# A New Modelling Paradigm for Strategic Planning

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#### Abstract:

The need for new paradigms in transport planning has been debated for many years. The shortcomings of the traditional quantitative models are widely accepted in academia, and practitioners are more and more coming to the same conclusions. The reasons for the wide application of the traditional models is that there have been no other mechanisms that can be usefully employed to solve the questions being asked This paper presents a new paradigm for Strategic Planning, based on a Futures Model that employs Genetic Algorithms (GA) to generate future scenarios and search for desired outcomes The results of some model runs are presented

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The need for new paradigms in transport planning has been stated over a number of years (Lee (1973), Starkie (1974), Atkins (1986, 1987), Orski (1987), Larson (1988) and Stephens (1991)). The shortcomings of the traditional quantitative models are widely accepted in academia, and practitioners are more and more coming to the same conclusions. The reason for the wide application of these models is that there are, as yet, no other mechanisms that can be usefully employed to solve the questions being asked. However, this paper describes a new paradigm for Strategic Planning, and the results of model runs using the new model are presented.

#### What We Need?

We need a modelling system that can be employed to develop broad strategic level forecasts of factors of interest (e.g. congestion, road deaths, pollution, etc) over some defined time horizon. We also need a mechanism that will inform us of the factors that require manipulation (e g fuel prices), by what degree (e.g. 23 per cent) and in which direction (eg increased) so as to bring about some defined 'preferred future' (we will not enter into the philosophical question as to who constructs these preferred futures) where desirable levels of given factors have been nominated (e.g. congestion, road deaths, pollution, etc) for some specified time in the future.

The model needs to be made aware of factors that are controllable so that recommended manipulations are only to factors that we consider within our power to affect. It should be possible to nominate a direction of allowable movement of any factor (e g it may be possible to decrease road taxes but not to increase them) We also require a system that will allow for the inputting of allowable ranges of factor movements (e g  $\pm$  13 per cent) and we need a system that will supply us with a number of possible ways of achieving the desired 'preferred future'

#### Why We Need It!

There are significant problems in the arena of policy formulation, especially where we are considering long-term proposals. Some of these problems are:

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Considering the prevalence of the types of impediments listed above it becomes obvious that if we are to ensure that long-term decisions are made in a structured, well informed, intelligent and needed manner then we must, of necessity, put systems in placed to eliminate, or at least, help ameliorate these problems. It is eminently possible that many of these, relatively new<sup>1</sup>, behavioural characteristics could precipitate the

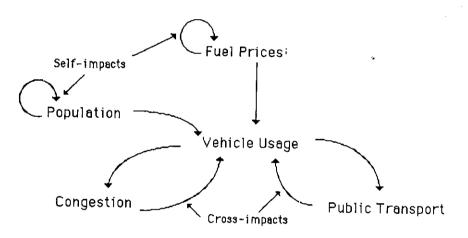
<sup>&</sup>lt;sup>I</sup> These behaviours, or many of them, are the direct result of the speed of information dissemination. In the past it was patently possible for a decision-maker to make decisions with longer term perspectives. The reasons are that it took time for the decision to be promulgated throughout the community. The populace was less informed than they are today and any response from them would take a commensurate 40

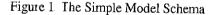
ultimate destruction of humanity or at least to a dramatic reduction in the quality and quantity of human life!

# What Is Available!

The Irrends Integration Procedure (IIP+) (Chambers and Taylor 1991) provided a modelling system that generated the future scenarios as described above. IIP+ was a valuable tool for planners and analysts to gain an understanding of the behaviour of complex systems and to obtain insights into the likely future impacts of policies and decisions. Valuable though these insights were they did not provide information on how desired outcomes might be achieved or how undesired outcomes might be avoided unless the analyst was prepared to perform tedious and extensive sensitivity analysis

Any model constructed has a schema of the environment being studies within itself. Given that this is true why should we not ask the model itself how to generate some 'preferred future' for that given model construct? In fact this is what the new manifestation of the TIP + model does! The TIP + model has now been enhanced, and renamed *Genie*. *Genie* includes a system that allows the user to construct a 'preferred future'. The model then applies a genetic algorithm to the model construct and generates a number of scenarios of factor manipulations that will bring about the 'preferred future' within the limits of the model construct.





The final factor trends from running the model are pethaps as shown in Figure 2:

amount of time to be fed back to the decision-makers These factors, and others, removed a number of the pressures that modern decision-makers must confront

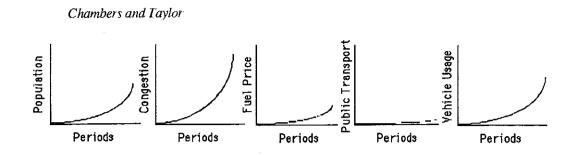


Figure 2 Final results from the TIP model run

Assume that we find the increases in congestion to be unacceptable. We might manipulate the ending value of congestion to be of the form shown in Figure 3:

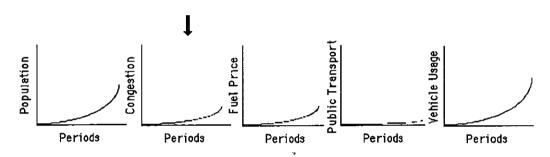


Figure 3 Modification of Congestion for a 'preferred future'.

The results from the genetic algorithm suggest that an increase in fuel prices by 14 per cent over the current levels provides one means to obtain the desired results for congestion levels. The results of one of the runs give the forecasted future as in Figure 4, by implementing the suggested increase in fuel prices, on the understanding that the model was notified that the only factor that could be manipulated was fuel prices.

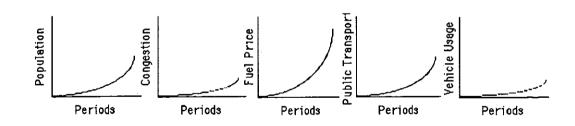


Figure 4 Final results from a genetic algorithm run with Congestion constrained and Fuel Price being the Manipulable Factor.

There are no changes in the population projections since the factor changed by the genetic algorithm (fuel prices) has no impact upon birth, death, immigration or emigration rates for the population. The outcomes as seen in Figure 4 are:

- (a) congestion has decreased in line with the desired changes;
  (b) fuel prices have been increased at the models suggestion
- (b) fuel prices have been increased at the models suggestion to bring about the decreased congestion;
- (c) the use of public transport has increased This is brought about by the reduction in vehicle usage that was caused by the increased fuel prices, and
- (d) the reduction in vehicle usage is a direct result of the increased price of fuel.

One of the major advantages of this type of model is that we can determine the side effects of policy proposals. Although in the example given the model is trivial, we are patently made aware of the side effects of increasing fuel prices. These side effects are a reduction in vehicle usage and an increase in the use of public transport. In more complex models there may be potential side effects that may not be as readily apparent *a priori* without a model of this type being employed, but which are of vital interest to the policy maker. Imagine introducing a policy that has a hidden side effect of increasing road deaths!

### Genetic Algorithms

Futures models are examples, in nontrivial cases, of highly complex systems that comprise many thousands of factors that are inter-related and which can affect each other and the final model results. The problem with structured searches of complex systems is that they ignore the realities that there are generally no simple algorithmic solutions to the problem. Traditional techniques rely heavily upon deterministic relationships between the input parameters and the final results. Where such a relationship does not exist genetic algorithms are one of the techniques available for obtaining a solution A broad understanding of genetic algorithms can be gleaned from Dawkins (1988), pp 43-74. Dawkins provides an entertaining and well informed introduction to genetic algorithms in two contexts:

- (a) the search for a given results (in his example the cumulative selection of a given phrase from Shakespeare's Hamlet), and
- (b) the evolution of a pseudo-species ('biomorphs') by genetic mutation.

One valid strategy for isolating a solution to a complex problem would be to test a number of randomly generated inputs to the system under consideration and concentrate further examination upon those inputs that produced the 'better' final solutions, as described in Dawkins (1988) This is the basic approach applied by genetic algorithms The major difference between this strategy and the traditional 'hunt-and-peck' process is that the inputs are coded as a binary string (chromosome). These binary strings are manipulated just as biological chromosomes are manipulated in nature, with a number of simplifying processes of reproduction being applied. The chromosomes provide us with information about the current state of the system and its elements. Logging descriptions of the chromosomes over a series of generations provides us with the history of the evolution of the system.

We now present an example of the process to illustrate its operations and capabilities. Assume that we wish to find the value of x, within the range (0,1), which will maximise some function f(x) of x The function is unknown to us, we could consider it to be hidden in a 'black box': when we supply a value of x, the box returns the value of the function f(x). The system will not reveal just what the function f(x) is, but it will provide us with a measure of the fitness of the current value. The *fitness value* is defined as a measure of the absolute difference between the target value (*say* d) and the most

recent value of the function, i.e. the fitness value is related to |f(x) - d| Our task is to find the maximum value of f(x) in the range. The following steps define the algorithmic procedure:

- (1) Generate, at random, a starting population of binary strings that can be decoded into values within the range of (0,1).
- (2) Test each of the decoded binary strings against the objective sought and supply a fitness value to each tested string.
- (3) Select two strings from the population, probabilistically, weighted by their fitness value (i.e. a string with a fitness value twice that of another has twice the probability of being selected as the other)
- (4) Select two points along one of the strings, at random
- (5) Swap the gene pattern between the two selected points between the two chromosomes, thus reproducing two new chromosomes.
- (6) Calculate the fitness of the newly created strings
- (7) Replace two chromosomes in the parent population with the two newly created chromosomes
- (8) If the fitness of any chromosome matches, or exceeds, the stopping value; stop else repeat from step 3.

The above process continues until such time as the fitness value of any particular chromosome reaches 1 0 or some preset stopping value. The above process is a necessary trivialisation of the real processes involved There are a large number of alternative mechanisms available to enhance and fine tune the performance of the genetic algorithm A worked example may help to show the basic approach:

(1) Generate the starting population (a string of ten bits is sufficient to define a value in the range 0-1)<sup>2</sup>. We will generate a starting population of only 10 chromosomes for this example; however, in reality a starting population much larger is generally required to ensure sufficient genetic diversity to guarantee that the result required is described somewhere within the total genetic pool available.

Chromosome	Chromosome	Chroosome	Chromosome
0100010101	0110010110	0110010011	1101111100
1011011011	1011000111	1011010101	0100001100
0000000111			1101111111

(2) Calculate the fitness of each chromosome. We decode each string into its numerical representation, feed it through the 'black box', and measure the result against our objective function to obtain the fitness.

Chromosome	Decoded Value	Fitness	Chromosome	Decoded Value	Fitness
1011011011 1101111100 0100001100 110111111	0 713867 0 871094 0 261719 0 874023 0 006836	0 588255 0.363934 0.994591 0.360107 0.696464	0110010110 1011000111 0100010101 01100100	0.396484 0.694336 0.270508 0.393555 0.708008	0.969690 0.617384 0.996659 0.971432 0.596996

<sup>2</sup>A string of nine bits can be decoded like so: 111111111 =  $2^{-10} + 2^{-9} + 2^{-8} + 2^{-7} + 2^{-6} + 2^{-5} + 2^{-4} + 2^{-3} + 2^{-2} + 2^{-1} = 0.9990234375$ (approximates to 1.0)

The bit pattern 0000100110 would decode to  $0^{-10} + 0^{-9} + 0^{-8} + 0^{-7} + 2^{-6} + 0^{-5} + 0^{-4} + 2^{-3} + 2^{-2} + 0^{-1} = 0$  390625 44

45

(3) Select two strings, probabilistically. In general steps 3-6 are repeated for the size of the population rather than just for two chromosomes, before we loop back to step 2.

Chromosome	Decoded Value	Fitness
0110010110	0 396484	0 96969
0000000111	0.006836	0.696464

(4) Select crossover points at random along the length of the chromosome

Chromosome
011 0010 110
000i0000l111

(5) Perform crossover.

(	Chromosome
(	011 0000 110
(	00010101111

(6) Insert these new children into the parent population by replacement of randomly selected chromosomes

Parent Pop	New Pop
Chromosome	Chromosome
0110010110	0110010110
1011000111	1011000111
0100010101	0100010101
0110010011	0110000110 New Insert '
1011010101	1011010101
1011011011	0000010111 New Insert
1101111100	1101111100
0100001100	0100001100
1101111111	1101111111
0000000111	000000111

(7) No chromosome has reached the stopping value of unity for its fitness, so continue to step 8

(8) End of first generation Loop back to step 3

The following graph (Figure 5) clearly, in this case, indicates the general form of the function under investigation We were lucky enough to stumble on two very good chromosomes in the seed population. These being the chromosomes 0100010101 and 0100001100 with fitnesses of 0.996659 and 0.994591 respectively. (There are actually 10 points on the graph although it appears that there are only nine. Two of the points, at this resolution, appears as one point on the graph.) this resolution, appear as one point on the graph )

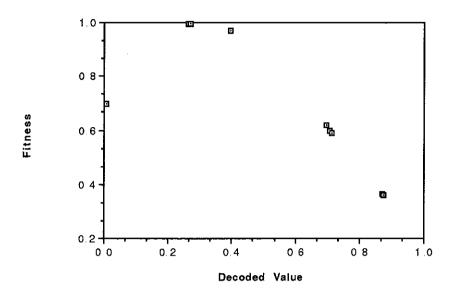


Figure 5. Plot of Fitness Values vs Decoded String for Seed Population

Continuing this process for another 10 generations yields the following results

Chromosome	Decoded Value	Fitness	Chromosome	Decoded Value	Fitness
0100011000 0100111111 0100110011	0.273438 0.311523 0.296875	0.997227 0.999220 0.999628	$\begin{array}{c} 0110010001\\ 0100011000\\ 0100110100 \end{array}$	0.391602 0.273438 0.300781	0.972566 0.997227 0.999661
010010011 0100000110 0100000110	0 294922 0 255859	0.999028 0 999572 0 992906	0100110100 0100110011 0100101111	0 299805 0 295898	0 999662 0 999663

It is obvious from the above population that the genetic algorithm has settled on the first four bits as being 0100 (only the first chromosome in the population has a 1 in the third bit position and this will, in all probability, be breed out in the next few generations). It may take a few more generations to settle on a chromosome pattern that is the closest that can be achieved (0100110011 chromosome 4 in the above list) which is a decoded value of 0 3, or at least is as close as this length chromosome can get to 0.3, which generates a maximum for the function  $f(x) = \cos^2(2x) + \sin(3x)$  within the range (0,1) for x.

The ability of a genetic algorithm to converge towards a desired solution is amply demonstrated by the above example. For a complex problem the chromosome length could be 100's of bits, could require a seed population of 100's and may require 100's of generations to converge; however, the algorithm has proved successful in converging for a thirty five (35) factor model of the Western Australian transport sector where nine (9) factors were manipulable. However, it was this very model which displayed the need for *Genie* to be programmed in a faster language, this run having taken in excess of 10 hours to complete on a Macintosh portable where the program was written in HyperTalk.

#### Where We Are Today

The original *TIP*+ program was coded in HyperTalk (the HyperCard scripting language) on a Macintosh computer; however, the computational requirements of the genetic algorithm have forced a rethink of the language to be employed if *Genie* is to be able to work in a think-tank environment, as was originally proposed HyperTalk is simply too slow a language *Genie* is now in the process of being developed in THINK Pascal and when completed should be a usable and useful product

At this stage there has been demand for and interest in *Genie* from a number of universities, consultants, municipalities and planning agencies around the world The product is currently offered free to any who are interested in being beta testers for the product. There appears to be enough demand to justify producing a version of *Genie* for IBM-compatible computers, probably employing Turbo Pascal for Windows If you are interested in *Genie* or *TIP*+ (Apple or PC) then contact Lance Chambers.

# Where We Need To Go From Here

We feel that there is great potential for models of this type to assist in the area of policy formulation. The genetic algorithm employed in the current manifestation of Genie is what is generically termed a Simple Genetic Algorithm (SGA). There is obvious potential to investigate more complex implementation of GA's to the problem of factor value searches within the problem space. There are questions as to whether the application of mutation, fitness scaling, sexual determination, genetic dominance or any number of other biological operators should be mimicked. Further, new methods of chromosome manipulation could be invented or imagined that could prove of value in the search procedure.

There is a need for more flexible mechanisms of model specification, data acquisition, inputting of specific factor forecasts over time that have been arrived at by some other mechanisms (eg population forecasts). Better reporting of results, the outputting of results so as to allow for simple pre-formated importation into other application (e.g. spreadsheets, statistical programs, word processors, etc), improved graphing capabilities; all of these need investigation and implementation where appropriate

The validity of the approach needs to be tested on a number of real-world problems and in a number of differing policy areas (the model paradigm does not limit its use to transport but is rather a generic tool to address a wide range of policy problems in any arena). This step is in all probability the most important. It would be a wasted exercise to continue with further development if there were no demand for the product or if it failed to adequately reflect reality. Yet the potential of this method should surely lead to a strong demand for its application on a wide scale?

#### Some Actual Results

A case study application of *Genie* for the transport system of metropolitan Perth was undertaken by the WA Department of Transport (WADoT). This study involved the creation of a 35 factor model with cross-impacts between factors. Assumed coefficient values were set by the analyst (see Newton and Taylor (1985) for information on the inherent processes in this task). The definition of the model, including a description of the factors, their interactions, and the interaction coefficients is given in Chambers and Taylor (1992). The model was run over a 20 year period, and the results of the model, including

a selection of intermediate outputs (i.e. status of the model factors at intermediate time intervals), are given in Appendix A. The genetic algorithm was then used to modify this scenario in the search for the most appropriate set of policies.

The following section outlines the genetic seed population parameters and factor phenotype limits You will note that limits were set on three of the factors Road Supply, Urban Sprawl and Taxes The implications of these limits are:

- Road Supply can be increased but not decreased
- Urban Sprawl can decrease slightly and can increase unfettered
- Taxes can be decreased, without limit, but can only be increased slightly

These limits were considered sensible and it is believed that the example demonstrates the flexibility of the Genie modelling system.

# **Population** Population

10		
- •		
50		
0.800	00	
8		
-		
0.00	то	1.00
-0.10	TO	1.00
-1 00	TO	0.20
-1.00	TO	1.00
	0 800 0.100 0.000 8 0 00 -0.10 -1 00	10 50 0.8000 0.1000 0.0000 8 0.00 TO -0.10 TO -1.00 TO

The following results represent the 'improved' results from the Genetic Algorithm run (those in borders are analysed in detail):

Gene ID	Fitness	Road Supply	Urban Sprawl	Decen Wrk Place	Taxes	Police Force	Attract of P/T	P/T Avail	P/T Fares	Flexible Wrk Hours
29	0.2081	0.1373	.0.1243	-0.4510	-0.6141	0.2392	-0.9843	-0.2549	0.1373	-0.7961
30	0.1479	0.1529	0.1329	-0.4510	0.1812	0.4588	-0.8588	-0.3176	0.5137	011701
39	0 2133	0 0745	0 1071	-0 9529	-0 0024	0.3020	-0.3098	0 9373	0 1451	-0.7882
40	0.1994	0.2471	0.1071	-0.7020	-0.6235	0.3098	-0.0431	0.6863	0.1451	-0.7882
45	0.2019	0.1294	0.1114	-0.0980	-0.5953	-0.2627	0.0039	-0.4275	0.1451	
46	0.1993	0 0118	0.1243	-0.2314	-0 5765	0 0 1 9 6	-0 9216	-0.7569	0.2000	
49	0.1865	0.1843	0.1157	-0.7020	-0.4729	0.2471	-0.1686	0.6784	+ 0	-0 7961
50	0.2025	0.0118	0.1243	-0.1843	0.6094	0.1137	-0.9843		0.1451	-0.7882
52		0.1686	0.1071	-0.8275	-0.0165			-0.7882	0.1059	
53	0.2006	0.2000				0.2863	-0.3098	0.5529	0.1529	0.2157
55	0.2000	0.2000	0.1071	-0 2000	-0 0024	0 3647	-0 4353	-0.5686	0.2078	-0.7569

From the above the following genes were selected for further analysis: Gene ID's 30, 45, 50 and 52. They were selected because each offers different mechanisms for achieving the given objectives of improving the economic climate and increasing the use of public transport It will be noticed, in the following graphs of results, that in a number of cases the objective of increasing public transport usage could not be achieved and in the case where it was achieved the recommendations could not, in reality, be implemented 48

for reasons that will be outlined

The differences in the recommendations may now be examined All the results suggested that Road Supply and Urban Sprawl should be increased, Decentralisation of Workplace should be constrained (i.e. centralised) and Public Transport fares should be increased. The following table indicates the main differences in the four selected recommendations.

Ta        Std Default      0 0        Gene ID 30      0 18        Gene ID 45      -0.59        Gene ID 50      -0 66        Gene ID 52      -0.01	812 0.4588 953 -0.2627 994 0.1137	Attract P/T 0.0 -0 8588 0 0039 -0 9843 -0 3098	P/I Avail 0.0 -0.3176 -0.4275 -0.7882 0.5529	Flex Wrk Hrs 0 01 -0.7882 -0.7725 0.0824 0.2157
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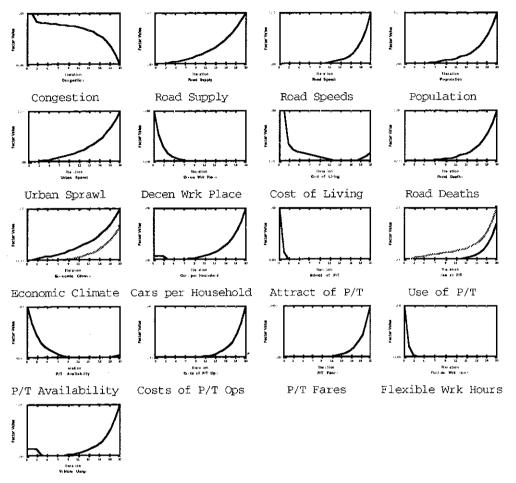
The table below shows the differences between the suggested possible answers.

Std Default Gene ID 30 Gene ID 45 Gene ID 50 Gene ID 52	Taxes 0.0 • (•) (•)	Size Police Force 0 0 • • •	Attract P/T 0.0 (••) (••) (•)	P/T Avail 0.0 (•) (•) (•)	Flex Wrk Hrs 0.01 (••) (••)
• Small posi (•) Small nega			Large posi Large nega	itive change ative change	

(blank) no change

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The following graphs represent the first run on the WADoI Model (described above) and the graphical outputs from running specific genes from the GA results.

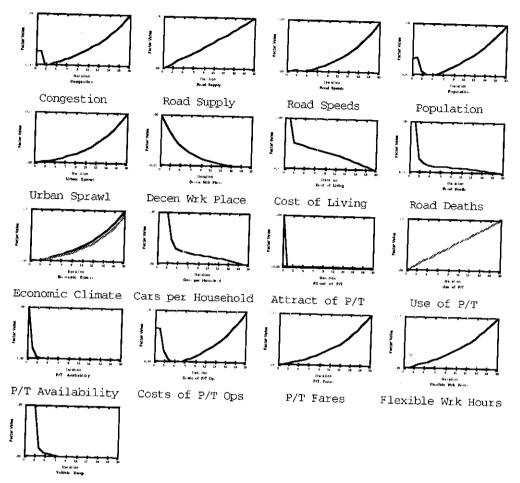


Vehicle Usage

# Figure 6: Solution offered by Gene ID 30

The solution offered by Gene ID 30 is shown in Figure 6. Although this is the 'best solution' the increase in road deaths makes it an untenable solution. Note that the grey lines on the graphs for 'Economic Climate' and 'Use of P/T' are the objectives for these factors.

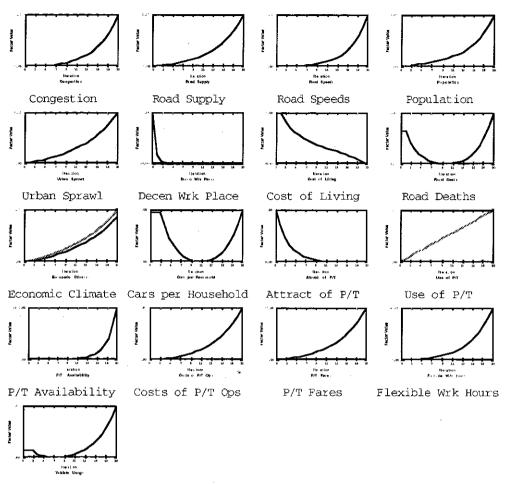
New Modelling Paradigm



Vehicle Usage

Figure 7: Solution offered by Genes ID 45 and ID 50

Genes ID 45 and ID 50 offer a common alternative solution (see Figure 7), but the need to reduce taxes by very significant levels makes this solution untenable.



Vehicle Usage

Figure 8: Solution offered by Gene ID 52

Gene ID 52 is in all probability the most useful and usable result (see Figure 8) The negative effects of the changes will not arise for some years giving the government time to develop new initiatives to overcome the forecasted increases in road deaths after year 11 or if not in power, at that time, to be able to regale the new government, for the increased road deaths, from the relative safety of the opposition benches

A small tax reduction is recommended, the police force should be increased (politically attractive), there should be a reduction in the attractiveness of public transport (e.g. less passenger comfort) but public transport availability and fares should be increased. Finally, there should be a push for more flexible working hours. All these factor changes can be seen as plausible and possible. This scenario avoids unpopular side effects whilst providing the desired outcomes

#### Conclusions

This paper introduced an alternative 'post-modernist' model of a complex, dynamic system suited to transport policy and decision making. The model structure is data efficent, and offers the analyst the means to explore the implications of a particular decision and to modify it to achieve desired goals. A number of illustrations of the method were given, culminating in a model of the transport system for metropolitan Perth that could be used to test a number of alternative policies. The modelling system identified as *Genie* offers considerable promise as a tool for rapid deployment in broad policy analysis and decision making, and its application by other researchers and analysts is recommended to aid in its further refinement.

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# Appendix A: Scenario Generation Output from Model WADoT

The following section lists the forecasted factor values from the simulation, using the model outlined in this paper and defined in Chambers and Taylor (1992)

Congestion Veh Purch Cost Enviro Aware Tourism Econ Climate P/T Fares	Fuel Costs Veh Run Cost Improved Techn Migration Cars per H'hold Flex Wrk Hrs	Fuel Avail Urban Sprawl o Alter've Fuels Depend on SS Attract of P/T Vehicle Usage	Road Supply Aging of Pop Cost of Living Road Deaths Use of P/T Bicycle Usage	Road Speeds Decen Wrk Place Family Size Size Pol Force P/T Availability Walking	Population Fitness Issues Taxes Education Cost of P/TOps
Period 1 1.00000000 1.00000000 1.00800000 1.02000000 1.00000000 1.00000000	$\begin{array}{c} 1.00000000\\ 1.00000000\\ 1.02000000\\ 1.00800000\\ 1.00000000\\ 1.00000000\\ 1.01000000\end{array}$	$\begin{array}{c} 1.00000000\\ 1.00300000\\ 1.00000000\\ 1.00000000\\ 1.00000000\\ 1.00000000\\ 1.00000000\\ \end{array}$	1.00000000 1.02000000 1.00000000 1.00000000 1.00000000 1.00000000	$\begin{array}{c} 1.00000000\\ 1.00500000\\ 0.99600000\\ 1.00000000\\ 1.00000000\\ 1.00000000\\ 1.00020000\\ 1.00020000\end{array}$	1.00100000 1.01000000 1.00000000 1.00000000
Period 2 1 00011160 1 00000014 1 01671560 1 04040000 1 00112000 1 00000000	$\begin{array}{c} 1.00000700\\ 1.00008960\\ 1.04034100\\ 1.01615400\\ 1.0000000\\ 1.02010000\\ \end{array}$	1.00000000 1.00610900 1.00000000 1.00000000 1.00000000 1.00000000	1.00000000 1.04040140 0.99960320 1.00011000 1.0000000 1.00077916	1.00000000 1.01002500 0.99206000 1.00006240 1.00000000 1.00039404	1 00203280 1 02001260 1 00000000 1 00114000 1 00000000
Period 5 1.00045184 0.99997134 1.04334156 1.10415093 1.00462235 0.99999988	$\begin{array}{c} 1.00001009\\ 1.00032408\\ 1 10386462\\ 1.04101974\\ 0.99998821\\ 1.05099990 \end{array}$	1.00000132 1.01549697 0.9999993 0.99998574 0.9999976 0.99998515	0.99999933 1.10408999 0.99835211 1.00020561 0.99999551 1.00191699	1.00000381 1.02526190 0.98030120 1.00025196 0.99999996 1.00097421	1 00514014 1 05064122 0 99996655 1 00479784 0 99999956
Period 10 1.00104008 0.99991940 1.08935982 1.21920057 1.01091068 0.99999966	$\begin{array}{c} 1 \ 00001232 \\ 1 \ 00072003 \\ 1 \ 21847274 \\ 1 \ 08381148 \\ 0 \ 99996565 \\ 1 \ 10459385 \end{array}$	1.00000377 1.03134232 0.99999993 0.99995985 0.99999982 0.99995264	0.999999817 1.21901815 0.99607500 1.00037533 0.99999687 1.00381557	1 00000773 1 05117073 0 96101005 1 00059795 0 99999996 1 00194192	1.01034851 1.10372148 0.99990379 1.01126461 0.99999834
Period 20 1 00230103 0 99979855 1 18793961 1 48650964 1 02538297 0 99999913	1.00000671 1.00152824 1.48461895 1.17470262 0 99991228 1 22012017	1.00000948 1.06379396 1.0000001 0.99990019 1.0000003 0.99986846	0.999999567 1 48600656 0 99071647 1 00076308 1 00000163 1.00762079	1 00001668 1 10497513 0 92354962 1 00139950 0 99999997 1 00387866	1 02088206 1 21795458 0 99975447 1 02573496 0 99999542