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Applications of Fuzzy Logic in Multi-Objective Decision Support for Transport Infrastructure Options

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Abstract

This paper outlines some recent applications of fuzzy logic in the context of multi-objective decision support (MODS) for transport infrastructure options or projects. Fuzzy logic (along with other methodologies such as neurocomputing) falls under the rubric of soft computing which seeks to accommodate the pervasive imprecision of the real world. Fuzzy logic provides a basis for MODS systems involving soft (linguistic) expressions of the performance of a discrete set of options (characterised in terms of multiple objectives and attributes) and soft expressions of attribute importance. In this paper, fuzzy additive weighting, fuzzy rule-based systems (involving multiple conditional (if ... then) propositions or rules), fuzzy relational equations, and quantifier guided linguistic ordered weighted averaging operators are outlined in the context of discrimination and choice between multi-attributed options. It is concluded that fuzzy logic-based methods have potential to form an effective basis of MODS for transport infrastructure options.

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Introduction

Transport planners are regularly faced with the problem of selecting from a wide range of infrastructure options or projects, a subset of options (or a single option) that are in some sense 'best' relative to a budget constraint. The choice is often complex in the sense that the range of options is wide in terms of project type, cost, impacts or objectives to be achieved; impacts and objectives are often difficult to quantify in precise numerical terms; and there are multiple stakeholders rather than a single decision-maker. Increasingly, the assessment of transport options involving the same mode are being replaced by multimodal assessments involving, for example, comparison of additional road capacity along some freeway or arterial, the construction of a busway or light rail system, etc (NCHRP, 1994). In addition, emphasis is progressively on ecologically sustainable development factors in the assessment of transport infrastructure options, particularly as related to the conservation of non-renewable resources, reduced emission of greenhouse gases to abate global warming, and the promotion of energy conservation and low energy use transport modes (ESD Working Groups, 1991). Clearly opportunities for the application of *multi-objective decision support* (MODS) to compare and discriminate between alternative transport options present themselves.

Recently, Schwartz and Eichhorn (1997) have advocated a simple form of multi-attribute utility analysis as a means to involve multiple stakeholders in a collaborative MODS process. Schwartz et al. (1998) further elaborated this approach in a comprehensive evaluation of modal options for a congested highway segment just outside Portland metropolitan area. Key project objectives related to the satisfaction of *local travel needs* (including those of low-income and disadvantaged groups), satisfaction of *commuter, freight, recreation/tourist travel needs, health and safety, environmental quality, community economic activity, socio-cultural quality*, minimisation of *costs* (in terms of tolls/fares and from other sources), and maximisation of the *likelihood of implementation*.

The MODS methodology elaborated by Schwartz and Eichhorn (1997), essentially *additive weighting*, is substantially that proposed by Schimpeler and Grecco (1965) and Jessiman et al. (1965). More recently, other MODS methodologies have been developed albeit mostly by academics and researchers rather than transport professionals. In particular, a MODS methodology that has had increasing application in the assessment of transport options is the *analytical hierarchy process* (AHP) (Saaty, 1977a). Brief outlines of some of these applications of the AHP in transport planning to problems involving multiple objectives or attributes have been given (Saaty, 1995). The AHP is a MODS method for presenting the elements (objectives, sub-objectives, attributes, options) involved in a decision in a multi-level hierarchy. Pertinent data (weights for objectives, sub-objectives and attributes, relative performance measures for options) are derived through pairwise comparisons.

Saaty (1977b) first applied the AHP in the Sudan transport study. In this study, air, road, rail, and port transport options (103 in all) satisfying economic, social and political constraints

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were prioritised. Pak et al (1987) used the AHP to discriminate between alternative public transport systems for implementation in a newly planned city while Ulengin (1994) used the AHP to discriminate between possible infrastructure options which might solve the overall transportation problem between the European and Anatolian regions of Istanbul including maintaining the existing state (the Bosphorus and Faith bridges and the boat and ferryboat system), construction of a third bridge, improvements to connection roads to the Bosphorus and Faith bridges, a tunnel under the Bosphorus, and an improved boat and ferryboat system

A range of other MODS methodologies have also found application in a transport context. Van Huylenbroeck (1991) used a methodology (*average value ranking*), a variant of additive weighting, to compare the environmental effects of alternative routes for a high speed train (HST) through Belgium. In this application attributes included *loss of biotopes, barrier effects, hydrological effects, loss of agricultural land, intersection of farms, traffic disturbance*, etc. Rogers and Bruen (1996) used *ELECTRE* to rank options within an environmental appraisal of by-pass options for a single carriageway running through Kilmacanogue village in Ireland. *ELECTRE* establishes the degree of dominance or *outranking* that each option has over another. Unlike additive weighting, *ELECTRE* is *non-compensatory* in that changes in one attribute cannot be offset by opposing changes in another attribute. Attributes included *effect on existing land use, severance, effect on open space and sporting facilities, visual intrusion, road traffic noise, construction disturbance*, and *cost*. Tzeng and Shiau (1987) also used *ELECTRE* to assess energy conservation strategies in urban transportation.

However, the above methods do not readily acknowledge the uncertainty, vagueness, and imprecision pervasive in the context of the assessment of projects with environmental consequences. Inherently vague, imprecise, intangible, or subjective attributes (e.g. *visual intrusion, wildlife impact, loss of landscape value, social disruption*) are common impacts associated with transport infrastructure options. Outcomes along these attributes might be more authentically represented in linguistic terms (e.g. *high, low, moderate, etc.*), facilitated by fuzzy sets, rather than by conversion to numerical values. Imprecision may also result from the complexity and/or limited knowledge of systems (social, human, technological, ecological, etc.), or resource constraints which may mean that only limited, unreliable, partial, or imprecise data is available. Even for fully quantitative attributes (e.g. *savings in travel time*), outcomes of options are commonly *predicted* or *estimated* magnitudes, perhaps based on system models, and are likely to be to some extent uncertain through a multitude of possible causes (e.g. limited predictive accuracy of models, model specification error, unreliable sources of data, measurement error, etc. (Mackie and Preston (1998)). In such circumstances a range of possible values, perhaps with a modal or most-likely value, might more appropriately acknowledge the limitations of magnitude estimation and predictive models. Again fuzzy sets facilitate such representation.

Thus, from the above perspectives, methods which demand less precise input are desirable.

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MODS based on fuzzy logic explicitly acknowledges uncertainty and provides an alternative to the above methods permitting the inclusion of objectives and attributes that are uncertain, vague, imprecise or difficult to quantify in numerical terms. Fuzzy logic based methods also avoid inappropriately high levels of discrimination between options that differ only slightly in their attributes. Further, fuzzy logic methods can often be constructed to integrate *soft* qualitative data, subjective judgement and opinion, and *hard* quantitative data.

Fuzzy logic based MODS methodologies are also applicable in the context of *screening problems* in which there exists a large set of possible options each of which is represented by a minimal amount of information supporting its appropriateness as a 'best' solution. This minimal amount of information provided by each option is used to select a subset to be further investigated, perhaps by the commitment of more substantial resources for data collection and analysis.

Vagueness, imprecision or fuzziness is viewed as a type of *deterministic uncertainty* in contrast to randomness or *statistical uncertainty*. The latter is modelled by probability and measures event *occurrence* whereas the former describes event *ambiguity* (Kosko, 1992). Though probability theory has been advocated as a means to more explicitly acknowledge uncertainty in MODS (e.g. *stochastic additive weighting* (Kahne, 1975)), it is believed that this uncertainty is essentially of a deterministic nature more appropriately modelled by fuzzy set theory. Thus fuzzy set theory and fuzzy logic provide a more convincing and defensible foundation for the representation of deterministic uncertainty.

A range of fuzzy logic based methods for MODS exist including *fuzzy additive weighting* (Schmucker, 1984; Smith, 1992; Liang and Wang, 1991; Teng and Izeng, 1996), *fuzzy rule based systems* (Smith, 1995-6, 1997a,b), *fuzzy relational equations* (Smith, 1999a) and *ordered weighted averaging operators* (Bordogna et al., 1998; Smith, 1999b).

Fuzzy sets and fuzzy logic

Fuzzy logic can be considered as a multivalent generalisation of classical bivalent logic. In classical logic, a proposition is true or false whereas in fuzzy logic, a proposition may be true to a degree and false to a degree. Fuzzy sets provide the basis for fuzzy logic. If X is a classical universal set, a real function defined on X , $A: X \rightarrow [0,1]$ is called the *membership function* or *grade of membership* of A and defines the fuzzy set (or more precisely, *fuzzy subset*) A of X . This is the set of pairs $(x, A(x))$, $x \in X$. A discrete fuzzy subset is represented as $\sum A(x)|x$. *Fuzzy numbers* are fuzzy subsets which are assumed to be normal (maximum membership equal to 1) and convex. Particular types of fuzzy numbers include *trapezoidal*,

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triangular, and bell-shaped Gaussian and Π or piecewise quadratic fuzzy numbers (based on the concept of S and 1 - S functions) (Zadeh, 1975).

The power of fuzzy sets lies in their ability to represent soft linguistic variables rather than quantitative variables. A *linguistic variable* is one whose values are words or sentences in a natural or artificial language (Zadeh, 1975). The concept of a linguistic variable provides a means for the approximate characterisation of phenomena too complex or too ill-defined for description in conventional quantitative terms. In addition, linguistic values are intuitively easy to use in expressing the subjectiveness and vagueness of an individual's judgements. A linguistic variable is defined in terms of a base variable, whose values are assumed to be real numbers within a specific interval of the real numbers, \mathbb{R} , e.g. $[0,1]$ or $[0,100]$. In MODS, important base variables are the *performance* (of options with respect to attributes) and the *importance* (of attributes). Linguistic terms (e.g. *low*, *medium*, *high*) approximate the actual values of the associated base variable. Their meanings are captured by fuzzy numbers.

A linguistic term set defines the *information granularity* or the finest level of distinction between different quantifications of uncertainty. Term sets should be small enough so as not to impose useless precision but rich enough to allow meaningful discrimination between options. Usually term sets have odd cardinality of 7 or 9 with a middle term *approximately 0.5* (assuming base set $[0,1]$). This is consistent with limits on the information processing of individuals (Miller, 1956). Examples of possible linguistic term sets for performance (*poor*, *very low*, *low*, *medium*, *high*, *very high*, *superior*) and importance (*negligible*, *very low*, *low*, *medium*, *high*, *very high*, *critical*) are shown in Figure 1.

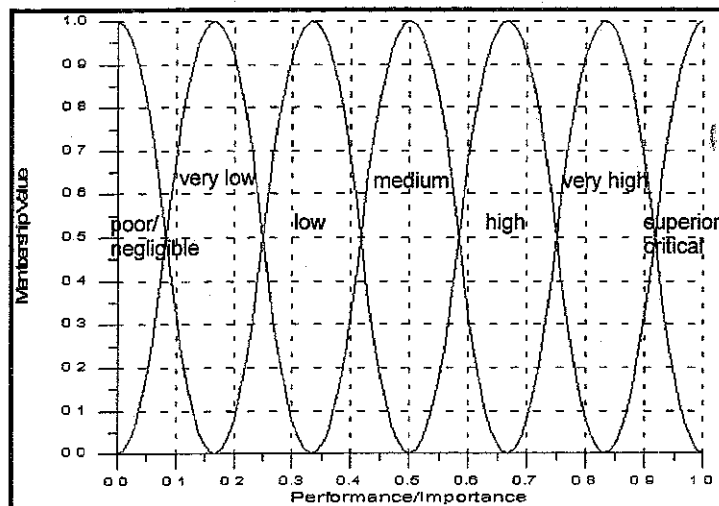


Figure 1 Possible term set for performance/importance

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Fuzzy additive weighting

Additive weighting derives the fuzzy aggregate performance of option i (denoted V_i) as the sum of the products of the fuzzy weight (w_j) and the fuzzy performance of option i with respect to attribute j (ϕ_{ij}) as $V_i = \{\oplus_{j=1}^J w_j \otimes \phi_{ij}\} \ominus \{\oplus_{j=1}^J w_j\}$ ($i = 1, \dots, I$). Fuzzy weights and performances may be discrete or continuous. Schmucker (1984) used discrete fuzzy subsets to represent the performance of options with respect to attributes and the importance of attributes. Addition was based on the *extension principle* (Zadeh, 1975). More recently, fuzzy additive weighting involving standard fuzzy arithmetic (Smith, 1995) has been presented. This is represented as $V_i = (1/I) \otimes \{\oplus_{j=1}^J w_j \otimes \phi_{ij}\}$ ($i = 1, \dots, I$). Here fuzzy weights (w_j) and fuzzy performances (ϕ_{ij}) are expressed in terms of continuous (triangular, trapezoidal, or Π) fuzzy numbers. However, it has recently been shown that a computationally efficient approach to fuzzy additive weighting involves *defuzzifying* the fuzzy numbers (representing linguistic values of performance and attribute importance) to *crisp* values prior to the use of conventional additive weighting (Tseng and Klein, 1992; Chen and Klein, 1997).

Consider four transport options assessed against six objectives/attributes (based on Smith, 1997a) as shown in the table below

	SIT	SI	NI	FFI	AQI	CC
Option 1	poor	superior	very high	low	medium	poor
Option 2	medium	low	superior	very low	superior	medium
Option 3	very low	poor	very low	superior	very low	very low
Option 4	superior	poor	very low	poor	poor	superior
Importance	high	very high	high	high	medium	low

Here SIT = *savings in travel time*, SI = *social impact*, NI = *noise impact*, FFI = *flora/fauna impact*, AQI = *air quality impact*, and CC = *capital cost*. Option 4 is minimally environmentally sensitive emphasising predominantly engineering/economic factors; other options satisfy the environmental factors each to a varying extent, but perform less satisfactorily with respect to engineering/economic factors. Thus

$$V_1 = (1/6) \otimes \{\text{high} \otimes \text{poor} \oplus \text{very high} \otimes \text{superior} \oplus \dots \oplus \text{low} \otimes \text{poor}\}$$

$$V_2 = (1/6) \otimes \{\text{high} \otimes \text{medium} \oplus \text{very high} \otimes \text{low} \oplus \dots \oplus \text{low} \otimes \text{medium}\}$$

$$V_3 = (1/6) \otimes \{\text{high} \otimes \text{very low} \oplus \text{very high} \otimes \text{poor} \oplus \dots \oplus \text{low} \otimes \text{very low}\}$$

$$V_4 = (1/6) \otimes \{\text{high} \otimes \text{superior} \oplus \text{very high} \otimes \text{poor} \oplus \dots \oplus \text{low} \otimes \text{superior}\}$$

are fuzzy subsets (shown in Figure 2) which may be defuzzified to order options in terms of

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preference. In terms of *total utility* values (Smith, 1995), option 2 is selected as 'best' with preferences as $O_2 > O_1 > O_4 > O_3$

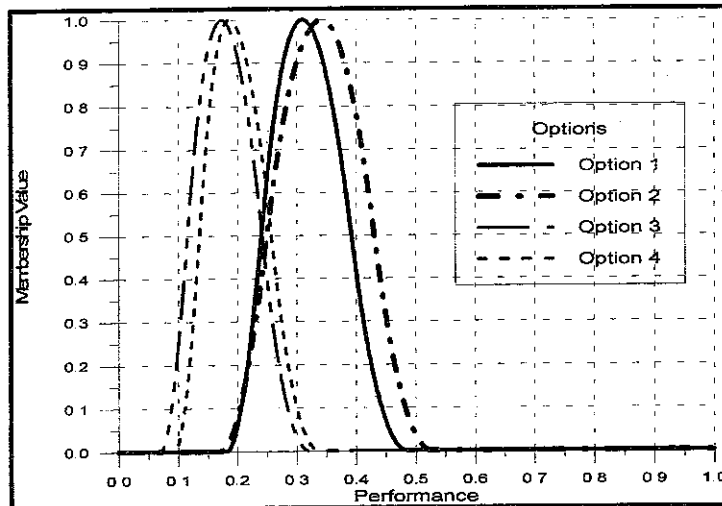


Figure 2 Performance of options using fuzzy additive weighting

Fuzzy rule-based systems

A *fuzzy rule-based system* involves knowledge represented in terms of *if ... then* rules. In general, m rules each with n antecedents, may be expressed as

if V_1 is A_{11} and V_2 is A_{12} and ... and V_n is A_{1n} then U is B_1

V_1, V_2, \dots, V_n and U are linguistic variables, A_{11} is a fuzzy subset of X_1 (i.e. linguistic value of V_1), A_{12} is a fuzzy subset of X_2 (i.e. linguistic value of V_2), etc., B_1 is a fuzzy subset of Y (i.e. linguistic value of U). Rules are aggregated using 'and', or 'or' (Lee, 1990). Then, given inputs ' V_1 is A_{01} and V_2 is A_{02} and ... and V_n is A_{0n} ', the rule-based system infers output ' U is B_0 '. Such systems of *if ... then* rules each with multiple antecedents and a single consequent are referred to in the fuzzy logic control context as *multiple input, single output* (MISO) systems (Lee, 1990) or as *fuzzy systems* (Kosko, 1992). Inputs to a system are often assumed to be crisp or *fuzzy singletons* though fuzzy inputs are possible (Lee, 1990).

An example of a fuzzy system might be

if V_1 is **very high** and V_2 is **low** and V_3 is **low** and V_4 is **low**
and V_5 is **low** and V_6 is **medium** then U is **very strong**
if V_1 is **very high** and V_2 is **very low** and V_3 is **very low** and V_4 is **very low**
and V_5 is **very low** and V_6 is **very low** then U is **definite**

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if V_1 is **high** and V_2 is **low** and V_3 is **low** and V_4 is **low** and
 V_5 is **medium** and V_6 is **medium** then U is **moderate**
 if V_2 is **not low** or V_3 is **not low** then U is **weak**

where V_1 = savings in travel-time, V_2 = social impact, V_3 = noise impact, V_4 = flora/fauna impact, V_5 = air quality impact, V_6 = cost. The consequent in each rule, U , represents preference with term set {none, very weak, weak, moderate, strong, very strong, definite} defined analogous to those for performance. Here, it is assumed that the base sets for the antecedents (attributes) are quantitative, so that quantitative outcomes can be assessed for each option. The base set for preference might be arbitrarily [0,1] or [0,100]. In practice, each rule need not include all antecedents (i.e. attributes). Given crisp values for $V_1 - V_6$ for each option, the associated preference may be calculated from the knowledge embodied in the rules. However, a simplified fuzzy system using the set of options as the base set for each antecedent facilitates the relative assessment of options with respect to attributes according to the extent to which the antecedent condition is satisfied (Smith, 1995-6, 1997a).

Fuzzy systems are in a sense parallel processors (Cox, 1995). Given input values, all rules that have any truth in their premises will fire and contribute to the output fuzzy subset. Fuzzy systems have been predominantly used in the context of control (de Silva, 1995), though other applications in the context of evaluation have appeared (Levy et al., 1991; Levy and Yoon, 1995). Numerous alternative structures for fuzzy rule-based systems are possible (Mizumoto, 1994; Yager and Filev, 1994; Kosko, 1992) including the *approximate analogical reasoning* method which facilitates non-quantitative assessment of options with respect to attributes (Smith, 1997b).

Fuzzy relational equations

Given an assessment of the importance of attribute j , ζ_j , expressed as a fuzzy subset of discrete base set Z and the performance of option i with respect to attribute j , η_j^i , expressed as a fuzzy subset of discrete base set Y , then a system of *fuzzy relational equations* may be established as $\zeta_j = \eta_j^i \circ \phi_{ij}$ ($j = 1, \dots, J$). ϕ_{ij} is an (unknown) *fuzzy relation* between the importance of attribute j and the performance of option i with respect to attribute j , defined on base set $Y \times Z$. In membership the j th relational equation or *fuzzy composition* (Terano et al., 1987) is $\zeta_j(z) = \bigvee_{y \in Y} (\eta_j^i(y) \wedge \phi_{ij}(y, z))$.

The solution of this system (if it exists) is given as $\hat{R}_i = \bigcap_{j=1, J} \hat{\phi}_{ij}$ where $\hat{\phi}_{ij} = ((\eta_j^i)^{-1} \circ_\alpha \zeta_j)$ is the largest ϕ_{ij} satisfying $\zeta_j = \eta_j^i \circ \phi_{ij}$, expressed in membership terms as $\phi_{ij}(y, z) = \eta_j^i(y) \alpha \zeta_j(z)$ (Sanchez, 1976). Here, \circ_α denotes the α -composition, where $a \alpha b$ is the α -relative pseudocomplement of a in b (Gödelian implication), defined as $a \alpha b = 1$ if $a \leq b$ and $a \alpha b = b$ if $a > b$ ($a, b \in [0, 1]$). This is a measure of the relative degree of containment of one grade of membership (a) in another (b). For example, if $\zeta_j = \{0.7|z_1, 0.3|z_2, 1.0|z_3\}$ and $\eta_j^i = \{0.2|y_1, 0.0|y_2, 0.8|y_3, 1.0|y_4\}$, then $\phi_{ij} = \{1.0|(y_1, z_1), 1.0|(y_1, z_2), 1.0|(y_1, z_3), 1.0|(y_2, z_1), 1.0|(y_2, z_2),$

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1.0|(y₂,z₃), 0.7|(y₃,z₁), 0.3|(y₃,z₂), 1.0|(y₃,z₃), 0.7|(y₄,z₁), 0.3|(y₄,z₂), 1.0|(y₄,z₃)} where, for example, $\varphi_{ij}(y_1, z_1) = \eta_j^i(y_1) \alpha \zeta_j(z_1) = 0.2 \alpha 0.7 = 1$ and $\varphi_{ij}(y_3, z_1) = \eta_j^i(y_3) \alpha \zeta_j(z_1) = 0.8 \alpha 0.7 = 0.7$. Note that $\zeta_j = \eta_j^i \circ \varphi_{ij}$ or, in membership terms, $\zeta_j(z) = \bigvee_{y \in Y} (\eta_j^i(y) \wedge \varphi_{ij}(y, z))$.

Assuming that the importance of satisfying all attributes is given by fuzzy subset **critical**, then in the relational equation, **critical** = $\eta^i \circ \check{R}_i$, find $\hat{\eta}^i$ (the largest η^i satisfying the equation), given as $\hat{\eta}^i = (\check{R}_i \circ_a \text{critical})^{-1}$ or, in membership terms, $\hat{\eta}^i(y) = \bigwedge_{z \in Z} [\check{R}_i(y, z) \alpha \text{critical}(z)]$. This method was proposed by Wilhelm and Parsaei (1991) who adopt the *Hamming distance* to identify the fuzzy subset, $\hat{\eta}^i$, representing the closest to fuzzy subset **superior** (performance)

Thus, for the above transport options,

$$\begin{aligned} \check{R}_1 &= (\text{poor}^{-1} \circ_a \text{high}) \cap (\text{superior}^{-1} \circ_a \text{very high}) \cap \dots \cap (\text{poor}^{-1} \circ_a \text{low}) \\ \check{R}_2 &= (\text{medium}^{-1} \circ_a \text{high}) \cap (\text{low}^{-1} \circ_a \text{very high}) \cap \dots \cap (\text{medium}^{-1} \circ_a \text{low}) \\ \check{R}_3 &= (\text{very low}^{-1} \circ_a \text{high}) \cap (\text{poor}^{-1} \circ_a \text{very high}) \cap \dots \cap (\text{very low}^{-1} \circ_a \text{low}) \\ \check{R}_4 &= (\text{superior}^{-1} \circ_a \text{high}) \cap (\text{poor}^{-1} \circ_a \text{very high}) \cap \dots \cap (\text{superior}^{-1} \circ_a \text{low}) \end{aligned}$$

from which the relational equations, **critical** = $\eta^i \circ \check{R}_i$ are solved for $\hat{\eta}^i$ ($i = 1, 2, 3, 4$). These fuzzy subsets (representing performance) are assessed relative to fuzzy subset **superior**. However, a problem with this method is that, depending on fuzzy subset representation of linguistic terms and the number of attributes, the intersection of fuzzy relations may be empty and no solution is available. Variations on this approach have been given elsewhere (Smith, 1999b)

A further method for aggregating fuzzy relations, ψ_{ji} , (defined on base set $Z \times Y$) between the performance of option i with respect to attribute j (defined on base set Y) and the importance of attribute j (defined on base set Z), has been presented by Eldukair and Ayyub (1992). In membership terms, this is $\psi_{ji}(z, y) = \zeta_j(z) \wedge \eta_j^i(y)$. This definition of $\psi_{ji}(z, y)$ is a solution to the fuzzy relational equation, $\eta_j^i = \zeta_j \circ \psi_{ji}$, or in membership terms, $\eta_j^i(y) = \bigvee_{z \in Z} [\zeta_j(z) \wedge \psi_{ji}(z, y)]$ (Terano et al., 1987). A more consistent variation on this method (Smith, 1999b) is, given $\psi_{ji}(z, y)$, let

$$\Psi_{ji} = \max_{z \in Z} \{r_{ji}(z)\} \max_{y \in Y} \{c_{ji}(y)\}$$

where $r_{ji}(z) = z \sum_{y \in Y} \psi_{ji}(z, y) / \sum_{z \in Z} \sum_{y \in Y} \psi_{ji}(z, y)$ and $c_{ji}(y) = y \sum_{z \in Z} \psi_{ji}(z, y) / \sum_{z \in Z} \sum_{y \in Y} \psi_{ji}(z, y)$. The fuzzy subset $\Psi_i = \{\Psi_{i1}|\theta_1, \Psi_{i2}|\theta_2, \dots, \Psi_{ij}|\theta_j\}$ may be construed as a representation of the degree to which attributes (denoted $\{\theta_1, \theta_2, \dots, \theta_j\}$) meet option i having regard to the importance of those attributes. Membership values for this fuzzy subset may be divided by $\hat{\Psi}_i$ for $\hat{\Psi}_i = \hat{\zeta}^{-1} \times \hat{\eta} = \hat{\text{critical}}^{-1} \times \hat{\text{superior}}$ (i.e. $\hat{\Psi}_i = 0.203$). The transformed fuzzy subsets $\hat{\Psi}_i = \{\hat{\Psi}_{i1}|\theta_1, \hat{\Psi}_{i2}|\theta_2, \dots, \hat{\Psi}_{ij}|\theta_j\}$ will not necessarily be normal (i.e. maximal membership may not equal 1)

Aggregation of the $\hat{\Psi}_{ji}$ is achieved using an *ordered weighted averaging* (OWA) operator

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(Yager, 1988) represented as $OWA(\hat{\Psi}_{1i}, \hat{\Psi}_{2i}, \dots, \hat{\Psi}_{ji}) = \sum_{k=1}^j \alpha_k b_k$ where b_k is the k th largest element of $\{\hat{\Psi}_{1i}, \hat{\Psi}_{2i}, \dots, \hat{\Psi}_{ji}\}$. Weights $\{\alpha_1, \alpha_2, \dots, \alpha_j\}$ (such that $\alpha_k \in [0,1]$ and $\sum_{k=1}^j \alpha_k = 1$) are associated with the position of b_k . The OWA operator includes the minimum and maximum when $\alpha = \{0, 0, \dots, 1\}$ and $\alpha = \{1, 0, \dots, 0\}$, respectively. Thus the extreme OWA operators are 'and' (no compensation) and 'or' (full compensation) operators. An average of the $\hat{\Psi}_{ji}$ ($j = 1, \dots, J$) corresponds to the OWA with weights $\{1/J, 1/J, \dots, 1/J\}$. The degree of *orness* of a OWA operator is defined as $orness(\alpha) = \sum_{k=1}^j \alpha_k (j - k) / (j - 1)$. Orness is an indication of the inclination for the OWA operator to give more weight to higher membership grades than lower ones. Orness is zero for 'and' and unity for 'or'. Thus, OWA weights reflect the degree of *optimism* or *pessimism* of the decision maker with 'or' selecting the most optimistic and 'and' selecting the most pessimistic value. For equal weights, $orness(\alpha) = 0.5$. The *Hurwicz* strategy involves a convex combination of the optimistic and pessimistic solutions with weights $\alpha = \{\lambda, 0, \dots, (1 - \lambda)\}$ where the parameter, λ , is such that $0 \leq \lambda \leq 1$. In this case, $orness(\alpha) = 1 - \lambda$. Thus, $\lambda = 1$ yields the optimistic and $\lambda = 0$ yields the pessimistic solution. To illustrate the above approach, assume that the $[0,1]$ interval evenly divided into (say) 18 sub-intervals. Discrete fuzzy subsets may be identified to represent the linguistic labels of performance and importance. Thus, base set Y is represented as $Y = \{y_1, y_2, y_3, y_4, \dots, y_{19}\} = \{0, 0.056, 0.111, 0.167, \dots, 1\}$ and fuzzy subset **poor** (performance), for example, is represented as **poor** = $\{1|y_1, 0.778|y_2, 0.222|y_3, 0|y_4, \dots, 0|y_{19}\} = \{1|0, 0.778|0.056, 0.222|0.111, 0|0.167, \dots, 0|1\}$. Similarly, base set Z is represented as $Z = \{z_1, z_2, z_3, z_4, \dots, z_{19}\} = \{0, 0.056, 0.111, 0.167, \dots, 1\}$ and fuzzy subset **negligible** (importance), for example, is represented as **negligible** = $\{1|z_1, 0.778|z_2, 0.222|z_3, 0|z_4, \dots, 0|z_{19}\} = \{1|0, 0.778|0.056, 0.222|0.111, 0|0.167, \dots, 0|1\}$. These definitions follow from the linguistic terms defined by Π fuzzy numbers in Figure 1. Then $\hat{\Psi}_i = \{\hat{\Psi}_{1i}|\theta_1, \hat{\Psi}_{2i}|\theta_2, \dots, \hat{\Psi}_{6i}|\theta_6\}$ for each option i is shown in Figure 3.

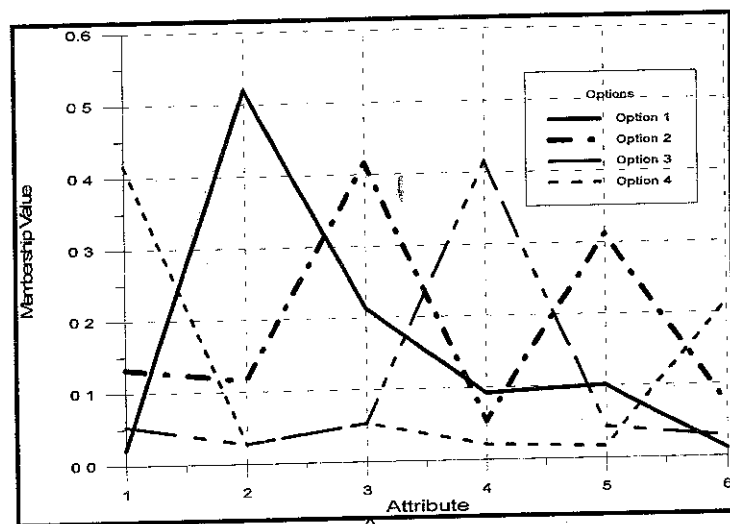


Figure 3 $\hat{\Psi}_i$ ($i = 1, 2, 3, 4$)

Results are given in the following table

Option	Pessimistic	Optimistic	Average	Hurwicz ($\lambda = 0.5$)
1	0.01	0.52	0.16	0.27
2	0.05	0.42	0.18	0.23
3	0.03	0.42	0.10	0.22
4	0.02	0.42	0.13	0.22

* denotes 'best' option under decision criterion

Thus, for the Hurwicz criterion, $O_1 > O_2 > (O_3 = O_4)$. The average yields option O_2 as 'best'

Ordinal and linguistic ordered weighted averaging operators

Fuzzy methods involving additive weighting and relational equations yield fuzzy subsets that do not necessarily correspond to any term in the term set requiring either the application of some defuzzification or linguistic approximation method (i.e. finding the closest term in the term set to the output fuzzy subset). Alternative methods aggregate linguistic labels by direct computation on the labels. Terms are distributed on a scale on which a *total order* is defined. For example, let $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{\text{poor, very low, low, medium, high, very high, superior}\}$ be a set of linguistic labels for *performance* where $s_0 < s_1 < \dots < s_{\max} (= s_6)$. s_0 and s_{\max} are the lowest and highest elements, respectively. Similarly, for *importance*, a linguistic term set might be $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\} = \{\text{negligible, very low, low, medium, high, very high, critical}\}$. These labels may be used to represent the fuzzy numbers in Figure 1. Let $\max = \#S - 1$ where $\#S$ is the *cardinality* (the number of elements) of S . Usually it is required that the linguistic term set satisfy the following conditions that $s_i \vee s_j = s_i$ if $s_i \geq s_j$ and $s_i \wedge s_j = s_j$ if $s_i \leq s_j$. In addition, a negation operator for a linguistic label is defined as $\text{neg}(s_i) = s_{\max - i}$. Thus, for example, $\text{neg}(s_2) = s_{6-2} = s_4$ (i.e. $\text{neg}(\text{low}) = \text{high}$) and $\text{neg}(s_0) = s_{\max}$ (i.e. $\text{neg}(\text{negligible}) = \text{critical}$, $\text{neg}(\text{superior}) = \text{poor}$). Linguistic expressions of the performance of options with respect to attributes are drawn from linguistic term set S . Thus $\phi_{ij} \in S$ is the performance of option i with respect to attribute j . In addition, weights $w_j \in S$ reflect the importance of attributes.

In classical logic, *quantifiers* in statements or propositions may be used to represent the number of items satisfying a given predicate. However, classical logic permits only two quantifiers, *for all* and *there exists (not none)*. Zadeh (1983) introduced *linguistic quantifiers* represented as fuzzy subsets in *linguistically quantified statements*. The general form of a quantified statement is ' Q X 's are A ', where Q is a linguistic quantifier (e.g. *few, most, at least n*), X is a class of objects and A , a fuzzy subset of X , is some property associated with the objects. For example 'most objectives are satisfied by option i ' is a quantified statement where X is a set of objectives, Q is the quantifier *most* and A is a fuzzy subset of X indicating the extent to which option i satisfies each objective.

Absolute quantifiers, defined on the set of non-negative reals, \mathbb{R}^+ , are used to represent

amounts that are absolute in nature (*about 5, more than 10*) and are closely related to the concept of the count or number of elements. *Proportional* quantifiers (*most, few, at least half*), defined on the unit interval, represent relative amounts. Yager (1993) defines *regular increasing monotone* (RIM) quantifiers (e.g. *all, most, many*) such that $Q(0) = 0$, $Q(1) = 1$, and $Q(r) \geq Q(s)$ if $r > s$. Q is a fuzzy subset in the unit interval. For example, the quantifier *most* might be represented as, for example, (A) $Q(r) = 1$ ($0.8 \leq r \leq 1$), $Q(r) = 2r - 0.8$ ($0.3 < r < 0.8$) and $Q(r) = 0$ ($0 \leq r \leq 0.3$), or as (B) $Q(r) = r^2$, $r \in [0, 1]$

Linguistic quantifiers are formalised by OWA operators, in particular, *ordinal* OWA operators (Yager, 1992). The ordinal OWA operator involves weights $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_J\}$ (where $\alpha_j \in S$, $\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_J$ and $\bigvee_{j=1, J} \{\alpha_j\} = s_{\max}$) is defined as $OWA(\phi_{i1}, \phi_{i2}, \dots, \phi_{iJ}) = \bigvee_{j=1, J} \{\alpha_j \wedge b_j\}$ where $\{b_1, b_2, \dots, b_J\}$ is associated with $\{\phi_{i1}, \phi_{i2}, \dots, \phi_{iJ}\}$ such that b_j is the j th largest ϕ_{ij} . The orness of an ordinal OWA operator is given as $orness(\alpha) = \bigvee_{j=1, J} (\alpha_j \wedge \text{Label}((J - j)/(J - 1)))$ where $\alpha_j \in S$ and where $\text{Label}(\cdot)$ maps a numeric value, $x \in [0, 1]$, to a linguistic label, $s_k \in S$, defined as $\text{Label}(x) = s_k$ ($k/\#S \leq x < (k + 1)/\#S$), $k = 0, \dots, \max$, and $\text{Label}(1) = s_{\max}$.

OWA weights $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_J\}$ are associated with the a priori definition of the RIM quantifier, Q . $\alpha_j \in S$ is obtained by applying the $\text{Label}(\cdot)$ function as $\alpha_j = \text{Label}(Q(j/J))$ ($j = 1, \dots, J$). For six attributes, OWA weights are $\alpha = \{\text{Label}(Q(1/6)), \text{Label}(Q(2/6)), \dots, \text{Label}(Q(1))\} = \{s_0, s_0, s_2, s_5, s_6, s_6\}$. The importance of attributes, $w = \{\text{high}, \text{very high}, \text{high}, \text{high}, \text{medium}, \text{low}\} = \{s_4, s_5, s_4, s_4, s_3, s_2\}$, may be included by modifying the values to be aggregated (Yager, 1992), for example, as follows

$$h_{ij} = (w_j \vee (\text{neg}(\text{orness}(\alpha)))) \wedge (\phi_{ij} \vee \text{neg}(w_j)) \wedge (\phi_{ij} \vee (\text{neg}(\text{orness}(\alpha))))$$

where $w_j \in S$, $\phi_{ij} \in S$, and $orness(\alpha) \in S$. For six attributes, $orness(\alpha) = \bigvee (\alpha_1 \wedge \text{Label}(1), \alpha_2 \wedge \text{Label}(4/5), \dots, \alpha_6 \wedge \text{Label}(1/5)) = \bigvee (s_0 \wedge s_6, s_0 \wedge s_5, s_2 \wedge s_4, s_5 \wedge s_2, s_6 \wedge s_1, s_6 \wedge s_0) = s_2$. Given h_{ij} ($i = 1, \dots, I$; $j = 1, \dots, J$), the OWA operator is calculated as $OWA_Q(h_{i1}, h_{i2}, \dots, h_{iJ}) = \bigvee_{j=1, J} \{\text{Label}(Q(j/J)) \wedge b_j\}$ ($i = 1, \dots, I$).

In terms of the ordinal OWA operator using definition (A) as the quantifier *most*, preferences for options are $O_1 = s_3$, $O_2 = s_3$, $O_3 = s_2$, $O_4 = s_2$ (i.e. $(O_1 = O_2) > (O_3 = O_4)$). Thus a more coarse discrimination between options is obtained consistent with the decreased granularity of uncertainty adopted in the manipulation of linguistic terms or labels.

It is also possible to aggregate linguistic values in a numeric environment (Bordogna et al., 1997). That is, linguistic values $\phi_{ij} \in S$ and $w_j \in S$ are mapped into numbers in $[0, 1]$ by applying a linguistic label to numeric function defined as $\text{Label}^{-1}(s_k) = \text{index}(k)/\max$ ($k = 0, 1, \dots, \max$) where $\text{index}(k) = k$, ($k = 0, \dots, \max$). Thus for linguistic weights $\{s_4, s_5, s_4, s_4, s_3, s_2\}$, numeric equivalents are $\{0.67, 0.83, 0.67, 0.67, 0.5, 0.33\}$. Then, the numeric OWA operator is $OWA_Q(\phi_{i1}, \phi_{i2}, \dots, \phi_{iJ}) = \sum_{j=1, J} \alpha_j b_j$. Here, the OWA_Q operator weights are determined by a RIM quantifier, Q , so that $\alpha_j = Q(\sum_{k=1, j} u_k) - Q(\sum_{k=1, j-1} u_k)$ where u_j is the weight associated with b_j . If the importance weights are not normalised such that $\sum_{j=1, J} w_j = 1$, then $\alpha_j = Q(\sum_{k=1, j} u_k / \sum_{k=1, J} u_k) - Q(\sum_{k=1, j-1} u_k / \sum_{k=1, J} u_k)$. $\text{Label}(OWA_Q(\phi_{i1}, \phi_{i2}, \dots, \phi_{iJ}))$ yields a linguistic expression of

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performance for option i In terms of the numeric OWA operator, $O_1 = s_3$, $O_2 = s_3$, $O_3 = s_1$ and $O_4 = s_0$ (i.e. $(O_1 = O_2) > O_3 > O_4$)

Discussion

This paper has reviewed some recent applications of fuzzy logic in the context of multi-objective support (MODS) for transport infrastructure options. Fuzzy logic falls under the rubric of soft computing which seeks to accommodate the pervasive imprecision of the real world. Fuzzy logic provides a basis for MODS involving soft (linguistic) expressions of the performance of a discrete set of options (characterised in terms of multiple objectives and attributes) and soft expressions of attribute performance. In this paper, fuzzy additive weighting, fuzzy rule-based systems (involving multiple conditional (if ... then) propositions or rules), fuzzy relational equations, and quantifier guided linguistic ordered weighted averaging operators have been outlined as a possible basis for the discrimination and choice between multi-attributed options. In terms of the example and the weights assigned to attributes, the methods illustrated select O_1 or O_2 above O_3 and O_4 . Further, one instance of a fuzzy rule-based system involving the rules expressed above (though not detailed here) also yields O_1 and O_2 above O_3 and O_4 .

Clearly, however, much more investigation and comparative assessment of the merits of alternative fuzzy logic based methods is warranted. In particular, the appropriate representation of linguistic terms for the assessment of performance and importance, the appropriateness of aggregation processes, and the possibility for combining both linguistic and numeric assessments of performance (readily achieved for fuzzy additive weighting and fuzzy systems) requires further attention.

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