Heavy Vehicle Safe Cornering Speeds
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Introduction

The Austroads (1997) publication provides a useful benchmark to the evaluate nominal safe cornering speed for a particular curve. The same specifically provides a safe design procedure for road curvature design and predicts benchmark cornering speeds for curve signage purposes. However this procedures, due to its application as a design recommendation, doesn't examine typical actual cornering speeds which professional heavy vehicle drivers safely negotiate a given curve.

The characteristics of a professional heavy vehicle driver are:

- Consistently fully utilize the available lane space through the curve
- effect set up for the curve a considerable distance from curve's apex and typically prior to the curve's entry transition point
- gradually apply increasing cornering lock (subject to steering system stability, dead spots, slack, state of repair and stiction and vehicle understeer and oversteer characteristics)
- familiar with the curve and particular road camber and the optimal line through each route curve
- familiar with the actual lane width and pavement infill at the curve's apex
- very familiar with the road surface and road conditions.

The ability to negotiate corners at speeds in excess of the curve's nominal safe road speed becomes second nature as a result of substantial driving experience and continuous on the job practice with a particular vehicle and with each specific curve.

To explain the seemingly excessive cornering speed which professional drivers safety negotiate a particular curve, especially should the curve attract a signed advisory speed, analysis techniques outside the scope of the Austroads design manual must be resorted to. In simple terms the Austroads manual predicts that professional drivers should invariably experience loss of control on each and every curve attracting a speed advisory sign.

This paper presents one contributing factor for this seemingly contradiction. This contributing factor will be identified by the following sequential analysis. This analysis, for a particular corner of constant mid lane radius connecting directly two straight road sections void of transition curves, will include:

1) No lane movement with the curve negotiated via a path of constant curvature or radius (Austroads 1997),
2) Effect lane movement subject to the available lane space and negotiate the curve via a path of constant curvature. For this case a sensitivity analysis will examine the effect of a driver initiating cornering at +/-, 0, 1, 3, 6, 9, 12, 15 from the

curve's entry inflection point. Here a positive sign is used to indicate commencement of cornering after the curve's entry geometrical transition point, whereas, the negative sign indicates early set up. It should be noted that fitting of constant radius turns from the start positions in excess of approximately 3 m early lack physical significance.

3) Effect lane movement subject to the available lane space and negotiate the curve along a Fresnel function or symmetrical Cornu spiral. Here a optimal spiral and a spiral satisfying the given curve's specific geometric boundary conditions will be examined.

Driver control and road safety implications of the various analysis predictions will then be discussed.

Austroads (1997) Constant Mean Radius Approach

As a vehicle traverses a circular curve, it is subject to a centripetal force which must be sufficient to balance the inertial forces associated with the circular path. For a given radius and speed, a set force is required to maintain the vehicle in this path and, in road design, this is provided by side friction developed between the tyre and pavement and by superelevation.

For normal values of superelevation, the following formula is accepted :

$$E + f = v^2 / gR \text{ or } V^2 / 127 R \qquad (1)$$

where

E	=	Pavement superelevation (m/m or tangent of angle) This is taken as positive if the pavement falls towards the center of the curve.
f	=	Coefficient of side frictional force developed between vehicle and tyres and the road pavement. This is taken as positive if the frictional force on the vehicle acts towards the center of the curve.
g	=	Acceleration due to gravity = 9.81 ms^{-2}
v	=	Speed of vehicle (m/s)
V	=	Speed of vehicle (km/h)
R	=	Curve radius or radius of curvature (m)

Curves are generally designed, so that a positive f is required for the range of vehicle speeds likely to occur.

Lane Movement and Constant Radius Paths

The fitting of a constant radius bend to an approach and exiting straight road sections, void of transition curve sections, involves major discontinuity of the path curvature, hence radius of curvature, particularly at both the entry and exit transition points. At the points of curvature discontinuity the vehicle theoretically is subject to infinite jerk. The occurrence of large magnitude jerk mars the stability and opportunity to maintain the desired path and so increases the risk of loss of control. A constant radius bend does, however, attract simple geometry and the shortest path through a curve. The same implies the path distance from the entry transition point to the curve's apex is a minimum. A further advantage of fitting a constant radius path through a curve is that the curve fit parameters can readily be determined using simple geometry and be effected conveniently using spreadsheet goal seek functions. This constant radius path geometry can be readily effected for both constant lane clearance and maximal nominal lane movement.

Typically found heavy vehicle drivers will exploit the available lane space by moving out prior to entering the curve and then move across the lane to position the vehicle close in at or near the curve's apex. This cutting in of the curve may extend to the rear most inside trailer tyre overhanging the pavement at the curve's apex or utilizing pavement infill should such exist.

Fresnel Function Path

In comparison a Fresnel path through a curve can be selected to satisfy the specific curve's geometric boundary conditions and most strategically maintain continuity of the curvature, hence radius of curvature, along the path. The continuity in the curvature through the curve implies, most importantly, finite jerk applies whilst negotiating the curve. In this investigation symmetrical Fresnel curves will be assumed. This implies the path on the entry side of the curve is identical to that on the exit side with a theoretical line of symmetry passing through the curve's apex and instantaneous center of curvature. Hence the distinct advantage of a Fresnel path through a curve is that the path is the smoothest path. This path, in this simplified analysis, from the entry transition point to the vehicle's position at the curve's apex.

A particular difficulty of analytically fitting a Fresnel path through any general road curve, from a given transition point to the curve's apex, is that the analysis is complex and numerically demanding (Bower (1994)). This complexity is eased somewhat for fitting a Fresnel path between a straight line and a given circle and between two straight lines (Meek and Walton 1989).

The Fresnel Function

Spackman & Tan (1993) define the Fresnel integral or Cornu Spiral as follows;

$$x = \text{integral}\ (\cos(0.5\ \pi\ v^2),\ 0,u) \qquad (2)$$
$$y = \text{integral}\ (\sin(0.5\ \pi\ v^2),0,u) \qquad (3)$$

The Cornu spiral can easily be evaluated and plotted using spreadsheet packages. One simply needs to evaluate the following tabular information in succession. Namely run v in desired increments, then evaluate $0.5\ \pi v^2$, then evaluate $\cos(0.5\ \pi v^2)$ then integrate, using say Simpson's Rule, to determine the coordinates for x. Followed likewise for the y component, namely evaluate $\sin(0.5\ \pi v^2)$ then integrate using say Simpson's Rule. The graphical presentation of the Cornu spiral is then obtained by plotting y against x as depicted in the Figure 1 following. An examination Figure 1 indicates Cornu spiral automatically satisfy radius of curvature continuity should the spiral originate from tangency with a straight line.

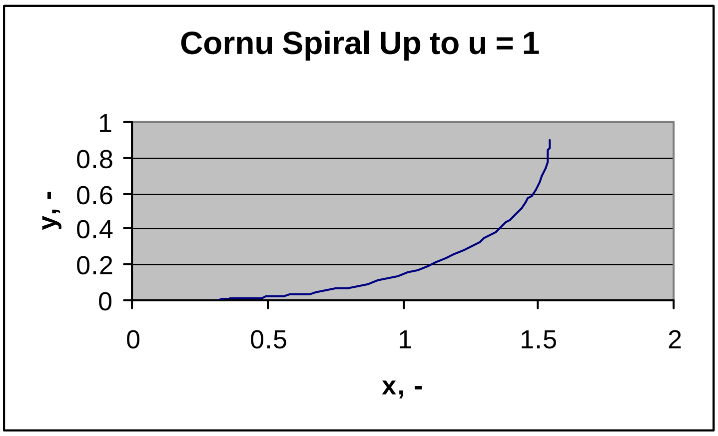

Figure 1 Typical Plot of Fresnel Function up to u = 1 with a unit scale factor.

The turning effectiveness of the Cornu spiral is apparent by comparing the turning effected by a unit circle to that effected by a Cornu Spiral. The cornering effectiveness of the Cornu spiral relative to a unit circle is apparent from an examination of Figure 2 following.

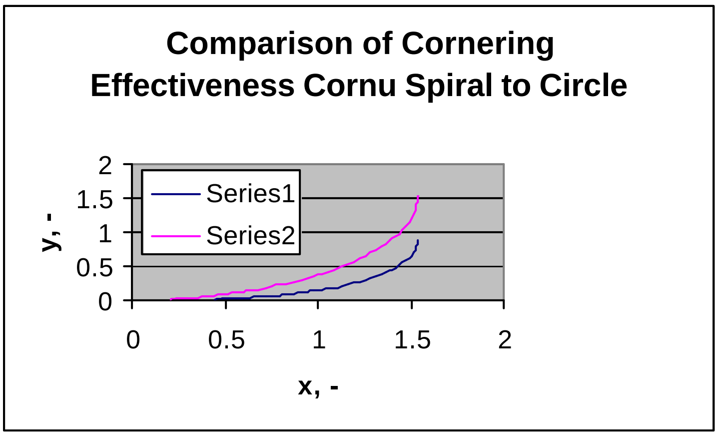

Figure 2 Comparison of Turning Effectiveness between a Fresnel function path (Series 1) and a constant radius bend (Series 2).

Further manipulation of the Cornu spiral expressed in integral form as mathematically described by Eqns (1) and (2) proves difficult. This analytical difficulty can be surmounted by use of graphical analysis techniques. Fortunately the integrand form of the Fresnel function can be suitably approximated by the following polynomial expressions (Tuma 1979).

$$S(x) = (1.x^3/(1!3) - 1\,x^7/(3!7) + 1.x^{11}/(5!11) - 1.x^{15}/(7!15) + \ldots)$$

$$= 0.333333\,x^3 - 0.02381\,x^7 + 0.000758\,x^{11} - 0.000013\,x^{15} \ldots \qquad (4)$$

and

$$C(x) = (1.x/(0!1 - 1x^5/(2!5) + 1.x^9/(4!9) - 1x^{13}/(6!13 + \ldots)$$

$$= 1.00000x - 0.1\,x^5 + 0.00463\,x^9 - 0.000107\,x^{13} \qquad (5)$$

where $x = (0.5\,\pi)^{0.5}\ v$.

Here the coordinates of the Cornu spiral are adequately approximately by

$$Y(x) = S(x) \quad \text{from equation (4)}$$

$$X(x) = C(x) \quad \text{from equation (5)}$$

Differentiation of equation (4) and (5) yields the approximate slope of the Cornu spiral to be

$$dy / dx = S'(x) / C'(x) \qquad (6)$$

From which it follows, from the differential of a quotient rule, that

$$d^2y / dx^2 = (C'(x)\,S''(x) - S'(x)\,C''(x))/\,(C'(x))^2) \qquad (7)$$

Knowledge of the differential polynomial expressions, equations (6) and (7), allows evaluation of the radius of curvature ($\rho(x)$) as a function of distance along the path. The typical variation in the radius of curvature along a Cornu spiral up to u = 1 is given in Figure 3 following. An examination of this Figure indicates the Cornu attains, after commencing from infinite radius of curvature (ie as straight line), attains the radius of curvature of a unit circle at approximately u = 0.5. Examination of Figure 3 also confirms the Cornu spiral exhibits radius of curvature continuity when fitted tangent to a commencing straight line.

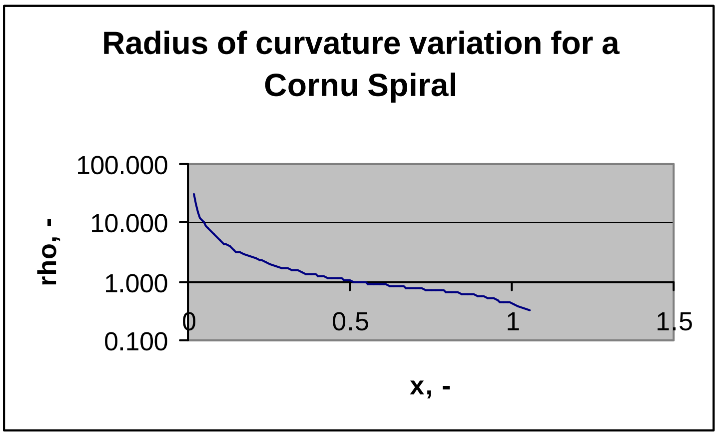

Figure 3 Variation in the dimensionless radius of curvature along a Cornu spiral (u = 1)

Improved understanding of the significance of the Cornu spiral is effected by simultaneously plotting the instantaneous turn angle (which fortunately corresponds to the change in direction from the start to a specific point along the Cornu spiral) along with the radius of curvature for the spiral. This double variation is presented in dimensionless variables in Figure 4 following.

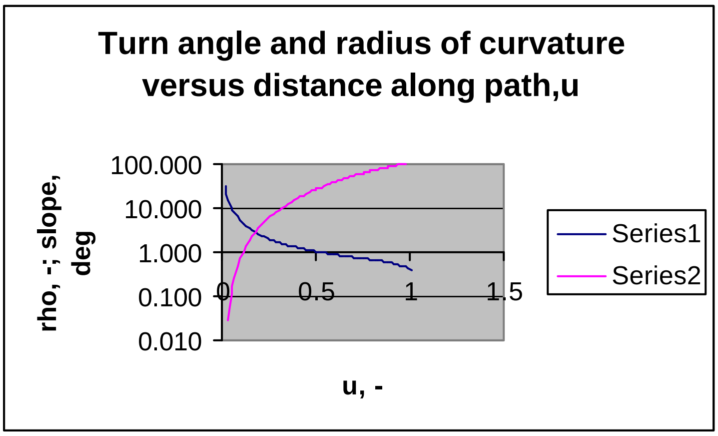

Figure 4 Variation in dimensionless radius of curvature (Series1) and the turn angle (Series 2) along a Cornu spiral.

An examination of Figure 4 confirms that the Cornu spiral effects efficient turns by progressively decreasing the radius of curvature to below that for a constant radius bend through the same turn angle. However, for the task at hand the curve geometric boundary conditions are specified. Namely, the start (transition point with the lead in straight road), curve apex and end coordinate (transition point with lead out straight road) of the curve are specified. Hence for a given turn angle the Cornu spiral will effect the required turn more effectively than a constant radius bend. As a consequence for a given turn angle, subject to specified curve geometric end points, the Cornu spiral path which can be adopted through a given curve will exhibit a larger radius of curvature compared to that of the best fit constant radius bend through the same end points. Most importantly, for given curve start and end points a Cornu spiral can be fitted. However, this curve fitting, using the integral expressions (Equations (2) and (3)), respectively is extremely complex (Bower 1994). This numerical difficulty is considerably eased by use of the polynomial expressions (Equations (4) and (5)), geometric scaling and spreadsheet goal seek operations.

Particular Properties of the Cornu Spiral

Meek and Walton (1989), have defined a few important formulae for the Cornu spiral possessing scaling parameter a. These formulae include:
Angle of tangent: $\pi u^2/2$
Curvature: $\pi u/a$
Arc length : $ds = adu$
Centre of circle of curvature : $(a/u\, C(u),\ a/u(S(u) + 1/\pi))$

A stated by Meek and Walton (1989) two line segments forming a known angle Ω in planar space $(0, \pi)$ can be connected by a pair of symmetrical Cornu spirals with the value of the parameter t_o where the spirals meet is

$$t_o = (1 - \Omega / \pi)^{0.5} \qquad (8)$$

with the distance along the lines from their intersection to where the cornu spiral meets the lines is

$$C(t_o) + S(t_o) \cot(0.5\Omega) \qquad (9)$$

Example

Consider a vehicle effecting a 90^o turn around a 81.85 m, mid lane, radius bend with a constant lane width of 3.7m. Assume also a truck width of 2400 mm and ignore the undercutting of the semi trailer at the apex of the curve. Assume the combined pavement and super elevation factor is 0.4.

Based on the Austroads (1997) model the predicted nominal cornering speed is 64.52 km/h. In comparison using the available lane space for different curve commencement positions and effecting a constant radius turn the following safe turn speeds are predicted.

Table 1 Theoretical constant radius safe cornering speeds

Set up distance, m	Optimal constant radius, m	Theoretical curveing speed, km/h	Factor Theor./Austroads -
-15 (early)	110.7	67.4	1.045
-9	81.95	64.56	1.006

-6	81.95	64.56	1.0006
-3	92.2	66.07	1.024
-1	90.7	66.06	1.024
0	85.85	66.08	1.024
+1 (late)	89.3	66.11	1.024
+3	87.9	66.25	1.027
+6	86.32	66.26	1.027
+9	86.58	66.36	1.028
+12	86.87	66.47	1.030
+15	79.3	66.6	1.032

The information presented in Table 1 is conveniently summarized in Figure 5 following. An examination of Figure 5 reveals that effecting a constant radius turn through a curve only provides a small increase in the safe cornering speed. Furthermore the magnitude of the same indicates only minor sensitivity to the curve set up position.

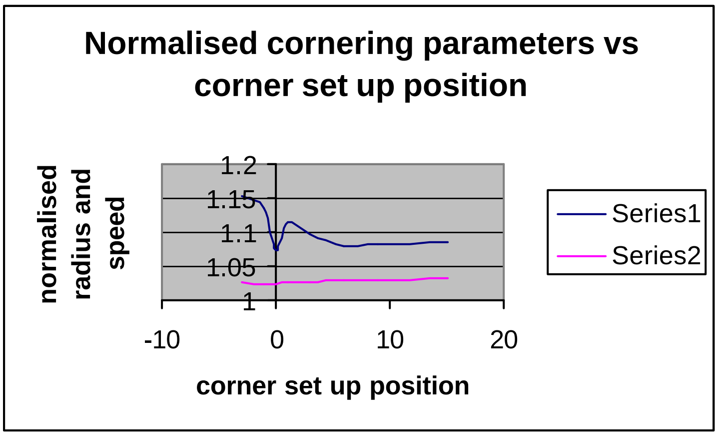

Figure 5 Predicted variation in relative curve radius of curvature (Series 1) and relative safe cornering speed (Series 2) versus curve set up position for constant radius path with variable lane clearance through curve.

In comparison the theoretical symmetrical Fresnel analysis predicts the following safe cornering speeds as summarized in Table 2 following.

Table 2 Theoretical optimal cornering details

Fresnel parameter, -, (a =1)	Curve set up at, m	Smallest radius of curvature, m	Theoretical v_{max} km/h	Factor Theor/Austroads -
0.70712	-35.00	181.8	96.6	1.497
0.8862	0	81.85	68.53	1.062

An examination of Table 2 reveals, when adopting Fresnel paths, that boundary conditions are critical in controlling the magnitude of the curve's apex curvature which, in turn, governs the theoretical safe cornering speed for a given curve. The same, in turn, implies that correct set up by drivers is essential for 'above nominal curve speed' negotiation of particular curves. This correct set up requires the driver to commence turning a relatively large distance before the curve's transition point, continue to apply smooth increasing lock and pass the curve's apex along the tangent to the geometrical curve of constant radius. These requirements imply considerable driver skill which, in turn, implies considerable practice and experience. Table 2 implies also a typical heavy vehicle driver can, subject to favourable road, weather and vehicle conditions, safety negotiate a typical turn at between 1.06 to 1.50 times the nominal speed based on constant lane clearance and a constant radius path.

Implications

This analysis reveals adoption of constant radius turns utilizing the available lane space associates with only a small predicted increase in the safe cornering speed. Furthermore this small increase is relatively insensitive to the curve set up position.

In comparison the set up position is critical in effecting a (symmetrical) Fresnel path through a given curve. Notably safe cornering speeds in access of 45% of the nominal safe curve speed is possible provided the driver commences the turn extremely early, continues to apply steadily increasing lock and then passes along the tangent to the curve's apex with simultaneous zero lane clearance. Whereas should a driver commence a turn at the curve's geometric entry transition point and effect a path which exactly coincides with the curve's minimum turn radius the safe cornering speed is only some 6% higher.

The ability to effect above nominal speed cornering is very dependent on driver skill. Notably the ability to commence a turn well before the curve's entry geometric transition point, to apply steady smooth increasing lock, to identify the curve's entry transition point, the curve's apex and tangent at the apex. In addition the driver must alternatively, with respect to the side of the vehicle, and in smooth succession establish zero clearance, at the commencement point, at the corner's apex and finally exiting the curve. The satisfaction of these multiple simultaneous requirements implies drivers must develop considerable skill to regularly effect above nominal speed cornering. This skill is established by experience and practice and demands drivers be familiar with specific curves. The same also implies that the vehicle must be stable entering the curve. Should a vehicle instability occur at or near the curve's entry transition point a deviated path will invariably occur. Any deviation from a smooth path suggests high risk of a heavy vehicle accident. Indirectly the same suggests that should a driver enter a curve, in excess of 6% greater than the curve's nominal cornering speed, and then experience a relatively minor deviation, due to say numerous unspecified causes, the subsequent path will deviate grossly from the optimal Fresnel path. This gross deviation will expose the vehicle to high risk of complete loss of control.

The sensitivity to the set up position and in turn 'smoothness' vividly suggest single vehicle heavy vehicle accident investigations closely examine the actual curve set up conditions, vehicle operation characteristics and geometrical implications. Such information may for example be evident from post accident heavy vehicle tyre scuff marks. Unfortunately, based on the author's experience, details of the vehicle pre curve or entry transition point actual set up position is typically grossly deficient from post accident photographic evidence.

The need for proper set up and smooth paths through curves, by association, implies that single vehicle accidents on curves with which drivers are intimately familiar warrant closer examination of possible contributing factors relative to that involving single vehicle accidents at curves with which drivers are unfamiliar.

This analysis confirms that commencing a turn late significantly diminishes the safe curve speed whereas commencing turns early is generally considerably safer. The implied suggestion, for road design personnel is to provide heavy vehicle drivers with improved signage / pavement markings to indicate the curve entry braking zone commencement and curve entry transition point.

An indirect implication of this analysis and tools developed is the possible extension of the same to the optimal geometric design of road curves (Meek and Walton (1989) using widespread pc based spreadsheet software. Other significant applications would be to enhance driver cornering skills in heavy vehicle driver education programs and for accurate on going monitoring driver skills. These long term applications will yield long term and effective contributions to general road safety.

Conclusion

This analysis confidently accounts for and predicts a contributing factor to explain heavy vehicle practical cornering speeds. It should be noted this symmetrical Cornu spiral analysis assumes typical curve cambers and details with the same ignored in this initial investigation. Further the same ignores the true three dimensional camber angle established by the combination of the road super elevation and gradient (ie general up or down grade). These latter second order effects are currently under investigation. This analysis reveals that above nominal speed cornering demands that drivers be adequately skilled and be familiar with the geometry of each specific curve along the haul route. Such cornering requires drivers, on a regular reliable basis, commence turning early, apply steady smooth increasing curve lock and satisfy multiple simultaneous exacting geometric boundary conditions. This professional heavy vehicle driving operation demands unyielding vigilance. The analysis presented has significant implications in regard single vehicle heavy vehicle accident investigations and prompts suggestion for improved curve signage for heavy vehicle drivers. Longer term applications of this analysis include optimal design of road curves using spreadsheet predicted Cornu spiral coordinates, explaining optimal cornering mechanics in driver education programs and for ongoing monitoring of driver skills. These applications will provide long term and significant contributions toward the goal of a zero road toll.

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