Framework for Probabilistic Method for Road Network Asset Management

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1 Introduction

Realistic estimates of short- and long-term costs for maintenance and rehabilitation of road asset management should take into account the stochastic characteristics of asset conditions of road networks. The probability theory has been widely used in assessing life-cycle costs for bridge infrastructures by many researchers such as Zayed et.al. (2002), Kong and Frangopol (2003), Liu and Frangopol (2004) and Noortwijk and Frangopol (2004). Very few studies were reported for road networks (Salem et. al. 2003, Zhao et. al. 2003). In the existing studies, analysts usually made assumptions about the variability and probability distributions of input variables and maintenance/rehabilitation costs in estimating life-cycle costs. By taking into account the variability of the stochastic characteristics of road asset conditions, variation in the cost estimates can be investigated. The output statistical information of the cost estimates produced useful information for further analysis in selecting cost estimates with a reasonable degree of reliability (e.g. 90th or 95th percentile).

This paper presents the results of research projects conducted by The Australian Cooperative Research Centre for Construction Innovation, Queensland University of Technology, RMIT University, Queensland Government Department of Main Roads and Queensland Department of Public Works. The research project aimed at developing a methodology for assessing variation and risk in investment in road network, including the application of the method in assessing road network performance and maintenance and rehabilitation costs for short- and long-term future investment.

The objectives of the paper are:

- To present a methodology for predicting the variation and likelihood (probability) of whole-of-life outcomes for short- and long-term investments in maintenance of road works given the natural variability of asset properties affecting road asset performance;
- To demonstrate the feasibility of the method on an Australian road network.

The expected outcomes of the study include:

- Greater confidence in predicting future whole-of-life costs for short- and long-term investment in road assets;
- Greater confidence in predicting the nature of future maintenance required and return periods (time intervals) for maintenance and rehabilitation;
- Greater confidence in economic outcomes;

2 Methodology Framework

The aim of the methodology is for predicting the likelihood (probability) of short-term and whole-of-life investments for road networks given the natural stochastic characteristics (or variability) of asset properties affecting road asset performance. The method for the analysis is used for network or strategic analysis. Steps for the analysis are given below. Figure 1 shows the schematic chart of the framework.

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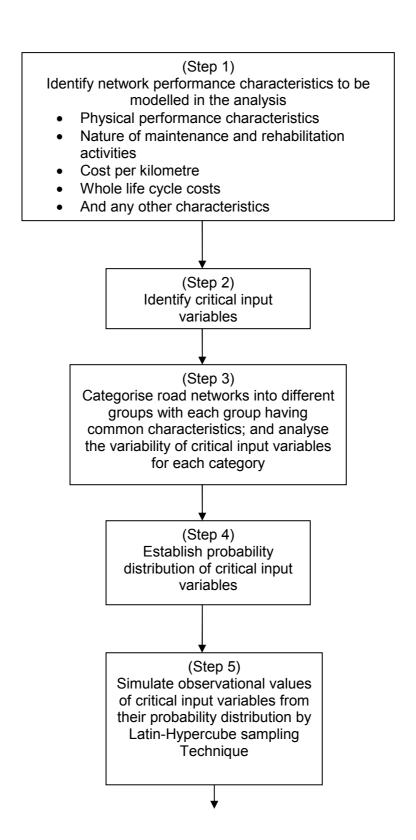


Figure 1 Flow chart for assessing variation in life-cycle costs

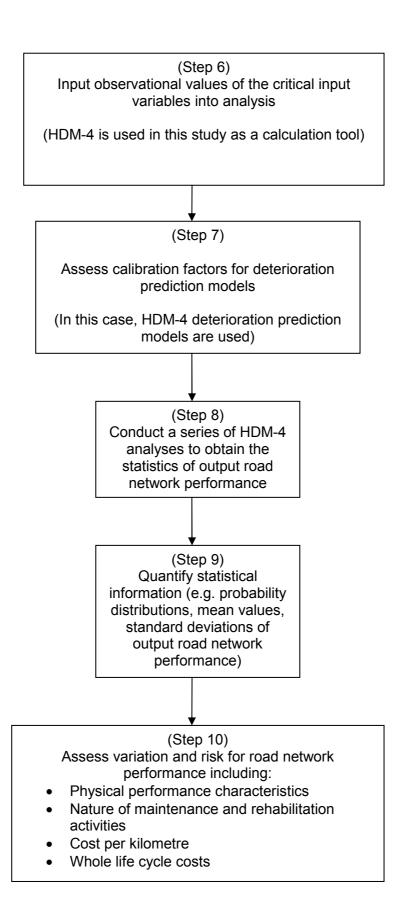


Figure 1 Flow chart for assessing variation in life-cycle costs (continued)

- 1) The first step is to identify network performance characteristics to be modelled in the analysis.
- 2) Identify critical input variables that significantly affect road network performance and, hence, cost estimates.
- 3) Categorise road networks and variability assessment. In this step the analysed road network is categorised into different categories, so that each category has common characteristics. The variability of critical input variables is assessed for each category. This step allows the possibility of incorporating the variability of road data for road networks into the analysis.
- 4) Establish probability distributions and statistical information (means, standard deviation and etc.) of the stochastic characteristics of the critical input variables of the road network.
- 5) Use Latin-Hypercube Sampling Technique to sample data from the probability distributions of the identified critical input variables.
- 6) Use a calculation tool to predict network performance characteristics (HDM-4 is used in this study). At this stage, it is necessary to calibrate HDM-4 prediction models to reflect observed local road asset condition.
- 7) Calibrate deterioration prediction models to reflect the rate of change in road pavement condition for local condition.
- 8) Conduct a series of HDM-4 analyses to obtain the statistics of output road network performance characteristics.
- 9) Quantify the statistical information (e.g. probability distribution, mean, standard deviation, etc) of the output road network performance characteristics.
- 10) Investigate the degrees of variation for the established probability distributions of the output road network performance characteristics.

3 Analysis Process

The first step in the analysis is to identify network performance characteristics to be modelled in the analysis. In this study, the network performance characteristics to be investigated include:

- Whole life cycle costs
- Physical performance characteristics
- Cost per kilometre
- Nature of maintenance and rehabilitation activities

The second step is to identify critical input variables. It must be recognised that it is not feasible to incorporate the variability of all input variables into the analysis. Piyatrapoomi et al (2005) adopted the probabilistic technique to identify input variables that are critical in predicting variation in road network performance. These critical input variables include pavement strength, pavement roughness, annual average daily traffic (AADT) and rut depth. The details of the analysis are given in Piyatrapoomi et al 2005.

The third step is to categorise road networks into different groups of common characteristics. Primary inter-city road networks in Queensland of approximately 4,500 km were chosen as a case study. Road networks of approximately 2,295 km, 1195 km and 1408 km were represented for wet non reactive soil, dry non reactive soil and dry reactive soil, respectively. These road networks were categorised into different groups, with group having common characteristics. The criteria used for categorising roads are shown in Table 1. Three hundred and seventy-eight road categories were generated from the combination of the categorising criteria. However, only sixty-five road categories were obtained from the categorisation of the 4,500 km road networks. Table 2 shows examples of road categories obtained from the categorisation. For simplicity an identification is given for each road category, for instance 'WNR-Good-Bt-Flx-(1.5k-3k)' refers to a road category located in wet non reactive soil, IRI < 2.31, bitumen surfacing, flexible pavement type, ADDT between 1501-3000.

Table 1 Criteria used for categorising road pavements

	Table 1 Criteri	a used for catego	rising road pav	rements
Annual Average Daily	Pavement Roughness	Surface Types	Base Types	Climatic and Soil Types
Traffic	(IRI)		. , , , , ,	.,,,,,,,
< 500	Good (IRI<2.31)	Bitumen	Flexible	Wet non Reactive soil
501-1500	Fair (2.31 <iri>4.2)</iri>	Asphalt concrete (AC)	Semi Rigid	Dry non Reactive Soil
1501-3000	Poor (IRI>4.2)		Full Depth Asphalt	Dry Reactive Soil
3001-5000				
5001-10000				
10001-25000				
>25000				

Table 2 Road categories

	rabie	2 Road cat	egories		
Description	Climatic	Surface	Pavement	Roughness	AADT
	Zone	Туре	Туре	ĬRI	
WNR-Good-Bt-Flx-(1.5k-3k)	Wet Non Reactive	Bitumen	Flexible	< 2.31	1501-3000
WNR-Fair-Bt-Flx-(1.5k-3k)	Wet Non Reactive	Bitumen	Flexible	2.31-4.2	1501-3000
WNR-Poor-Bt-Flx-(1.5k-3k)	Wet Non Reactive	Bitumen	Flexible	>4.2	1501-3000

The fourth step is to quantify the variability of critical input variables for each road category. As mentioned, the critical input variables include roughness, AADT, rut depth and pavement strength. Roughness data, AADT and rut depth data were extracted from ARMIS database (a

Road Management Information System Database) of the Queensland Government Department of Main Roads. The variability of these critical input variables were quantified by the means, standard deviation values and probability distributions for each road category. For roughness, the majority of probability distributions of the sixty-five categories were found to have goodness-of-fit with beta general distributions. For average rut depth, the most common probability distributions that have goodness-of-fit with the data were found to be log normal and gamma distributions. For annual average daily traffic, the data showed goodness-of-fit with triangular and exponential distributions. Beta general distribution and log normal distribution can be used for modelling the probability distribution of pavement roughness and rut depth, respectively. Triangular or exponential distribution can be used for modelling the probability distribution of average daily traffic (AADT). Figures 2, 3 and 4 show mean values, standard deviations of pavement roughness (IRI), AADT and rut depth, respectively for the identified sixty-five road categories.

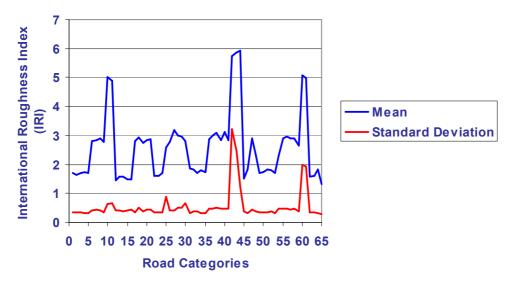


Figure 2 Means and standard deviations of roughness (international roughness index IRI) for 65 road categories

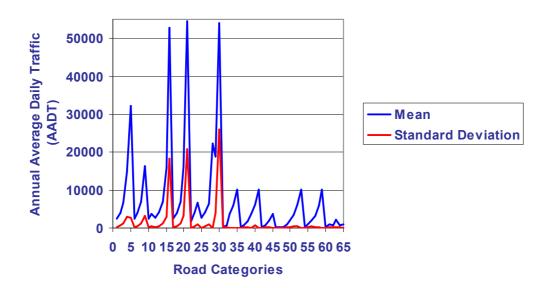


Figure 3 Means and standard deviations of annual average daily traffic (AADT) for 65 road categories

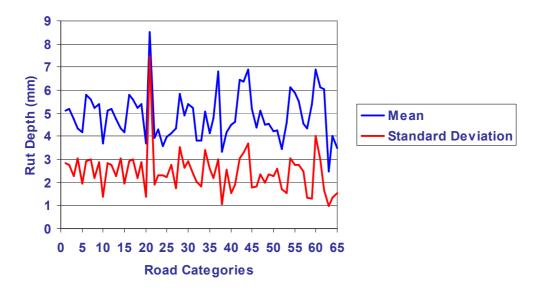


Figure 4 Means and standard deviations of rut depth for 65 road categories.

Pavement strength was identified as one of the critical input variable. Road agencies do not usually monitor pavement strength at the network level. Pavement strength is usually monitored by the Falling Weight Deflectometre (FWD). It is time consuming and high in cost in collecting pavement strength data at the network level. An analysis method for optimising longitudinal test intervals for pavement strength was developed by Piyatrapoomi et al (2004). Three case studies were conducted using the developed method in assessing optimal intervals for pavement strength data collection for three climatic and soil conditions in Queensland. The results found that road agencies could reduce strength test sampling rates by 75 to 80 per cent compared to current practice without losing statistical relevance for network applications.

Based on the above findings, pavement strength data were collected at an affordable cost for the 4,500 km network used for the analysis in this study. The probability distribution of pavement strength was established for each road category. In this study, pavement deflections obtained from the FWD test were transformed into the Structural Number (SN) which represents pavement strength by Robert's function. Using Australia and New Zealand FWD data, Robert's function yielded a reasonably close relationship to $r^2 > 0.9$ (Salt and David 2001). The Robert's function is given below;

$$SNP = 12.992 - 4.167 Log(D_o) + 0.936 Log(D_{900})$$
 (1) Where; $SN =$ the Structural Number $Do =$ pavement deflection under load cell $D900 =$ pavement deflections at locations 900mm from the load cell

Figure 5 shows mean values and standard deviation values of the structural numbers (SN) for the identified sixty-five road categories.

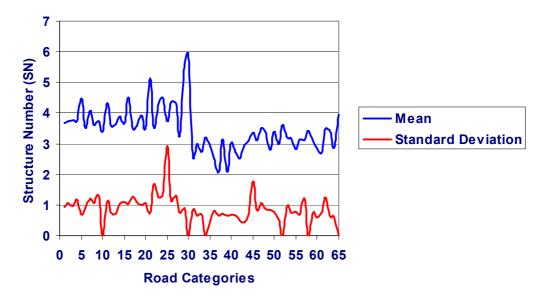


Figure 5 Means and standard deviations of structure numbers for 65 road categories.

Step five is to incorporate the variability of the critical input variables into the analysis. In this study, Latin-Hypercube sampling technique was used for simulating sample values from the probability distributions of the critical input variables for the analysis. In analysing a complex system, the Latin-Hypercube sampling technique is more appropriate than the well-known Monte Carlo simulation technique. This is because the Latin-hypercube sampling is the simulation technique that can be used for assessing the relationship of the variability of input and output variables by simulating small sampled data sizes for the analysis. The Latin-Hypercube sampling technique can substantially reduce analysis tasks that may not economically be viable to conduct using the Monte Carlo simulation. Since the Latin-Hypercube sampling technique has been popular in recent years for assessing quantitative risk, the technique has been incorporated in commercial risk analysis softwares, such as @Risk software. Piyatrapoomi (1996) found that sampling observational values of thirty data points were enough to obtain good estimates of the means, standard deviations and probability distribution functions of output variables. To obtain better results, forty sample data were simulated for each critical input variable and for each road category. @Risk software was used for simulating the variability of critical input variables for the analysis.

Step six is to model road network for network performance analysis. In this study, HDM-4 was used as a calculation tool. HDM-4, developed by the International Study of Highway Development and Management (ISOHDM), is a globally accepted pavement management system. It is a computer software package used for planning, budgeting, monitoring and management of road systems. There are three analysis options in HDM-4, which include: (1) Strategy Analysis, (2) Program Analysis and (3) Project Analysis. The Strategy Analysis Option was employed in this study in assessing the effects of the variability of pavement strength on the estimate of maintenance and rehabilitation budgets. Forty HDM-4 input files were prepared for a series of HDM-4 analysis. Each file represents the randomness in the variability of the critical input variables.

Step seven is to calibrate performance models. Since HDM-4 was used in this analysis, HDM-4 performance models were to be calibrated to reflect network performance for local condition. Piyatrapoomi and Kumar (2004) used the probability-based method in calibrating road performance models. The method is based on the probability theory and the Monte Carlo simulation technique. In this method, the stochastic characteristics of input variables of the deterioration prediction models were quantified by probability distributions. Monte Carlo simulation method was used to simulate the variability of the input variables of the prediction

model to predict the variability of the model output. The model output is then tuned so that the predicted variability demonstrated by modelled deterioration closely replicates actual variability of measured deterioration. In this method, the degree of goodness-of-fit between the calibrated function and recorded road data can be explicitly assessed and identified. Thus, this method gives a higher degree of confidence in using the calibrated models. Two case studies were conducted to assess the calibration factors of annual change in roughness for HDM-4 models in the previous study (Piyatrapoomi and Kumar (2004). Road data of 1688 kilometres of a National Highway (Bruce Highway) located in the tropical Northeast region of Queensland were used in the analysis. And road data of 1034 kilometre from Landsborough Highway located in central Queensland were used in the second analysis. The calibration factors for annual change in roughness found in that study were used in this analysis. Tables 3 and 4 show the calibration factors for annual change in pavement roughness for wet and dry regions. It must be noted that other prediction models need to be calibrated. In this study, only annual change in pavement roughness model was calibrated for demonstration purposes.

Table 3 Wet tropical region of Queensland

Thickness	Calibration Factor for HDM-4 annual change in roughness model
200-300 mm	0.55
300-400 mm	0.35
400-500 mm	0.25
500-600 mm	0.20

Table 4 Dry region of Queensland

Thickness	Calibration Factor for HDM-4 annual change in roughness model
100-200 mm	0.78
200-300 mm	0.48
300-400 mm	0.48
400-500 mm	0.43

Step eight is to conduct a series of HDM-4 analysis in order to predict the statistics of the output network performance. Details for maintenance and rehabilitation treatment choices and the intervention criteria for treatments that will be invoked are given in Piyatrapoomi et al 2006.

Step nine is to quantify the statistical information (e.g. mean, standard deviation, probability distribution, etc.) of output network performance.

Step ten is to assess risk and variation in road network performance.

4 Results

From the statistical information of the outputs, we can investigate risk and variation of road network performance. As established earlier, road network performance of interest includes:

- Whole-of-life cycle costs
- Physical performance characteristics
- Nature of maintenance and rehabilitation activities
- Cost per kilometre

Discussions of the results are given below.

4.1 Whole-of-life cycle costs

Information on risk and variation of whole-of-life-cycle costs allow road asset managers to investigate the likelihood or probability of costs of not being exceeded for a certain degree of confidence. From such information, road asset managers can choose an appropriate level of confidence in the cost estimates. For instance, they may choose the mean cost estimate with a probability of occurrence of approximately 50 per cent. Or they can choose high level of probability of occurrence, for instance, 95th or 90th percentile for which there is respectively 5 and 10 per cent probability that the cost will be exceeded. Decision-makers will have informed knowledge on the variation and limitation of the cost estimate which is very helpful in preparing a realistic budget for road network maintenance and rehabilitation. Figure 6 shows mean values of cumulative whole-of-life cycle costs for a 25-year period. Figure 7 shows standard deviation values of the whole-of-life cycle costs. The total mean costs for the network investment for the next 25-year period would be approximately A\$1.8 billion. It is noted that the estimated cost had not yet been discounted. Cost variation of one standard deviation shown in Figure 7 indicated sudden increases in the calculated standard deviation values. This characteristic indicated that there are possibilities of major maintenance and rehabilitation or major spending occurring in those years. It is observed that there are between 3 to 5 years for sudden increases in the standard deviation values.

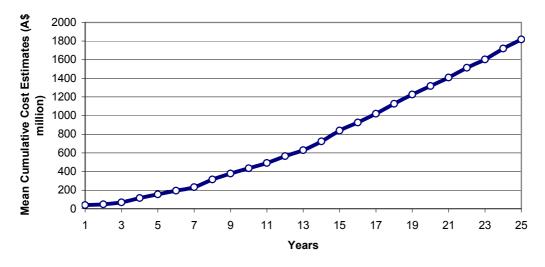


Figure 6 Mean estimates of cumulative costs for road maintenance and rehabilitation for a whole life cycle of 25 years for 4,500 km road network



Figure 7 Standard deviations of cumulative cost estimates for road maintenance and rehabilitation for a whole life cycle of 25 years for 4,500 km road network

The statistical information including the mean, standard deviation and probability distributions can be used to assess levels of risk in cost estimates. Table 5 shows the calculation of levels of risk (probability of occurrence) when it is assumed that costs were blown out by 10 and 20 per cent from the mean estimates. These levels of risk assessment were calculated for 5-year period.

Table 5 Risk or probability of occurrence of cost blowouts

Years of Cumulative Cost	Mean Cost Estimates (A\$ Million)	10% Blow out Costs	Risk or (probability of	20% Blow out Costs	Risk or (probability of
Estimates	(At Willion)	(A\$ Million)	occurrence) (%)	(A\$ Million)	occurrence) (%)
1 st year	39.95	43.29	36.0%	47.22	29.7%
2 nd year	48.61	53.47	37.4%	58.33	28.8%
3 rd year	69.46	76.40	35.2%	83.34	25.6%
4 th year	115.30	126.83	32.6%	138.36	20.1%
5 th year	115.23	170.73	32.2%	186.30	18.7%

4.2 Physical performance characteristic

Figure 8 shows an example of whole-of-life cycle performance for pavement roughness for a road category. The figure shows mean and mean plus one standard deviation of whole-of-life pavement roughness for a 25-year period for a road category in wet non reactive soil having initial roughness of less than 2.31, bitumen surfacing, flexible pavement and carrying traffic of AADT between 1500 to 3000 vehicles. The figure shows that mean IRI values are less than 4 for the whole-life-cycle. When we take the mean value for consideration, it represents approximately 50 per cent probability of occurrence. When we consider the IRI values of mean plus one standard deviation which represents approximately 83.33 per cent of occurrence, the maximum value of mean plus one standard deviation of most road categories are below 5 IRI. From this information, road asset managers can investigate in detail the variation and probability of occurrence of pavement roughness for the whole-of-life cycle for each road category.

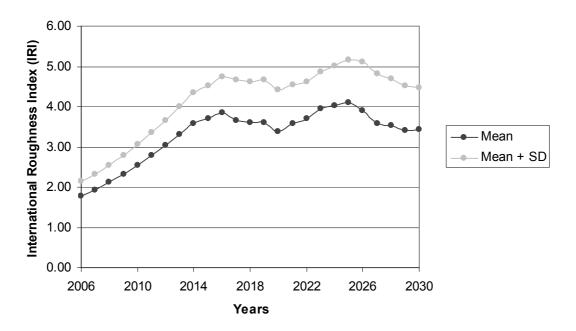


Figure 8 Mean and mean plus one standard deviation for pavement roughness for a whole life cycle of 25 years for road category of WNR-Good-Bt-Flx-(1.5k-3k)

4.3 Nature for maintenance and rehabilitation activities

Table 6 shows typical results of time intervals for maintenance and rehabilitation treatments for a road category. The table shows means and standard deviations of the time-intervals. In the Table, WNR-Good-Bt-Flx-(1.5k-3k) represents road category in wet non reactive soil, with pavement roughness of IRI less than 2.31, bitumen surfacing, flexible pavement and carrying traffic of AADT between 1501 and 3000 vehicles.

The first time interval is the time required for a treatment from the start of the analysis year. The start of the analysis year for this analysis is 2006. The second time interval is the return period for a major rehabilitation after the first treatment has carried out. The percentage (%) of treatment types represents possibility or percentage of a selected treatment that is likely to occur. There may be different possibilities of selected treatments for a road category. These different possibilities of treatment resulted from the variability of critical input variables and the random combination of the variability represented in the analysis and random simulation. This information allows road asset managers to be aware that there are other possibilities in the treatments that can occur or can be selected. Mean and standard deviation values of the time intervals presented in the tables provide flexibility of the variation in selecting a time for treatments.

4.4 Cost per kilometre

Cost per kilometre was calculated from the whole-life cycle cost of 25-year period of each road category divided by the number of kilometres of road length within that category. Table 5 shows typical results of means and standard deviations of costs per kilometre. This information can assist road asset managers to make informed decisions in relation to investment costs and the degree of variation in the predicted costs for each road category.

Table 6 Mean and Standard Deviation Values for Time-Interval for Maintenance and Rehabilitation for Wet Non Reactive Soil

			_	1 st Time Interval (Years)	al			2 nd Time Interval (Years)	ərval
Description	Last Surfacin g	Mean (Years)	SD (Years)	% of Treatment	Treatment Types	Mean (Years)	SD (Year s)	% of Treatme nt	Treatment Types
WNR-Good-Bt-Flx-(1.5k-		6.4	1.2	34%	Reseal-15% Cracked	13.5	1.7	34%	Granular Overlay- SIRI& 10% Cracked
		11.1	1.9	%99		9.4	4.0	%99	Granular Overlay-
					Granular Overlay- 5IRI& 10% Cracked				5IRI& 10% Cracked

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Coefficient of Variation	0.241	0.316	0.398
Cost per Kilometre (SD) A\$ million	0.1068	0.1483	0.2152
Cost per Kilometre (Mean) A\$ million	0.4439	0.4701	0.5404
(Mean , SD) of Structure Number (SN)	(3.66, 0.92)	(3.73, 1.06)	(3.76, 0.97)
Æ	269	181	154
Description	WNR-Good-Bt-Flx-(1.5k-	مد) WNR-Good-Bt-Fix-(3k-	JR.) WNR-Good-Bt-FIx-(5k- 10k)

5 Conclusion

This paper presents a ten-step framework for assessing risk and variation in whole-of-life cycle road network performance for maintenance and rehabilitation investment of road networks. Queensland Inter-city road networks of approximately 4,500 km were used as a case study. The expected outcomes of the project included:

- Greater confidence in predicting future whole-of-life costs for short- and longterm investment in road assets:
- Greater confidence in predicting the nature of future maintenance required and return periods (time intervals) for maintenance and rehabilitation;
- Greater confidence in economic outcomes;
- Greater accuracy in predicting the relationships among the variability of critical input variables, such as traffic, environmental zones, roughness, etc. on predicted outcomes (i.e. roughness, costs)
- Greater confidence in assessing cost per kilometre for each road category.

The case study has achieved the following:

- Identified critical inputs that influence the reliability of road investment model outputs.
- Incorporated stochastic properties of critical model inputs for chosen road networks into the investment analysis process.
- Improved and applied a methodology for calibrating road investment models that make use of the variability input properties. A calibrated model reliably predicts the variability of the roughness model outputs when compared with the network variability of actual roughness of the chosen network.
- Analysed and demonstrated the variability of predicted network performance (both physical and financial performance) for a chosen primary inter-city road network in Queensland, The Queensland National Land Transport Network.

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