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# Public-Transport Automated Timetables using Even Headway and Even Passenger Load Concepts 

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#### Abstract

Public-transport (PT) timetables and their compliance mirror the quality of the PT service provided. Hence, vehicles departing too early or ahead of schedule need to be restrained, just as those leaving late must be scheduled or rescheduled to be on time. Because of existing problems of PT reliability, there is need to improve the correspondence of vehicle-departure times with passenger demand instead of assuming that passengers will adjust themselves to given timetables (excluding situations characterized by short headways). With the advance in technology of passenger information systems, the importance of even and clock headways is reduced. This allows for the possibility to create more efficient schedules from both the passenger and operator perspectives. This work contains a methodology framework with developed algorithms for the derivation of vehicle departure times (timetable) with either even headways or even average loads and with a smoothing consideration in the transition between time periods. The procedures presented are accompanied by examples and clear graphical explanations. It is emphasized that the PT timetable is one of the predominant bridges between the operator (and community) and the passengers, and thus its improvement will increase the level-ofservice for the PT passengers.


Main subject and keywords: Public transport, timetables, passengers’ level of service.

## INTRODUCTION

The Public Transport (PT) timetable is one of the predominant bridges between the operator (and community) and the passengers. Therefore more attention should be provided for the construction of the timetable in order to improve its correspondence with fluctuating passenger demand.

In general terms, the PT operational planning process includes four basic components performed in sequence: (1) network route design, (2) setting timetables, (3) scheduling vehicles to trips, and (4) assignment of drivers (crew). It is desirable for all the four components to be planned simultaneously to exploit the system's capability to the greatest extent and maximize the system's productivity and efficiency. However this planning process is extremely cumbersome and complex, and therefore seems to require separate treatment of each component, with the outcome of one fed as an input to the next component. This work is related to Component (2) of the planning process.

## Literature Review

The problem of finding the best dispatching policy for PT vehicles on fixed routes has a direct impact on constructing timetables. This dispatching-policy problem, which has been dealt with quite extensively in the reviewed literature, can be categorized into three groups: (1) models for an idealized PT system, (2) simulation models, and (3) mathematical programming models.

The first group, idealized PT systems, was investigated by, among others, Newell (1971), Osana and Newell (1972), Hurdle (1973), Wirasinghe (1990, 2003), and De Palma and Lindsey (2001). Newell (1971) assumed a given passenger-arrival rate as a smooth function of time, with the objective of minimizing total passenger waiting time. He showed analytically that the frequency of transit vehicles with large capacities (in order to serve all waiting passengers) and the number of passengers served per vehicle each varied with time approximately as the square root of the arrival rate of passengers. Osana and Newell (1972) developed control strategies for either holding back a transit vehicle or dispatching it immediately, based on a given number of vehicles, random round-trip travel times with known distribution functions, and uniform passenger-arrival rates with a minimum waiting-time objective. Using dynamic programming, they found that the optimal strategy for two vehicles and a small coefficient of variation of trip time retained nearly equally spaced dispatch times. Hurdle (1973), investigating a similar problem, used a continuum fluid-flow model to derive an optimal dispatching policy while attempting to minimize the total cost of passenger waiting time and vehicle operation.

Wirasinghe (1990, 2003) examined and extended Newell's dispatching policy while considering the cost components initially used by Newell (1973). Wirasinghe considered the average value of a unit waiting time per passenger $\left(\mathrm{C}_{1}\right)$ and the cost of dispatching a vehicle ( $\mathrm{C}_{2}$ ) to show that the passenger-arrival rate in Newell's square root formula is multiplied by $\left(\mathrm{C}_{1} / 2 \mathrm{C}_{2}\right)$. Wirasinghe also showed how to derive the equations of total mean cost per unit of time by using both uniform headway policy and Newell's variable-dispatching policy.

De Palma and Lindsey (2001) develop a method for designing an optimal timetable for a single line with only two stations. The method is suitable for a situation in
which each rider has a precise time in which they want to travel; travelling earlier or later than desired increases the total cost. The objective is to minimize riders' total scheduledelay costs. Two cases are analyzed with respect to passenger preferences. In the first case, all passengers treat a unit of delay equally. The second case assumes several rider groups, with different level of delay costs ascribed to riders from the different groups. In addition, the researchers compared two models: a "line" model, in which preferred travel times are uniformly distributed over part of the day and trips cannot be rescheduled between days; and a "circle" model, in which preferred travel times are uniformly distributed over the full 24 -hour day and trips can be rescheduled between days. Optimal timetables are derived for each of the models.

In the second group, simulation models were studied by, for example, Marlin, Nauss, and Smith (1988), Adamski (1998), and Dessouky et al. (1999). Marlin et al. (1988) developed a simulation model for dispatching t PT vehicles every day. They checked the feasibility of the results and used mathematical programming for vehicle assignments in an interactive computer-support system. Adamski (1998) employed a simulation model for real-time dispatching control of transit vehicles while attempting to increase the reliability of service in terms of on-time performance. His simulation implemented optimal stochastic control with linear feedback. The use of intelligent transportation systems was applied by Dessouky et al. (1999) in a study of bus dispatching at timed transfer points. The researchers used a simulation analysis to show that the benefit of knowing the location of the bus was most significant when the bus was experiencing a significant delay, especially when there was a small number of connecting buses at transfers point.

Mathematical programming methods, the third group for determining frequencies and timetables, have been proposed by Furth and Wilson (1981), Koutsopoulos et al.(1985), Ceder and Stern (1984), Ebelein et al.(1998), Galla and Di-Miele (2001), and Peeters and Kroon (2001). Furth and Wilson sought to maximize the net social benefit, consisting of ridership benefit and waiting-time saving, subject to constraints on total subsidy, fleet size, and passenger-load levels. Koutsopoulos et al. extended this formulation by incorporating crowding-discomfort costs into the objective function and treating the time-dependent character of transit demand and performance. Their initial problem consisted of a non-linear optimization program relaxed by linear approximations. Ceder and Stern addressed the problem with an integer programming formulation and a heuristic person-computer interactive procedure. Their approach focuses on reconstructing timetables when the available vehicle fleet size is restricted. Ebelein et al. (1998) studied a special dispatching problem for the purpose of introducing dead-heading trips in high-frequency transit operations. They solved their dispatching strategy optimally; they also determined the number of stops that could be skipped in order to minimize total passenger cost in the system.

Galla and Di-Miele (2001) produced a model for the special case of dispatching buses from parking depots. Their model is based on the decomposition of generalized assignments and design, non-crossing, and matching sub-problems. It can be extended to a case in which there is an overlap between arrival and departure- vehicle flows. Peeters and Kroon (2001) present a procedure for planning an optimal cyclic railway timetable; i.e., a timetable in which trains leave at the same minute every hour. The problem is represented through a constraint graph, in which each node is an event that needs to be
scheduled; cycles are examined according to a calculation of tensions and potentials. The model is formulated as a mixed-integer nonlinear program with the objectives of minimizing passenger time, maximizing timetable robustness, and minimizing the number of required trains. A solution procedure is suggested, by which the nonlinear part of the formulation is transformed into a mixed-integer linear problem that is an approximation of the original problem; further actions are taken in order to reduce the number of constraints.

In this paper another group of dispatching policy is introduced and called databased models initiated and described by Ceder (1986, 2003, 2007).

## STUDY FRAMEWORK

The bus timetable is perhaps the main reference for defining unreliable bus service. The assumption that passengers will adjust themselves to given timetables (with headways of, say, longer that 10 minutes) instead of adjusting the timetables to the passenger demand is one of the largest sources of unreliable service. When passenger load is higher than expected, the bus is slowing down (increased dwell time), behind schedule and entering the inevitable process of further slow down. This will eventually lead to the known bunching phenomenon with the buses behind (Ceder, 2007). Opposite to that is the situation of overestimating the demand which may result in buses running ahead of time. Both situations are not observed when the service is highly frequent and characterized by low variance of the headway distribution.

This work aimed at proposing and analyzing three different procedures for better matching the passenger demand with a given timetable while attempting to minimize the number of departures (leads to reducing the number of buses which is one of the main resources). This will result in a more reliable and comfortable service. Commonly, across almost all the PT agencies, the frequency is determined by the maximum load procedure. This max load procedure is established to ensure adequate space to accommodate the maximum number of on-board passengers, along the entire route, for a given time period (e.g. one hour). That is,

$$
\begin{equation*}
F_{j}=\max \left[\frac{l_{j}}{d_{j}}, F_{m j}\right] \tag{1}
\end{equation*}
$$

where $\mathrm{l}_{\mathrm{j}}$ is the average (over days) maximum number of passenger (max load) observed on-board in period $\mathrm{j}, \mathrm{d}_{\mathrm{j}}$ is the desired occupancy (load factor) in period j , and $\mathrm{F}_{\mathrm{mj}}$ is the minimum required frequency (number of buses) in period $j$. The minimum frequency is a standard set for a minimum level of service, and the desired occupancy is the planning standard of the acceptable load.

Figure 1 presents schematically the three procedures to be proposed and analyzed, along with their input and outcome. Procedure 1 produces departure times with evenly spaced headways while considering a smooth transition between adjacent hours. This procedure is based on the given standards $\mathrm{d}_{\mathrm{j}}$ and $\mathrm{F}_{\mathrm{mj}}$ for each hour j and on the j -th hourly max load, $\mathrm{l}_{\mathrm{j}}$. Procedure 2 determines departure times such that, in average sense, buses will carry on even $\mathrm{d}_{\mathrm{j}}$ loads at the hourly max load point. This procedure 2 is based on $\mathrm{d}_{\mathrm{j}}, \mathrm{F}_{\mathrm{mj}}$ and on individual bus loads at the hourly max load point where $\mathrm{l}_{\mathrm{j}}$ is observed. Procedure 3 derives the departure times such that, in average sense, the on-board
passenger load will not exceed $\mathrm{d}_{\mathrm{j}}$, and will be equal to $\mathrm{d}_{\mathrm{j}}$ at each individual bus max load point (as opposed to the $l_{j}$ points in procedure 2 ).

INPUT
PROCEDURE
OUTCOME

- Average

Hourly (j)
Max Load ( $\mathrm{l}_{\mathrm{j}}$ )

- Hourly (j)

Desired
Occupancy ( $\mathrm{d}_{\mathrm{j}}$ )

- Hourly (j)

Minimum
Frequency
( $\mathrm{F}_{\mathrm{m} \mathrm{j}}$ )


FIGURE 1 Overview of the procedures used in the study

## PROCEDURES

In this section the three procedures exhibited in Figure 1 are fully described. In addition, a numerical example is introduced to illustrate the principles and application of each procedure.

## Example Problem and Initial Analysis

The example problem is used as an explanatory device for three procedures. Table 1 contains the necessary information and data for a 2 -hour example $\mathrm{j}=1,2$ of a bus route from A to C with one stop at B. There are 5 departures observed. For each departure Table 1 contains the average observed on-board passenger at both boarding points A and B. The minimum required frequency is $\mathrm{F}_{\mathrm{mj}}=2, \mathrm{j}=1,2$. The desired occupancy is $\mathrm{d}_{1}=50$ and $\mathrm{d}_{2}=55$ passenger per bus, and the average travel time from A to B is 18 minutes.

TABLE 1. Input and observed data of the example

| Route: <br> Average <br> Travel Time |  | A <br> 18 min |  |  |  |  | B |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | Observed Departure Time at A | Average Observed Number of Passengers on-board the Bus |  |  |  |  | Hourly <br> Average <br> Maximum <br> Load <br> (HAML) <br> Point | Desired Occupancy (pass/bus) | Derived Values |  |  |  |
|  |  |  |  |  |  |  | Frequency (bus/hr) |  | $\begin{gathered} \text { Headway } \\ \text { (min) } \end{gathered}$ |  |
|  |  | $\begin{aligned} & \text { at } \\ & \mathbf{A} \end{aligned}$ | $\begin{aligned} & \text { at } \\ & \mathbf{B} \end{aligned}$ | Hourly Demand |  |  |  |  | H | I | H | I |
|  |  |  |  | $\begin{aligned} & \text { at } \\ & \text { A } \end{aligned}$ | $\begin{aligned} & \text { at } \\ & \mathbf{B} \end{aligned}$ | Ind. Max. (IM) |  |  | $\begin{aligned} & \mathbf{A} \\ & \mathbf{M} \\ & \mathbf{L} \end{aligned}$ | M | A $\mathbf{M}$ L | M |
|  | 7:15 | 30 | 65 |  |  |  |  |  |  |  |  |  |  |
| 7-8 | 7:45 | 80 | 35 | 125 | 148 | 193 | B | 50 | 2.96 | 3.86 | 20 | 16 |
| a.m. |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 8:10 | 25 | 80 |  |  |  |  |  |  |  |  |  |
| 8-9 | 8:30 |  | 72 |  |  |  |  |  |  |  |  |  |
| a.m. | 8:50 | 88 | 67 | 192 | 177 | 214 | A | 55 | 3.49 | 3.89 | 17 | 15 |

Minimum Frequency: 2 buses per hour
The estimated hourly demand is also included in Table 1. It is based on two basic assumptions: (a) the average load observed is a representative value of the actual demand and it is independent of the exact setting of departure times; (b) the passengers observed on-board are accumulated at a uniform rate. The first assumption can be realized when using the vast amount of data anticipated from equipment like APC (Automated Passenger Counters), or when the schedulers have reliable sources of information provided by road inspectors and supervisors. The second assumption usually holds when the observed headways are relatively small. For headways greater than 30 minutes, some of the passengers may time their arrival and, if data is available, this second assumption may not be needed.

Referring to the example in Table 1, the hourly demand for the first hour, between $7-8$ a.m., $j=1$, is based not only on the average loads observed, but also on part of the load observed on the first bus in the second hour, $\mathrm{j}=2$. That is, the average load on the
first bus in $\mathrm{j}=2$ is divided proportionally in order to reflect the demand at the end to the period $j=1$. Therefore, at point $A$ and $B$ the loads of 25 and 80 associated with the 8:10 departure, are divided into $3 / 5$ and $2 / 5$ where the $3 / 5$ portion is related to the $\mathrm{j}=1$ demand. This proportion is stem from the 25 minutes difference between the last departure of the period $j=1(7: 45)$ and $8: 10$ where 15 minutes of this time difference belongs to $\mathrm{j}=1$, and 10 minutes to $\mathrm{j}=2$.

The hourly demands at A and B are 125 and 148 passengers, respectively, for $\mathrm{j}=$ 1 , and 192 and 171 , respectively, for $\mathrm{j}=2$. This means that for $\mathrm{j}=1$ the hourly max load point is B with $\mathrm{l}_{1}=148$, and - point A for $\mathrm{j}=2$ with $\mathrm{l}_{2}=192$. In addition, the third column under hourly demand, in Table 1, includes a newly element called individual max hourly demand. This demand reflects the sum of the max on-board loads observed on each bus, in each hour, while considering also the proportion of max demand associated with the first bus of the next hour. The results are 193 and 214 passenger demand for $\mathrm{j}=1,2$, respectively, where, for the example of $\mathrm{j}=1$, one obtains $193=65+80+(3 / 5) \mathrm{x} 80$. The interpretation of this element is clarified under the description of procedure 3.

Finally in the last four columns in Table 1 there are the derived frequencies and headways based on equation (1). The headway $\mathrm{H}_{\mathrm{j}}$ for hour j , is simply the inverse of the frequency, and in minutes:

$$
\begin{equation*}
H_{j}=\frac{60}{F_{j}} \tag{2}
\end{equation*}
$$

Therefore, for $\mathbf{j}=1,2$ the frequencies based on the hourly max load points are 2.96 and 3.49 , respectively, and are 3.86 and 3.89 buses per hour for the individual max load hourly demand, respectively.

## Procedure 1

One characteristic of existing transit timetables is the repetition of the same headway in each time period. The scheduler, using $H_{j}$, is facing, however, a problem on how to set the departure times in the transition segments between adjacent time periods. In addition, the scheduler (or existing software) usually round-up the frequencies $\mathrm{F}_{\mathrm{j}}$ to the next integer, prior the use of equation (2). In this section it is shown that in order to save resources there is no need to round-up $\mathrm{F}_{\mathrm{j}}$ and moreover the transition between hours (or any other time periods) can be carried out in a simple and accurate manner.

## The Underlying Principle of Procedure 1

The simple way, used by many bus agencies, to smooth the headways during the transition time is to consider an average headway between two adjacent hours. This average rule may result in either undesirable overcrowding or underutilization. For example, using equations (1) and (2) one obtains $\mathrm{H}_{1}=25$ and $\mathrm{H}_{2}=9$ minutes with average of 17 minutes. Thus, a timetable can be set to 7:00, 7:25, 7:50, 8:07, 8:16, ...By assuming uniform arrival rate with $\mathrm{d}_{1}=50$ and $\mathrm{d}_{2}=60, \mathrm{j}=1$ contributes for the $8: 07$ departure $(10 / 25) \times 50=20$ passengers, for the remaining 10 minutes between 7:50 and $8: 00$, and $\mathrm{j}=2$ contributes (7/7) x $60=60$ passengers. The total is $20+60=80$ average passengers, on the 8:07 departure, representing overcrowding. In order to overcome this undesirable situation the following principle is employed.

Principle 1: Establish a curve representing the accumulative frequency versus the time (adding the non-integer value of the frequency determined with respect to time). Moving horizontally, for each departure, until intersecting the accumulative curve, and then vertically, results with the required departure times.

Proposition 1: Principle 1 provides the required evenly spaced headways with a transition load approaching the average of $d_{u}$ and $d_{u+1}$, where $d_{u}$ and $d_{u+1}$ are the desired occupancies for two consecutive time periods.

Proof: Figure 2 illustrates Principle 1 using the information in Table 1. Since the slopes of the lines are 2.96 and 3.49 for $\mathrm{j}=1$ and $\mathrm{j}=2$, respectively, the resultant headways are those required. The transition load is the one determined for the 8:01 departure, and is comprised of 20 minutes arrivals for $\mathrm{j}=1$, and 1 minute arrival for $\mathrm{j}=2$. Therefore (20/20) x $50+(1 / 17) \times 55=53$ approximately. This transition load is not the exact average between $\mathrm{d}_{1}=50$ and $\mathrm{d}_{2}=55$ since departures are made in integer minutes. That is, the exact determined departure after $8: 00$ is $(3-2.96) \times 60 / 3.49=0.688$ minutes, and inserting this value instead of 1 minutes in the above calculation yields a closer value to the exact average. Basically, the proportions considered satisfy the proof-by-construction of Proposition 1.

Figure 2 exhibits the resultant six departures for procedure 1 where the determined frequencies are kept non-integer. Principle 1, therefore, allows for saving some unnecessary bus runs and also stabilizes the average load during the transition segment between time periods.

## Procedure 2

While arriving with procedure 1 to a satisfactory timetable, with even headways, it is still unclear if the loads on individual buses will not exceed $\mathrm{d}_{\mathrm{j}}$, for all j . It is well-known that passenger demand varies even within a single time period, reflecting the business, industrial, educational, cultural, social and recreational transit needs of the community. This dynamic behaviour provides a basis for the scheduler to adjust the departure times. These adjustments are not done frequently unless there is a clear cut information (e.g. from the road supervisions) to support it. Nonetheless with the anticipated vast amount of passenger load data (e.g. from APCs) it is possible to construct procedures to better match the timetables (departure times) with the variable demand. This and the next section provide such procedures.

## The Underlying Principle of Procedure 2

The results of procedure 1 starts with the 7:20 and 7:40 departures for $\mathrm{j}=1$. The frequency required, based on the hourly max load point, is 2.96 for $\mathrm{j}=1$ (as is shown in Table 1). This frequency aimed at 50 passengers per bus while considering the entire hourly max demand. However, the assumption of uniform passenger arrival rate, between the observed departures, results in 65/15 $=4.3$ passengers/minute between 7:00 and $7: 15$ and $35 / 30=1.2$ pass $/$ min between $7: 15$ and $7: 45$ at the hourly max load point B . Therefore, the 7:20 departure (by procedure 1) may result in $65+1.2 \times 5=71$ passengers; significantly above the desired 50 passengers. In order to avoid this imbalanced situation the following principle is exploited.


FIGURE 2 Determination of the example departure times (at A) for evenly spaced headways with a smoothing process between time periods

Principle 2: Construct a curve representing the accumulative loads observed on individual buses at the hourly max load points. Moving horizontally per each $\mathrm{d}_{\mathrm{j}}$ for all j , until intersecting the accumulative curve, and then vertically, results with the required departure times.

Proposition 2: Principle 2 results in departure times such that the average max load on individual buses, at the hourly j max load point, approaches the desired occupancy $\mathrm{d}_{\mathrm{j}}$.

Proof: Figure 3 illustrates Principle 2 for the example problem appearing in Table 1. The derived departure times are unevenly spaced to obtain even loads at points $B$ for $j=1$, and point A for $\mathrm{j}=2$. These even loads are constructed on the accumulative curve to approach $d_{1}$ and $d_{2}$ for $\mathrm{j}=1, \mathrm{j}=2$, respectively. Assuming uniform passenger arrival rate between each two observed departures shows that the load (at B) of the 7:45 departure (at A), for example, is comprised of the arrival rate between 7:12 and 7:15 (65/15 = 4.3) and the rate between 7:15 and 7:45 ( $35 / 30=1.2$ ). Thus, $4.3 \times 3+1.2 \times 30=49$ which is approaching $d_{1}=50$. Moreover, in the transition between $\mathrm{j}=1$ and $\mathrm{j}=2$, the value of $\mathrm{d}_{2}=$ 55 is considered since the resultant departure is after 8:00. The load of the bus departing A on $8: 16$ at its hourly max load point $A$, is comprised of (25/25) x $25+(94 / 20) \times 6=$ 53.2 which is approaching $\mathrm{d}_{2}=55$. The exact value of $\mathrm{d}_{2}$ can be obtained only for departures with non-integer minutes. This completes the proof-by-construction of Proposition 2.

Figure 3 includes the results of procedure 2 with six departures. The last departure at $8: 52$ is determined using a slight extrapolation of the uniform passenger arrival rate between 8:30 and 8:50.

## Procedure 3

While procedure 2 ensures even average loads of $\mathrm{d}_{\mathrm{j}}$ at the j -th max load point, it does not guarantee that in other bus stops the average load will not exceed $d_{j}$ and, therefore, may result in overcrowding. The purpose of the procedure presented below is to derive the bus timetable provided that in an average sense all buses will have even loads (equal to the desired occupancy) at the max load stop of each bus. That is, for a given time period each bus may have a different max load point across the entire bus route with a different observed average load. The objective set forth is to change the departure times such that all observed average max loads will be same and equal to $\mathrm{d}_{\mathrm{j}}$ during all j . Certainly the adjustments in the timetable are not intended for highly frequent urban services where the headway is less than say, 10 minutes, or an hourly frequency of about 6 vehicles or more. Behind this procedure is the notion that passenger overcrowding situations (loads greater than $\mathrm{d}_{\mathrm{j}}$ ) should be avoided.

## The Underlying Principle of Procedure 3

The results of procedure 2 are exhibited in Figure 3. Considering in that Figure, for example, the resultant departure at 7:45 with 50 average passengers on-board at point B . From Table 1 it is clear that point B is the $\mathrm{j}=1$ max load point. However, one does not know what is the average load in the other stops, and in the example problem, it is referred to point A. Since the first departure is at 7:12, the accumulative load at A


FIGURE 3 Determination of the example departure times (at A) with even loads at the hourly maximum load point
between 7:12 and 7:45 is of interest. For that purpose the data in Table 1 is used while constructing an accumulative curve of the observed loads at A. The average load at A for the 7:11 departure results in 22 passengers ( $30 / 15=2$ pass/min arrival rate). The average load at 7:45 is combined from the remaining passengers between the observed 30 at 7:15 and 22, and those observed on the 7:45 departure. That is $(30-22)+80=88$ passengers. No doubt that the 7:45 departure faces, in an average sense, overcrowding at A while complying with $d_{1}=50$ at $B$. In order to overcome this undesirable possible overcrowding the following principle is employed.

Principle 3: Construct an accumulating passenger load curve at each stop (except the arrival point). Moving horizontally per each $\mathrm{d}_{\mathrm{j}}$, for all j , on each curve, until intersecting each of the accumulative curves, and then vertically to establish a departure time for each curve. The required departure time is the minimum one across all curves. Using the last determined departure time, set the loads across all the curves and add the considered or next $d_{j}$. Repeat until the end of the time span.

Proposition 3: Principle 3 results in departure times such that the average max load observed on individual buses approaches the desired occupancy $\mathrm{d}_{\mathrm{j}}$.

Proof: Figure 4 illustrates Principle 3 for the first three departure of the example problem in Table 1. Fig. 4 shows the accumulative load curves of the three buses where the curve at B is shifted by 18 minutes to allow for an equal time basis (at the route's departure point) in the analysis. At the initialization the value of 50 is coordinated with the two accumulative curves to obtain: 7:11 at B and 7:22.5 at A. According to principle 3 one selects the minimum time between the two to be the first departure at 7:11 (emphasized in Fig. 4). It means that the first bus is shifted backward by 4 minutes to have at $B$, in an average sense, 50 instead of 65 passengers. Then one adds $d_{1}=50$ to 50 at stop B curve, and to 22 at stop A curve. This results in 7:31 and 7:45 departures. Hence, 7:31 is the next departure, and the procedure continues and results in 7:56 as the last departure at the period [7:00-8:10]. Adding $\mathrm{d}_{\mathrm{j}}=50$ to 122 (at A) or to 134 (at B) results in departures beyond $8: 10$. The bus of $7: 11$ has its $d_{1}=50$ passengers at $B$ and the bus of 7:31-at A. This completes the proof-by-construction of Proposition 3.

Figure 4 includes at its bottom the complete set of departure times of the example problem. If extrapolating the accumulative curve, another departure can be set at approximately 9:00.

## Comparison

The comparison between the observed data of the example problem and the results of the three procedures is summarized in Table 2 and illustrates in Figure 5. In Table 2 the associated individual average max load and its corresponding stop appear in brackets under each departure. It can be seen, as expected, that only procedure 3 complies with balanced loads at the critical individual max load points.

Figure 5 presents the diversity of the individual max loads across all the procedures and the observed ones. Certainly this comparison is applied only to the specific example problem in Table 1, and varies from one situation to another. In


FIGURE 4 Determination of the first three departure times (at A) considering even loads at the individual bus maximum load point

TABLE 2. Departure times and loads of the observed data and for the three procedures

| Departure | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | Characteristic |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Observed | $7: 15$ | $7: 45$ | $8: 10$ | $8: 30$ | $8: 50$ |  |  | Observed |
|  | $(65, \mathrm{~B})^{*}$ | $(80, \mathrm{~A})$ | $(80, \mathrm{~B})$ | $(94, \mathrm{~A})$ | $(88, \mathrm{~A})$ | - | - |  |
| Procedure 1 | $7: 20$ | $7: 40$ | $8: 01$ | $8: 18$ | $8: 35$ | $8: 52$ |  | Even Headways |
|  | $(72, \mathrm{~B})$ | $(54, \mathrm{~A})$ | $(57, \mathrm{~B})$ | $(58, \mathrm{~B})$ | $(81, \mathrm{~A})$ | $(75, \mathrm{~A})$ | - |  |
| Procedure 2 | $7: 11$ | $7: 45$ | $8: 16$ | $8: 28$ | $8: 40$ | $8: 53$ |  | Even Load at |
|  | $(50, \mathrm{~B})$ | $(88, \mathrm{~A})$ | $(99, \mathrm{~B})$ | $(79, \mathrm{~A})$ | $(52, \mathrm{~B})$ | $(61, \mathrm{~A})$ | - | Hourly Max |
|  |  |  |  |  |  |  | Load Point |  |
| Procedure 3 | $7: 11$ | $7: 31$ | $7: 56$ | $8: 12$ | $8: 24$ | $8: 36$ | $8: 48$ | Even Load at |
|  | $(50, \mathrm{~B})$ | $(50, \mathrm{~A})$ | $(50, \mathrm{~B})$ | $(55, \mathrm{~B})$ | $(55, \mathrm{~A})$ | $(55, \mathrm{~A})$ | (55,A) | Individual Max |
|  |  |  |  |  |  |  |  | Load Point |

*( $\mathrm{i}, \mathrm{j}$ ) in bracket means: $\mathrm{i}=$ average individual max load associated with the cell's departure time, $\mathrm{j}=$ the stop where i is observed or determined
situations where the hourly max load point is usually coincided with the individual max load, the results of procedure 2 will be close to those of procedure 3 .

## CONCLUDING REMARKS

Different bus agencies use different scheduling strategies based primarily on their own schedulers' experience, and secondarily on their scheduling software (if any). As a result, it is unlikely that two independent bus agencies will use exactly the same scheduling procedures, at the detailed level. In addition, even at the same bus agency, the schedulers may use different scheduling procedures for different groups of routes. Consequently, there is a need when developing computerized procedures to supply the schedulers with alternative schedule options along with interpretation and explanation of each alternative. Three such alternatives are presented in this work. Also, undoubtedly, it is desirable that one of the alternatives will coincide with the scheduler manual procedure. In this way, the scheduler will be in a position not only to expedite manual tasks but also to compare the different procedures regarding the trade-off between passenger comforts and operating cost.

This work presents the creation of bus (and potentially rail) timetables with even headways and even average passenger loads on individual buses. Average even loads on individual buses can be approached by relaxing the evenly spaced headways pattern (rearrangement of departure times). It is known that passenger demand varies even within one hour, reflecting the business, industrial, educational, cultural, social and recreational public transport needs of the community. This dynamic behaviour can be detected through passenger load counts, and information provided by road supervisors. The adjustments of departure times, made in this work by three procedures, form the basis to improve the correspondence of bus departure times with the fluctuated passenger demand. These adjustments, resulting in a balanced load timetables, are based on a given bus desired occupancy at the maximum load point of each bus. The keyword here is to be


FIGURE 5 The example departure times and average maximum loads on individual buses as observed and obtained by the three procedures
able to control the loading instead of being exposed repeatedly to an unreliable service resulting from imbalance loading situations.

With the growing problems of PT reliability, and advance in the technology of passenger information system the importance of even and clock headways is reduced. This allows for introducing alternative timetables with the consideration of even average loads on individual buses. The construction of such timetables takes into account, in essence, the passenger perspective. The controlled procedures for adjusting the timetable, will eventually reduce one of the major sources of unreliable service, resulting also in the reduction of wait and travel times.

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