation steps in the Gipps model can change TTCs and headways. This hypothesis should be tested with a large number of vehicles. The Gipps CF model is implemented for all chosen NGSIM data. For NGSIM data it can be seen that increasing a simulation step, which means updating the safe distance of the Gipps model less often, causes more TTC events close to the observed numbers (Figure 5). This situation makes safety indicators considerably dependent on the leader vehicle's maneuvers. For instance in the time between simulation steps, if the leader breaks sharply the TTC decreases dramatically, because the Gipps model is not applying the safe distance until the next simulation step. As a result in Figure 5 the numbers of small TTCs increases once simulation step increases. In simulation steps of 0.5 second, the number of TTC occurrences is smaller than in observed. In simulation step 0.1 second, TTCs in model are almost one tenth of observed.



Figure 5: Real Vs Simulated TTC frequency, Simulation step left to right: 1, 0.5 and 0.1 seconds TTC Frequency Diagram TTC Frequency Diagram TTC Frequency Diagram

In terms of TTC distribution, according to the Figure 6 diagrams, the higher simulation step the higher portion the smaller TTCs will be. For instance, in the left hand figure for simulation step 1 sec, the number of small TTCs in the model is higher than real data. In simulation step 0.5 second the distribution of TTCs is the most similar distribution to a real situation. Although the distribution of 0.5 second matches very well with real data, the frequency of TTC events are about 25% (Figure 5) of a real situation. In simulation step 0.1 second the portion of small TTCs are much less than in reality. Overall simulation step should be calibrated for the Gipps model to present a better safety indicator reproduction.





A similar circumstance takes place for headway distributions. There would be a simulation time that presents the most appropriate distribution of headways and accordingly a more accurate number of short headways. In Figure 7 again the 0.5 second simulation step is illustrated showing the best agreement with reality distribution.

Figure 7: Simulated Vs Real headway distribution in overall NGSIM data simulated with the Gipps CF model. Simulation step left to right: 1, 0.5 and 0.1 seconds



Overall, examining the critical safety indicators distribution in the NGSIM data, it seems that drivers adjust their safe distance in half a second. Consequently choosing a best simulation step for safety metrics may not be best for other traffic metrics in the simulation model, namely for capacity, ramps performance, average speed. Up to now the Gipps model does not offer a right TTC frequency though it presents a good distribution, of course with a specific simulation step (here 0.5 second). In the next step it is expected that applying a modification especially according to driver behavior, can improve the Gipps model's manner in producing TTCs.

4. Modifications

To achieve the right number of critical TTCs in this Section a couple of modifications are applied to the Gipps CF model to improve the model abilities to represent safety indicators with a better accuracy.

4.1 First Modification: Human Perception limitation

The applied modification to the Gipps model is a psychophysical concept. Observing the individual trajectories (Figure 1), highlights that the real follower does not react to every small action of its leader. There is minute fluctuation in the real follower while the Gipps model follower reacts to every small action of its leader. The reason for the real follower behavior is because humans cannot detect any small changes of speed or spacing under certain thresholds. "In a car-following task, the abilities to detect changes in distance and velocity are critical" (Yang and Peng, 2010). It is not reasonable, that drivers respond to every small action of the leader and as a result a psychophysical notion saying humans cannot perceive speed differences under a specific threshold. This notion could be interpreted as oscillation situation which is the situation that driver will not pass any threshold. Psychophysical models such as Wiedmaan model (1992) expect such a behavior. Driver will alternate a small negative and positive acceleration alternatively. In real driving never acceleration of vehicle is zero. A small acceleration always exists. The modification that is implemented here is not exactly the same notion, but it is similar. In this modification to the Gipps model, driver will not adjust her/his safe distance unless she/he percept a recognizable change in speed. The range of 0.05 till 0.2 is reported in the literature for the noticeable speed difference (Harris and Watamaniuk, 1995, McKee, 1981, Brown, 1960). In this paper test 0.2 is tested. As a result Equation 6 is applied:

Equation 6:
$$v_n(t) = \begin{cases} v_n(t) & \text{if } \frac{\Delta v_n(t) - \Delta v_n(t-1)}{\Delta v_n(t-1)} \ge 0.2 \\ v_n(t-1) & \text{otherwise} \end{cases}$$

This means that if the driver's speed difference exceeds the driver cognition threshold, 0.2, the Gipps model should be applied to calculate new speed; otherwise the previous speed should be

kept. It obviously decreases the unrealistic noises in the speed profile (Figure 8). This method may help to create more realistic critical TTCs. However it does improve the speed and space profile. Trajectories at Figure 8 show a better match with observed. Figure 8 shows that the modified model has fewer fluctuations than the generic Gipps model. In this test, the simulation step is 0.5 seconds.



Figure 8: The Gipps model Vs the psychophysically modified model

To examine the overall impact of the modification as it is applied to Gipps CF model, for all chosen 251 vehicles, root mean square error, RMSE and mean error, EM are calculated (Table 1). The modification improves space, speed, headway and TTC values. Moreover the overall number of TTC events at Figure 9 is also slightly improved compared with the pure Gipps model in Figure 5. At the same time TTC and headway distribution are still good.

Figure 9: Results of the psychophysical modification, from left to right: TTC frequency bar chart, TTC distribution and Headway Distribution diagram



Table 1: RMSE for the modifications of the Gipps and the generic Gipps

Model Status	RMSE x	RMSE v	RMSE a	RMSETTC	RMSE Hdwy	EM x	EM v	EM TTC
The Original Gipps	6.9	1.20	1.97	1.31	0.12	0.38	1.95	1.72
1- Psychophysical	6.24	1.18	1.98	1.11	0.14	0.37	1.95	1.63
2- deceleration phase change	7.03	1.32	2.50	1.45	0.13	0.39	1.84	1.69

• x:space, v: speed, a:acceleration, Hdwy: headway,

• Error for TTC below 3 sec and headway errors for headways less than 1.5 sec is calculated

4.2. Second modification: speed adjustment depending on driving phase

Precisely examining several trajectories determines that in most of the time in deceleration phase (sharp breaking), followers have higher speed than leaders and in contrast in acceleration phase followers has a slower speed than leaders. The trajectories are shown in Figure 10 is a good example.



Figure 10: Speed profile for a pair of vehicles in both deceleration and acceleration phase

To just not rely on few trajectories, all speed distributions have been explored. Figure 11 shows real NGSIM data speed differences distributions only in deceleration time which it is interpreted as the time that the leader has negative acceleration rate. The mean of distribution of speed difference between leader and follower is -0.9 (m/s) and standard deviation of 1.59 (m/s) which means in deceleration time the follower has in average 0.9 (m/s²) higher speed than its leader. This can prove the hypothesis which has been observed in single trajectories. The generic Gipps model shows a good speed deviation, as a result only by shifting the speed values with $\mu = 0.9$ (m/s), It is expected that the frequency of TTCs improves.

Figure 11: Observed speed differences per (m/s) in deceleration phases in all the NGSIM data



To apply this notion, the following commands have been applied. If simulation step assumed to be equal to 0.5 seconds, then:

$$\begin{array}{ll} \textit{if} & v_{leader}(t) - v_{leader}(t-1) < \textit{speed difference threshold} \\ \textit{then} & v_{follower}^{modifed}(t) = v_{follower}^{Gipps}(t) + \mu \\ & \mu = +0.9 \left(\frac{m}{s}\right) \\ \end{array}$$
 Calibrated for the NGSIM data

The speed difference threshold in the above formula is a negative value which according to the observation once simulation step is equal to 0.5 second, -0.5 m/s is a reasonable value which means a deceleration rate sharper than -1 m/s². In Figure 12 the effects of applying this modification can be seen. The frequency of TTCs below 3 seconds increased compared with the generic Gipps in Figure 5. Number of TTC equal to 1 and 2 seconds are almost equal to observed. However TTCs equal to 3 second is higher than real figures. Overall for deceleration time according to Figure 12, although the distribution is not appropriate, the numbers of critical TTCs are closer to the observed. This modification improvement is that without increasing the simulation step, achieving higher frequencies of critical safety indicators are possible. On the other hand the RMSE as shown in Table 1 slightly worsens while the EMs improved for speed and TTC.

Figure 12: Results of modification in speed in deceleration phase, from left to: TTC frequency bar chart, TTC and Headway distribution diagram



4.3. Discussion

According to the results of the two modifications few points can be concluded. The first psychophysical modification does not significantly increase the frequency of critical events; however it greatly improves the speed and space trajectories. On the other hand the second modification which just targets deceleration phase, notably improves critical TTC frequencies, though it decreases the accuracy of the speed and space profiles. According to the effects from the both modifications, a combination of them may improve the Gipps CF model in two perspectives. A presumable combination of both changes on the Gipps CF model can enhance the speed and space profiles and on the other hand the frequency of TTC critical events.

5. Conclusion

The Gipps CF model has been precisely examined and explored to find out ways of improving the model for safety study purposes. The weakness of the Gipps CF model has been illustrated. It has been shown that the simulation step has a vital role in the Gipps model to reproduce safety indicators. The outcome of this research can aid evaluating and predicting safety by microscopic simulation at different traffic facilities. NGSIM trajectory data is used to mine its unsafe events. Short following headways (less than 1.5 seconds) and Time To Collision (less than 3 seconds) is used to evaluate serious safety events within the traffic stream.

By looking at the trajectory details a few hypothesis have been developed that can potentially improve the Gipps CF model for safety studies. The modifications have been introduced to the Gipps model to get better abilities to simulate unsafe vehicle movements. The modifications included:

- psychophysical modification and applying human perception limitation
- Improvement in deceleration phase of CF model

The result shows that the proposed modifications on the Gipps CF predict the unsafe safety indicators frequency better than the generic Gipps model. They can also improve or worsen some of generic performance metrics such as speed, space or acceleration profiles. The modifications also can cause that the Gipps model lose its simplicity.

Further research is needed to test the two modifications together. There are other potential improvements that could be found by a more detail exploration in the individual trajectories behaviour. Any CF model that includes human errors also makes the model more suitable as safety assessment tools. There is also a real demand to solve the simulation step and reaction time interference within the CF models performance results.

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