

AN EMPIRICAL STUDY OF THE DYNAMICS OF SYDNEY SUBURBAN  
RAIL PATRONAGE AND ITS IMPLICATIONS FOR  
BEFORE AND AFTER STUDIES

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ABSTRACT:

*This study investigates the way in which Sydney suburban rail patronage responds over time to changes in the determinants of patronage. Questions of the following form are addressed. If the real rail fare index changes in the current accounting period, how long does it take for patronage to respond? For how many periods following the initial response are subsequent responses observed? What is the long run effect? In addition to the real rail fare, such questions are addressed with respect to the real price of petrol, real household disposable income and 'on time running'.*

*One motivation for the dynamic analysis of patronage response arises from interest in the use of 'Before and After' studies to investigate market segment fare elasticities. This study permits an empirical investigation of the potential value of such studies for the Sydney suburban rail network. It is concluded that they are unlikely to yield fruitful results.*

ACKNOWLEDGEMENT: The author is indebted to Charles Nelson of ARRDO for his perceptive technical comments, to Geoff Carr of ARRDO for his editorial comments and to the State Rail Authority of N.S.W. for the provision of data.

## INTRODUCTION

This study investigates the way in which aggregate Sydney suburban rail patronage responds over time to changes in the determinants of patronage. In particular, questions of the following form are addressed. If the real rail fare index changes in the current accounting period how long does it take for patronage to respond? For how many periods following the initial response are subsequent responses observed? What is the cumulative effect of the real fare change? With regard to elasticity estimation, the distinguishing characteristic of this study from two earlier Sydney studies (Public Transport Commission of New South Wales 1978, State Rail Authority of New South Wales 1981), is an explicit focus on the dynamics of patronage response.

One motivation for a dynamic analysis of patronage response arises from interest in the use of 'Before and After' studies to investigate market segment fare elasticities. The 'Before and After' methodology must accommodate the dynamic nature of patronage response as well as seasonal, trend and economic effects coincident with the fare effect. This study provides information about the dynamics of patronage response and estimates the size of seasonal and other effects for Sydney suburban patronage. An operational 'Before and After' procedure is derived and an empirical analysis is conducted of its associated estimation error. This analysis enables conclusions to be drawn about the potential value of the 'Before and After' methodology.

## METHODOLOGY

### The General Estimating Equation

The approach adopted is that of an econometric time series analysis. The general estimating equation is presented in this section and is then discussed in considerable detail in the following sections. The most general equation estimated, relative to which every subsequent equation estimated is a 'special case', was of the following form:

$$\begin{aligned} \ln PAT_t = & a + \sum_{i=1}^2 b_i \ln PAT_{t-i} + \sum_{i=0}^{13} c_i \ln RRAILF_{t-i} + \sum_{i=0}^{13} d_i \ln RHDINC_{t-i} \\ & + \sum_{i=0}^{13} e_i \ln RGAS_{t-i} + \sum_{i=0}^{13} f_i \ln OTR_{t-i} + g \text{ STRIKES}_t \\ & + h \text{ ESD}_t + k \text{ TREND}_t + \sum_{i=0}^{12} p_i S_t^i \end{aligned} \quad (1)$$

where  $\ln$  denotes a natural logarithm

$t$  denotes the time period (accounting periods)

$PAT$  denotes suburban passenger journeys

$RRAILF$  denotes a real rail fare index

$RHDINC$  denotes a real deseasonalised household disposable income index

## DYNAMICS OF RAIL PATRONAGE

RGAS denotes the real price of petrol

OTR denotes the % of trains in the metropolitan network no more than five minutes late

STRIKES denotes the number of days duration of strikes affecting the suburban network

ESD denotes a binary dummy variable to capture the operation of the Eastern Suburbs railway. ESD = 0 for all periods prior to the opening of the Eastern Suburbs railway, ESD = 1 subsequently.

$S_t^i$  denotes a binary dummy variable to capture the seasonal effect in accounting period  $i$ . For example,  $S_t^1 = 1$  if period  $t$  coincides with accounting period one,  $S_t^1 = 0$  otherwise.

TREND denotes a linear time trend

### The Modelling of Patronage Reaction Dynamics

The general estimating equation (1) has been specified so as to have the capacity to capture a wide array of dynamic effects. In this section a brief discussion of the modelling of dynamic effects is provided with a view to explaining the capacity of equation (1) to capture dynamic behaviour.

Consider an equation depicting the relationship over time between two variables  $y$  and  $x$ :

$$y_t = v_0 x_t + v_1 x_{t-1} + \dots + v_m x_{t-m} \quad (2)$$

Using  $\Delta y_t$  to denote  $(y_t - y_{t-1})$  and  $\Delta x_t$  to denote  $(x_t - x_{t-1})$ , (2) implies that:

$$\Delta y_t = v_0 \Delta x_t + v_1 \Delta x_{t-1} + \dots + v_m \Delta x_{t-m} \quad (3)$$

Now assume that all of the  $\Delta x$ 's except  $\Delta x_t$  are zero, and  $\Delta x_t = 1$ . Then it may be seen that:

$$\Delta y_t = v_0 \quad \Delta y_{t+1} = v_1 \quad \dots \quad \Delta y_{t+m} = v_m \quad (4)$$

Thus as a consequence of a one unit change in  $x$  lasting only one period (i.e. an impulse) it is apparent that  $y$  responds in the fashion exhibited in (4). Comparing (4) with (2) it may be noted that these responses of  $y$  to an  $x$  impulse are provided by the coefficients on the equation relating  $y$  and  $x$ . If these  $v$ 's were known they could be graphed as shown in Fig. 1.

This graph, known as an impulse response function, represents the response in  $y$  over time to an impulse change in  $x$ . In this example there is little immediate response, a substantial response after three periods, smaller subsequent responses, and no response after ten periods. Notice that in this example a single monitoring

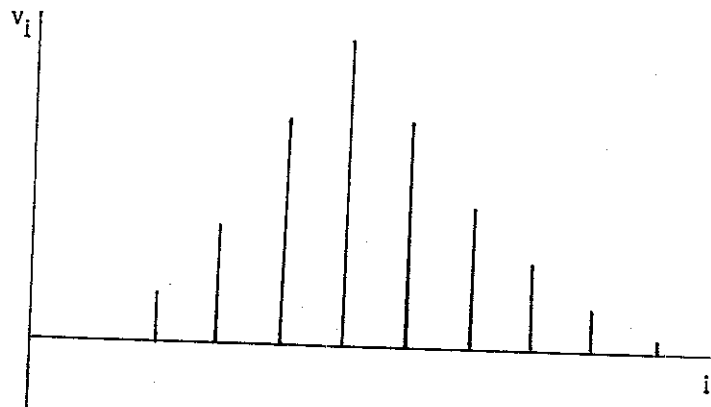


Fig. 1 An illustrative impulse response function

of the response after say two periods, would seriously understate the total response. To accurately measure the total response requires knowledge of this impulse response function. Once the y variable is thought of as patronage, and the x variable as the real rail fare, the importance of the impulse response function is readily apparent.

In principle, the impulse response function relating patronage to the real rail fare could have any one of several distinct shapes. Figure 2 illustrates three plausible cases.

Figures 2(a) and 2(b) require little explanation, however, the explanation of Fig. 2(c) is less obvious. In this case for the first few periods following a fare change, there is a negative patronage response. Thus, for example, following a fare increase, patronage falls as users switch to other modes. Five periods after the fare change however, positive responses emerge. In the case of a fare increase, this represents the switching back to rail of those users who switched in response to the original fare increase but have found the alternative mode unsatisfactory. It also represents 'reverse switching' as users of the non-rail mode switch to rail because of increased costs in the alternative mode arising from its increase in patronage. (Note that this is simply an example of a well known textbook case involving substitutes: if the price of butter increases, the price of margarine also increases and some margarine users will switch to butter.)

With respect to 'Before and After' studies, it is worth emphasizing once again in the context of Fig. 2(c), the importance of monitoring 'after' patronage at the correct time. Failure to wait long enough for the 'switch-back' or 'reverse switching' effect to arise, will lead to a serious overstatement of the negative response to a fare increase.

In Figs 2(a), 2(b) and 2(c), three quite distinct impulse response functions are illustrated. Any one of these may represent the actual dynamic relationship between rail patronage and real rail fares. Since it is desirable to allow the data to determine which of these (or of other equally plausible shapes) is appropriate, the general estimating equation (1) should be specified so as to allow any one of these shapes to be identified if it is consistent with the data. This has been a major consideration in the specification of the general estimating equation (1), as may be illustrated by the discussion of a stylized equation which has the same general form as (1).

# DYNAMICS OF RAIL PATRONAGE

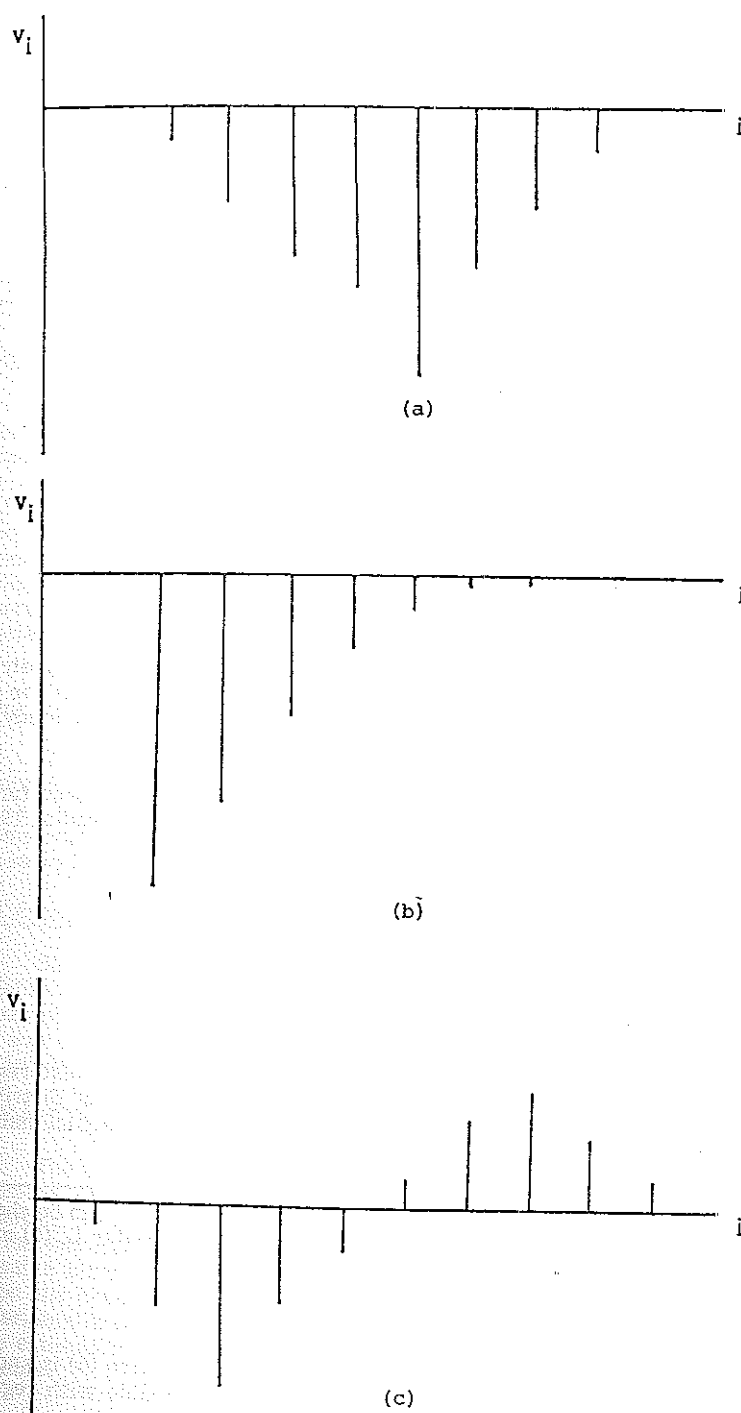


Fig. 2 Some possible rail fare-patronage impulse response functions

Consider the equation:

$$y_t = a + \sum_{i=1}^2 b_i y_{t-i} + \sum_{i=0}^{13} c_i x_{t-i} \quad (5)$$

Such an equation is extremely general in its implications for the form of the impulse response function.<sup>(1)</sup>

If  $b_i = 0$  for  $i = 2$

and  $c_i = 0$  for  $i = 1$  to 13

then (5) reduces to:

$$y_t = a + b_1 y_{t-1} + c_0 x_t$$

$$\text{or } y_t = a + c_0 \sum_{i=0}^{\infty} b_1^i x_{t-i}$$

This may be recognised as the widely used Koyck (or geometrically declining) distributed lag. The impulse response function in this case has the shape illustrated in Fig. 2(b) (assuming, as is reasonable in a patronage/fare study, that  $c_0 < 0$ ).

If the  $b_1$  and  $b_2$  coefficients are non-zero, the impulse response function can exhibit the oscillatory form illustrated in Fig. 2(c) (see Box and Jenkins 1976).

If  $b_i = 0$  for  $i = 1, 2$

(5) reduces to:

$$y_t = a + \sum_{i=0}^{13} c_i x_{t-i} \quad (7)$$

This form may either directly display the pattern of the impulse response in Fig. 2(a), or may be the basis of a subsequent constrained lag distribution estimation (such as an ALMON) which could generate a wide variety of patterns, including Fig. 2(a).

<sup>1</sup> The generality of this equation for the impulse response function can be shown in a relatively technical, but succinct fashion. Using the lag operator  $L$ , (5) may be rewritten as:

$$y_t = \frac{c(L)}{b(L)} x_t \quad (6)$$

where  $c(L)$  and  $b(L)$  are polynomials in the lag operator. Equation (6) is known as a 'rational' distributed lag model. Such a model is very flexible with regard to the shape of the impulse response function (see Harvey 1981).

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Not only can an equation with the structure of (5) capture a wide array of dynamic responses, it also embodies the static case of total 'within period' response. This would arise if all of the coefficients in (5) except 'a' and 'c<sub>0</sub>' were zero. In this case (5) reduces to:

$$y_t = a + c_0 x_t$$

From this discussion it is apparent that the general estimating equation (1) is capable of capturing a wide array of dynamic and 'within period' (or static) patronage response patterns to a fare change.

### Modelling Seasonal and Trend Effects

The coefficient on an additive seasonal dummy variable  $S_i^j$  in the estimating equation (when multiplied by 100) is approximately the percentage difference in patronage attributable to season  $i$  compared to the so called 'omitted' season. Note that since the analysis involves 13 accounting periods per year, equation (1) involves the specification of 12 seasonal dummies. Thus, for example, if no seasonal dummy has been defined for accounting period 13,  $S_1(x100)$  is the percentage difference in patronage in accounting period one compared to accounting period 13 arising from seasonal influences. By extension, the difference between two  $S_i$  coefficients (x100) is the percentage difference between the associated periods arising from seasonal differences.

The coefficient on the additive trend variable (when multiplied by 100) is approximately the trend percentage change in patronage per period.

The ESD dummy variable is designed to capture the patronage effect of the opening of the Eastern Suburbs railway. This variable is specified not because the aim is to estimate the magnitude of this effect but because failure to accommodate it would distort the estimation of the remaining parameters of (1). (The same is true of the STRIKES variable.)

### Data Approximation

Since neither a disposable income index nor a consumer price index is available for accounting periods, accounting period series were constructed from the published quarterly series using linear interpolation. The consumer price index used was the ABS Sydney 'all groups' index; the absence of a Sydney disposable income series prompted the use of a national disposable income series as a proxy.

### ESTIMATION RESULTS

The general estimating equation (1) was first estimated with a view to addressing the question: does the real rail fare have a detectable effect on accounting period patronage when allowance is made for dynamic effects? This question was also addressed with regard to the real price of petrol, real household disposable income and 'on time running'. Each question involves conducting an F test on the hypothesis that all of the coefficients in the distributed lag involving the variable in question are zero. The results of these tests are presented in Table 1; the estimation results for the lagged endogenous variables are also presented in this table.

Table 1 Hypothesis tests based on the general estimating equation

Variable	Coefficient	t statistic	p value of t statistic	F statistic	p value of F statistic
LPAT <sub>t-1</sub>	-0.0457	-0.3929	0.6962		
LPAT <sub>t-2</sub>	-0.0677	-0.6238	0.5358		
LRRAILF 0-13				2.1026	0.0297
LRGAS 0-13				3.9725	0.0002
LRHDINC 0-13				2.0271	0.0366
LOTR 0-13				2.6338	0.0068
LPAT 1-2				0.0578	0.9438

Estimation period: 1974.3 to 1983.5 (accounting periods)

Observations 120      Degrees of freedom 60

$R^2 = 0.8666$        $\bar{R}^2 = 0.7355$

Note: I-J denotes an unrestricted distributed lag from lag I to lag J.

The p value (or probability value) of the t statistic provides a convenient way of doing an instant hypothesis test. If the p value exceeds the level of significance of the hypothesis test then, for a two tailed test, the null hypothesis of a zero coefficient is accepted (for a one tailed test,  $p/2$  is compared to the level of significance). Similarly, if the p value of the F statistic exceeds the level of significance of the test, the null hypothesis is accepted.

Since the general estimating equation (1) involves lagged endogenous variables, the Durbin-Watson statistic is inappropriate for testing for the existence of an autocorrelated disturbance. Instead, Durbin's regression test is used (this test is described at length in Harvey 1981, p. 277). The test involves regressing the OLS residual from the estimation of (1)  $u_t$ , on a distributed lag  $u_{t-1}, \dots, u_{t-p}$  and all of the regressors from the original estimating equation (1). Either an MA(p) or an AR(p) disturbance structure can then be tested for by a joint F test on the coefficients of the lagged residuals  $u_{t-1}, \dots, u_{t-p}$ . Setting p at 13 the F statistic was 1.1269 which, with 13 and 20 degrees of freedom has a 'p' value of 0.3934. Hence the hypothesis of uncorrelated disturbances is accepted.<sup>2</sup>

From Table 1 it may be concluded that the real rail fare, the real price of petrol, real household disposable income and 'on time running', each has a statistically significant effect on accounting period rail patronage when dynamics are accommodated. From Table 1 it may also be concluded that the nature of the dynamic interaction does not require a model with an autoregressive component; both lagged endogenous variables are insignificant.

Having established that each variable of interest has some dynamic effect on patronage when it is accommodated in a very general model the duration of the effect of a change in each variable on patronage is next investigated. That is to say, the highest and lowest non-zero coefficient in the impulse response fraction for each variable are identified.

<sup>2</sup>The terms MA(p) and AR(p) are notation for stochastic time series models, see Harvey (1981). The usual first order autocorrelation of the disturbance for example would be referred to as an AR(1) structure. Here a test is being conducted for a much more general disturbance structure.



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The general estimating equation was re-estimated with the lagged endogenous variables omitted. The maximum lag for each variable was identified first. The constraint that the coefficient on lag 13 is zero was tested for a particular variable. If this hypothesis was accepted, the hypothesis that the coefficients on lags 12 and 13 are both zero was tested, and so on. This leads to a series of F tests. As soon as a constraint was rejected the lowest lag in the set constrained to zero was identified as the maximum lag for the variable under consideration. A sequence of such tests was conducted for each variable. Each sequence led to the identification of a maximum lag. The minimum significant lag for each variable was then identified in a similar fashion.

For a particular variable the constraint that the lag zero coefficient is zero was tested. If this was accepted the constraint that the coefficients of lag zero and lag 1 are both zero was tested, and so on. As soon as a constraint was rejected, the highest lag in the set constrained to zero was identified as the minimum lag. A sequence of such F tests was conducted for each variable leading to the identification of the minimum significant lag for each variable.

The results of the procedure for identifying minimum and maximum significant lags are presented in Table 2.

**Table 2 Maximum and minimum significant lags**

Variable	Minimum lag	F statistic for rejected constraint	p value of F statistic	Maximum lag	F statistic for rejected constraint	p value of F statistic
LRRAILF	4	1.8678	0.1167	4	1.8333	0.0786
LRGAS	2	6.0163	0.0014	12	3.8537	0.0277
LRHDINC	1	2.5879	0.0852	13	2.7133	0.0093
LOTR	0	7.3208	0.0092	11	3.0087	0.0387

Note: See text for an explanation of 'F statistic for rejected constraint'.

From Table 2 conclusions may be drawn about the timing of dynamic effects on patronage. The impact of a real rail fare change is detected four accounting periods after it has arisen. There is no further impact. The impact of a change in real petrol prices is detected two periods after the change. There is continued impact until 12 periods after the change. The impact of a change in real household disposable income is detected one period after the change and subsequent impact arises until 13 periods after the change.<sup>3</sup> Finally, the impact of a change in 'on time running' is felt immediately and subsequent impact arises until 11 periods after a change.

<sup>3</sup> Since the identified maximum lag coincides with the maximum lag specified it is possible that a higher maximum lag specification would lead to a higher identified maximum lag for RHDINC. Using the equation which generated Table 2 the maximum specified lag for RHDINC was extended from 13 to 26 and the hypothesis that the coefficients on lags 14 to 26 are all zero was tested. The value of the F statistic for this test was 0.5169 with a 'p' value of 0.8994. Hence, the hypothesis is accepted and the identified maximum lag of 13 is supported.

Having inferred something about the timing of effects on patronage, the magnitude of the effect was estimated. To this end the general estimating equation was constrained to reflect both the minimum and maximum lags of Table 2, and the absence of lagged endogenous variables suggested by the results of Table 1. The constrained equation was re-estimated; the results appear in Table 3.

Table 3 Estimation results from the constrained estimating equation

Variable	Coefficient	t statistic	p value of t statistic	F statistic	p value of F statistic
CONSTANT	8.0817	8.5459	0.0000		
LRRAILF	-0.2053	-3.0233	0.0035		
LRGAS $t^{-4}$ 2-12				4.2153	0.0000
LRHDINC 1-13				1.6583	0.0904
LOTR 0-11				3.1303	0.0013
STRIKES	-0.0160	-6.0616	0.0000		
ESD	0.0357	0.9165	0.3626		
TREND	-0.0022	-1.7473	0.0850		
Seasonals (omitted period: accounting period 9)					
AP1	-0.0751	-2.4950	0.0150		
AP2	-0.0236	-0.7906	0.4319		
AP3	0.0181	0.6177	0.5388		
AP4	-0.0553	-1.9687	0.0530		
AP5	-0.0517	-1.8748	0.0650		
AP6	-0.0226	-0.7887	0.4330		
AP7	-0.0885	-3.1020	0.0028		
AP8	-0.0567	-2.0706	0.0421		
AP10	0.0668	2.4021	0.0190		
AP11	0.0347	1.2283	0.2235		
AP12	0.0675	2.3994	0.0191		
AP13	-0.0076	-0.2691	0.7887		

Estimation period: 1974.3 to 1983.5 (accounting periods) where 1974 denotes 1974-5 financial year.

Observations 122

Degrees of freedom 69

$R^2 = 0.8833$

$\bar{R}^2 = 0.7954$

DW = 2.0387

$Q(33) = 41.9928$

'p' value of Q statistic = 0.1356

Note: 2-12, 1-13 and 0-11 each indicates an unconstrained distributed lag.  $Q(33)$  is the Box-Pierce statistic for 33 lags in the estimated disturbance correlogram; it is the appropriate test statistic for the hypothesis that the disturbance is not autocorrelated. The reported 'p' value implies acceptance of the hypothesis that the disturbance is not autocorrelated.

The real rail fare effect on accounting period patronage may be estimated directly from Table 3. The effect is felt four periods after the change, the entire effect arises within a single period and in this double log equation the patronage elasticity is estimated to be -0.2053. The boundaries of a 95% confidence interval are -0.0721 to -0.3384. The significance of reporting these boundaries arises from the proposition that any hypothesised value of the population elasticity lying between these boundaries would be accepted if tested using this data and a 5% two-tailed hypothesis test.

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Since the real petrol price, real household disposable income and 'on time running' each impact on patronage over several periods, the most detailed quantification of their effects involves estimating each coefficient in their respective impulse response functions. Multicollinearity however renders such estimation very imprecise, the standard errors of the estimated response coefficients are large. Instead of reporting the entire estimated impulse response functions here (they are reported in the Appendix), for each variable the cumulative effect on accounting period patronage of a 1% change sustained for one year is reported. This cumulative effect is estimated by summing the estimated coefficients in each impulse response function (for the constrained general estimating equation, this amounts to summing the estimated coefficients in the respective distributed lags). For the double log estimating equation used in this study this cumulative effect constitutes a one year sustained impulse elasticity. For each variable the hypothesis that the one year sustained impulse elasticity is zero was tested; the estimation and test results are reported in Table 4.

**Table 4 Estimated 'one year' sustained impulse elasticities from the constrained estimating equation**

Variable	Estimated elasticity	t statistic	p value of t statistic
RGAS	0.7902	4.1056	0.0001
RHDINC	-0.5953	-1.0056	0.3181
OTR	0.7680	3.4791	0.0009
RRAILF	-0.2053	-3.0233	0.0035
<u>95% confidence intervals for the estimated elasticity</u>			
	Lower limit	Upper limit	
RGAS	0.4131	1.1675	
RHDINC	-1.7556	0.5650	
OTR	0.3353	1.2007	
RRAILF	-0.3384	-0.0721	

Note: The RRAILF result is identical to that appearing in Table 3 because the associated distributed lag only involves one term. It is presented here for purposes of comparison.

From Table 4 it may be seen that each of the one year elasticities is significant except that of real household disposable income. It is notable however, that this point estimate of the long run accounting period income elasticity (-0.5953) derived here from an accounting period study, is consistent with the point estimate for annual patronage (-0.747) reported in the 1981 annual period study (State Rail Authority of New South Wales 1981).<sup>(4)</sup>

<sup>4</sup> Consider for example a 3% per annum percentage change in real household disposable income. Then an annual patronage income elasticity of -0.747 implies a change in annual patronage of -2.24%. Now a 3% per annum percentage change is equivalent to 0.23% per accounting period. Given an accounting period patronage income elasticity of -0.5953, this implies a change in accounting period patronage of -0.1369% per accounting period, or -1.77% per annum. This is notably similar to the -2.24% derived from the annual patronage elasticity estimate.

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To provide perspective on the elasticity results reported in Table 4, descriptive statistics for the data used in this study are reported in Table 5.

Table 5 Descriptive statistics for data in the form of percentage change per accounting period

Variable	Minimum	Maximum	Mean	Standard deviation
%ΔRGAS	-11.5294	13.8483	0.4224	3.3642
%ΔRHDINC	-1.3872	1.5317	0.1906	0.5437
%ΔOTR	-33.5561	32.9980	0.3152	6.9197
%ΔRRAILF	-20.3983	20.5844	0.0122	4.2367

Note: The period summarised is 1974-5 accounting period one to 1983-4 period 5.

The estimated trend effect from Table 3 is reported in Table 6. (It may be noted that the estimated 2.8% per annum decline is very similar to the 2.1% per annum decline reported in an earlier study; Hensher and Bullock (1979).)

Table 6 Estimated trend effect

Estimated trend per accounting period	t statistic	p value of t statistic
-0.22%	-1.7473	0.0850
Estimated trend per annum: -2.8%		
<u>95% confidence limits</u>		
Upper limit: -0.02% per accounting period (-0.26% p.a.)		
Lower limit: -0.46% per accounting period (-5.8% p.a.)		

The statistically significant estimated seasonal effects reported in Table 3 are presented in Fig. 3.

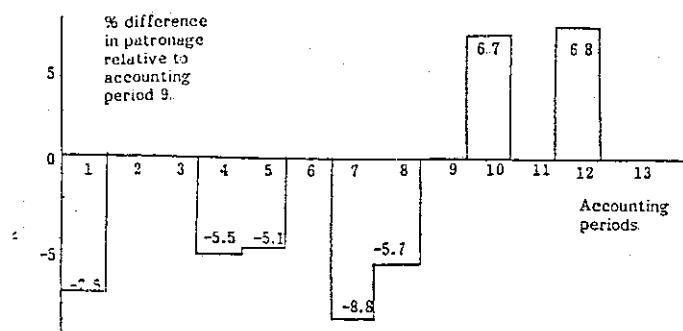


Fig. 3 Statistically significant seasonal effects

## DYNAMICS OF RAIL PATRONAGE

### THE IMPLICATION OF THE RESULTS FOR BEFORE AND AFTER STUDIES

#### A Description of the 'Before and After' Procedures Implied by the Results of this Study

From Fig. 3 it is evident that there is no differential seasonal effect between accounting periods 2 and 3. Hence, it is convenient for illustrative purposes to use these periods for a description of the 'Before and After' procedure.

On the basis of the preceding econometric results the population patronage regression equation is of the form:

$$\begin{aligned} \ln PAT_t = & a + c_4 \ln RRAILF_{t-4} + \sum_{i=1}^{13} d_i \ln RHDINC_{t-i} \\ & + \sum_{i=2}^{12} e_i \ln RGAS_{t-i} + \sum_{i=0}^{11} f_i \ln OTR \\ & + g \text{ STRIKES} + h \text{ ESD}_t \\ & + k \text{ TREND}_t + \sum_{i=0}^{12} p_i S_t^i \\ & + \epsilon_t \end{aligned} \quad (9)$$

where all variables are defined exactly as previously,  $\epsilon_t$  is a white noise disturbance and period  $t$  is assumed to coincide with accounting period 3.

Lagging equation (9) one period, subtracting the result from equation (9), multiplying by 100, and solving for the real rail fare elasticity  $c_4$ , provides:

$$\begin{aligned} c_4 = & \frac{\% \Delta PAT_t}{\% \Delta RRAILF_{t-4}} - \sum_{i=1}^{13} d_i \frac{\% \Delta RHDINC_{t-i}}{\% \Delta RRAILF_{t-4}} \\ & - \sum_{i=2}^{12} e_i \frac{\% \Delta RGAS_{t-i}}{\% \Delta RRAILF_{t-4}} - \sum_{i=0}^{11} f_i \frac{\% \Delta OTR_{t-i}}{\% \Delta RRAILF_{t-4}} \\ & - k \frac{100}{\% \Delta RRAILF_{t-4}} - (\epsilon_t - \epsilon_{t-1}) \frac{100}{\% \Delta RRAILF_{t-4}} \end{aligned} \quad (10)$$

where it is assumed that periods  $t$  and  $t-1$  have been chosen to avoid the occurrence of strikes.

Equation (10) is the basic 'Before and After' estimating equation. Setting the  $(\epsilon_t - \epsilon_{t-1})$  term to zero and replacing all the unknown coefficients with their estimated values from the constrained equation, provides an operational mechanism for estimating  $c_4$  (note that all of the  $\% \Delta$ 's on the right hand side of (10) are known when the estimation process is conducted in period  $(t+1)$ ).

### An Analysis of the 'Before and After Estimation Error

Since time series data exist for the Sydney suburban network there is obviously no need for the application of 'Before and After' methods to that context. Instead, the methodology is considered as potentially useful for the analysis of market segments for which patronage time series do not exist. For such analysis however, seasonal effects, trend effects and dynamic economic effects are all required for the market segment of interest. Since these are not known, one approach is to assume that in these regards the market segment behaves exactly as the aggregate market; in this case the estimated form of equation (10) is equally applicable to the market segment.

To the extent that market segments do not 'behave exactly as the aggregate market' with regard to seasonal effects etc., the use of the estimated form of equation (10) will promote error. Without analysis of the market segment (analysis which of course is currently infeasible because of data constraints) it is difficult to assess the extent of this error. Thus, in what follows, this source of error is ignored and the focus is on the error which arises because of the existence of the disturbance term in (10), and the error which arises because of the need to estimate the coefficients in (10).

Subtracting the estimated form of (10) with the disturbance set to zero from (10) provides the estimation error  $E_t$ :

$$\begin{aligned}
 E_t = (c_4 - \hat{c}_4)_t &= \sum_{i=1}^{13} (\hat{d}_i - d_i) \frac{\% \Delta RHDINC_{t-i}}{\% \Delta RRAILF_{t-4}} \\
 &+ \sum_{i=2}^{12} (\hat{e}_i - e_i) \frac{\% \Delta RGAS_{t-i}}{\% \Delta RRAILF_{t-4}} \\
 &+ \sum_{i=0}^{11} (\hat{f}_i - f_i) \frac{\% \Delta OTR_{t-i}}{\% \Delta RRAILF_{t-4}} \\
 &+ (\hat{k} - k) \frac{100}{\% \Delta RRAILF_{t-4}} \\
 &- (\epsilon_t - \epsilon_{t-1}) \frac{100}{\% \Delta RRAILF_{t-4}}
 \end{aligned} \tag{11}$$

Where a caret (^) denotes an estimated coefficient.

It is convenient to write (11) in a more concise form using vectors. Define a parameter estimation error vector:

$$P'_t = \left[ (\hat{d}_1 - d_1) (\hat{d}_2 - d_2) \dots (\hat{k} - k) \right]$$

and a data vector

$$D'_t = \left[ \% \Delta RHDINC_{t-1} \% \Delta RHDINC_{t-2} \dots 100 \right]$$

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Then (11) may be written as:

$$E_t = \frac{1}{\% \Delta RRAILF_{t-4}} D_t' P (\epsilon_t - \epsilon_{t-1}) \frac{100}{\% \Delta RRAILF_{t-4}} \quad (12)$$

From (12) assuming (as is reasonable) that  $\epsilon_t$  and  $\epsilon_{t-1}$  are uncorrelated with the elements of  $P$  the standard deviation of  $E_t$  may be derived:

$$\sigma(E_t) = \left[ \frac{1}{\% \Delta RRAILF_{t-4}} \left[ D_t' V[P] D_t + 100^2 (2) \sigma^2(\epsilon_t) \right] \right]^{\frac{1}{2}} \quad (13)$$

where  $\sigma^2(\ )$  denotes a variance and  $V[\ ]$  denotes a variance-covariance matrix.

Before proceeding it is worth emphasising that (13) implies that the precision of the estimation is directly related to the magnitude of the percentage change in the real rail fare; the greater the change the greater the statistical precision.

Now  $\sigma(E_t)$  is a population parameter and is thus unknown; it is estimated by:

$$s(E_t) = \left[ \frac{1}{\% \Delta RRAILF_{t-4}} \left[ D_t' \hat{V}[P] D_t + 100^2 (2) s^2(\epsilon_t) \right] \right]^{\frac{1}{2}}$$

where  $s^2(\ )$  denotes an estimated variance and  $\hat{V}$  denotes an estimated variance-covariance matrix. Using this expression the precision of a single 'Before and After' estimate may be assessed by evaluating  $s(E_t)$ .

The term  $s(\epsilon_t)$  is the standard error of estimate from the constrained regression and is thus readily available (0.0245). The vector  $D_t'$  is a data vector which depends on the period; here the data based on the period 1983.3 is used (the most recent data involving accounting period three). It is not hard to see that the elements of  $\hat{V}[P]$  are identical to estimated coefficient variances and covariances from the constrained regression thus they are also readily available. Finally, a  $\% \Delta RRAILF_{t-4}$  of 10% was selected. Based on these it is calculated:

$$s(E_{1983.3}) = 0.4730$$

The significance of this result for the potential value of 'Before and After' studies can be assessed by reference to the likely value of the parameter being estimated. Based on one very extensive review of the literature (Transport and Road Research Laboratory 1980) it may be concluded that estimated urban public transport fare elasticities have rarely exceeded 0.7 in absolute value; for particular market segments such as peak travel/work trips they have rarely exceeded 0.4 in absolute value. Relative to such magnitudes the standard error of estimate of

approximately 0.47 arrived at here, is substantial. Appealing to large sample normality for example, permits the estimation that there is approximately a 30% probability of making an error of in excess of 0.47 in magnitude; in comparison to the likely magnitude of the elasticity this would constitute a substantial error.

To summarize the position, assuming a  $\% \Delta RRAILF$  of 10% and using the pertinent data associated with the period 1983.3, the standard deviation of a 'Before and After' elasticity estimation error is estimated to be approximately 0.4730. In comparison to the probable value of the true elasticity, this standard deviation is substantial. This would be even more obviously the case had a real rail fare change of less than 10% been considered.

These results suggest that a single application of the 'Before and After' methodology is unlikely to yield a tolerably precise elasticity estimate. This conclusion is reinforced when it is recalled that in this analysis an additional source of error has been ignored; the error which arises in assuming that an aggregate market patronage equation is transferrable to a market segment.

#### An Analysis of the Estimation Error from an Averaged 'Before and After' Estimation

In this section an expression is derived for the standard deviation of the estimation error from an averaged 'Before and After' estimation. The derivation is based on a specific set of assumptions emphasised below. The expression is used to assess the precision of a 'Before and After' estimation based on averaging the estimates from several studies.

The estimation error from a single study conducted at time  $t$  is provided by equation (12):

$$E_t = \frac{1}{\% \Delta RRAILF_{t-4}} D_t' P - (\epsilon_t - \epsilon_{t-1}) \frac{100}{\% \Delta RRAILF_{t-4}} \quad (12) \text{ rptd.}$$

Assume that the vector  $D_t$  is the same for all  $t$  (i.e. the percentage change in all variables is constant through time; a growing steady state assumption). Also assume that there is at least one accounting period between each 'Before and After' study. Finally, assume that the same estimated trend effects, seasonal effects etc., are used in each 'Before and After' study (i.e. the estimates are not updated prior to each study).

Given these assumptions the covariance between the estimation error of a study conducted in period  $t$ , and one conducted in period  $s$ , may be derived from (12) as:

$$\sigma(E_t, E_s) = \frac{1}{\% \Delta RRAILF^2} D_t' V[P] D_s \quad (14)$$



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The variance of the estimation error from a single study may also be derived from (12) as:

$$\sigma^2(E_t) = \frac{1}{\% \Delta RRAILF^2} \left[ \underline{D}' V[\underline{P}] \underline{D} + 100^2(2) \sigma^2(\epsilon) \right] \quad (15)$$

Note that the assumptions ensure that the variance and covariances are constant, hence no time subscripts appear on the right hand side of equations (14) and (15).

The variance of an average estimation error may in general be written as:

$$\sigma^2 \left[ \frac{\sum E_t}{n} \right] = \frac{\sum \sigma^2[E_t]}{n^2} + \frac{1}{n^2} \sum_{\substack{t, s \\ t \neq s}} \sigma(E_t, E_s)$$

where n denotes the number of estimates averaged.

Given the constancy of both the variance and the covariances this expression can be rewritten as:

$$\sigma^2 \left[ \frac{\sum E_t}{n} \right] = \frac{\sigma^2[E]}{n} + \frac{n(n-1)}{n^2} \sigma(E_t, E_s)$$

Substituting for  $\sigma(E_t, E_s)$  then provides:

$$\sigma^2 \left[ \frac{\sum E_t}{n} \right] = \frac{\sigma^2[E]}{n} + \frac{(n-1)}{n} \frac{1}{\% \Delta RRAILF^2} \underline{D}' V[\underline{P}] \underline{D} \quad (16)$$

It may be noted that since  $V[\underline{P}]$  is a variance-covariance matrix,  $\underline{D}' V[\underline{P}] \underline{D}$  is a positive definite quadratic form so that  $V(\sum E_t/n)$  converges to a positive constant. Thus, given these assumptions, the averaged estimation is not statistically consistent.

From (16) an expression for the estimated standard deviation of the average error may be derived:

$$s(\sum E_t/n) = \left[ \frac{s^2(E)}{n} + \frac{(n-1)}{n} \frac{1}{\% \Delta RRAILF^2} \underline{D}' V[\underline{P}] \underline{D} \right]^{1/2}$$

Of the terms in this expression,  $\underline{D}' V[\underline{P}] \underline{D}$  was evaluated (using the data for period 1983.3) in order to evaluate the magnitude of  $s(E)$  reported in the previous section. Hence, it is possible to evaluate  $s(\sum E_t/n)$  for different values of  $\% \Delta RRAILF$  and of n. Selected evaluations are reported in Table 7.

Table 7 Estimated standard deviations of the fare elasticity estimation error from averaging the estimates provided by multiple 'Before and After' studies

	Number of studies							
	2	4	6	8	10	20	40	60
1%	2.3	2.79	2.94	3.01	3.06	3.14	3.18	3.19
% change in the real rail fare in each study								
10%	0.4046	0.3657	0.3517	0.3445	0.3402	0.3312	0.3267	0.3251

The first row of Table 7 suggests that a large number of 'Before and After' studies, based upon small percentage changes in the real rail fare, do not enhance estimation precision. In fact, because of the positive correlation between errors being amplified by the smallness of the percentage change in the real rail fare (see equation (14)), precision deteriorates.

The second row of Table 7 indicates that for substantial changes in the real rail fare, estimation precision can be enhanced by averaging the results of multiple studies. The extent of the enhancement however, is not substantial; even after 60 studies the estimation precision remains intolerably low. It should also be emphasised that a large number of substantial fare changes over a period sufficiently short to be useful, is operationally infeasible.

On the basis of this analysis it is concluded that the averaging of the results of multiple 'Before and After' studies is unlikely to yield a tolerably precise fare elasticity estimate. Given the results in the previous section for a single 'Before and After' study, it is concluded that the use of 'Before and After' studies is unlikely to prove fruitful for fare elasticity estimation.

### CONCLUSION

In this study, evidence has been provided that real rail fares, real household disposable income, real petrol prices and 'on time running' each has a detectable effect on Sydney metropolitan accounting period rail patronage. The timing of the effects differs appreciably across the variables. The real rail fare effect is first felt four periods after the change arises and the entire effect is confined to a single period. For both the real price of petrol and real household disposable income there is a similar delayed response, however, the eventual response is sustained over several subsequent periods once it has arisen. For 'on time running' the effect on patronage is immediate and is sustained for several subsequent periods. The details of the timing and magnitude of these effects are summarized in Table 8, and Table 9 provides the patronage response for various scenarios involving changes in the determinants of patronage. This study also provides estimates of seasonal and trend effects. Seasonal effects are presented in Table 3 and Fig. 3, the trend effect is summarised in Table 6. The estimated trend effect of a 2.8% decline per annum is notably similar to the 2.1% per annum decline reported elsewhere (Hensher and Bullock (1979)).

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With regard to the potential value of market segment 'Before and After' fare elasticity studies, the empirical results of this study suggest that they are unlikely to be fruitful. In addition to the inevitable error which arises in transferring trend, seasonal and dynamic economic effect estimates from the aggregate market to the market segment, this study suggests that there is a substantial probability of significant error in a 'Before and After' estimate arising from parameter estimation error and patronage noise. The results of this study also suggest that this error cannot be significantly reduced by averaging the estimates from multiple 'Before and After' studies.

**Table 8 Suburban patronage response to changes in some of the determinants of patronage**

Variable	No. of accounting periods before patronage response detected	No. of accounting periods after which no further patronage response detected	Estimated % change in accounting period patronage for a sustained 1% increase in variable (i.e. the 'long run' elasticity)
Real rail fare index	4	4	-0.2053
Real price of petrol	2	12	0.7902
Real household disposable income index	1	13	-0.5953
% of trains no more than 5 mins late	0	11	0.7680

Note: Patronage is measured in passenger journeys.

**Table 9 Patronage responses for selected scenarios**

Scenario	Patronage response
1. Trains run on time (i.e. 'on time running increases from the 88% average for 1983-84 to 100%')	10.5% increase
2. On time running performance improves from 1983-84 average of 88% to 93%	4.4% increase
3. A permanent single 10% increase in the real rail fare index	2.0% reduction
4. Real household income increases at a rate of 2.5% <u>per annum</u>	1.5% <u>decline per annum</u>
5. A permanent 5% increase in real petrol prices	3.9% increase
6. A permanent 5% decrease in real petrol prices	3.9% decrease

Note: All scenarios are on a 'ceteris paribus' basis. On time running refers to the percentage of trains in the metropolitan area no more than five minutes late.

This study concludes with a comment on the omission of some determinants of patronage. The focus here has been on patronage determinants which lend themselves to the econometric time series approach; an approach which was adopted because of its suitability for the investigation of dynamic relationships. This has meant that because of data constraints, or because of their unsuitability for this approach, some likely determinants of demand have been neglected: advertising, the incidence of vandalism, the cleanliness of trains and stations, timetable changes, changes in rollingstock, central business district employment, demographic changes etc. Such variables may serve to explain the trend in patronage identified in this study. Furthermore, whilst there is no reason to believe that such omitted variables are strongly correlated with the explanatory variables in this study and hence undermine its statistical validity, the known omission of variables must always serve to qualify the results of any econometric study.

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## APPENDIX: IMPULSE AND STEP RESPONSE FUNCTIONS

Table 1 Estimated on time running impulse response function

	Coefficient	t statistic
	0	0.2685
	1	-0.0775
	2	0.1128
	3	-0.1045
	4	-0.0219
Lag:	5	0.0719
	6	-0.0934
	7	0.2268
	8	-0.0596
	9	0.0544
	10	0.1272
	11	0.2623

Note: The reported t statistic is the appropriate statistic to test the hypothesis that the coefficient is zero.

Table 2 Estimated on time running step response functions

	t statistic
0 to 0	0.2685
0 to 1	0.1909
0 to 2	0.3038
0 to 3	0.1935
Sum of	0 to 4
estimated	0 to 5
coefficients:	0 to 6
	0 to 7
	0 to 8
	0 to 9
	0 to 10
	0 to 11

Note: The reported t statistic is the appropriate statistic to test the hypothesis that the sum of the coefficients is zero.

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Table 3 Estimated real price of petrol  
impulse response function

		Coefficient	t statistic
Lag:	2	0.7137	4.0453
	3	-0.2057	-0.9416
	4	0.0051	0.0231
	5	0.1434	0.6496
	6	-0.3495	-1.6037
	7	0.2966	1.3628
	8	-0.1981	-0.9161
	9	0.1480	0.6822
	10	-0.6202	-2.8835
	11	0.5882	2.7732
	12	0.2688	1.7080

Note: The reported t statistic is the appropriate statistic to test the hypothesis that the coefficient is zero.

Table 4 Estimated real price of petrol  
step response function

			t statistic
Sum of estimated coefficients:	2 to 2	0.7137	4.0453
	2 to 3	0.5080	2.7916
	2 to 4	0.5131	2.8552
	2 to 5	0.6565	3.4175
	2 to 6	0.3069	1.6037
	2 to 7	0.6036	3.1655
	2 to 8	0.4054	2.0509
	2 to 9	0.5534	2.6759
	2 to 10	-0.0668	-0.3069
	2 to 11	0.5214	2.1854
	2 to 12	0.7902	4.1055

Note: The reported t statistic is the appropriate statistic to test the hypothesis that the sum of the coefficients is zero.

With the exception of one term in each function, the terms in the real household disposable income impulse response function and the step response function are statistically insignificant. Hence, they are not reported.

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